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Sir Isaac Newton, 1642-1727

STELLAR MOTIONS

WITH SPECIAL REFERENCE TO MOTIONS DETERMINED BY MEANS OF THE SPECTROGRAPH

BY

WILLIAM WALLACE CAMPBELL, Sc. D., LL. D.
Director of the Lick Observatory, University of California

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THE SILLIMAN FOUNDATION

In the year 1883 a legacy of eighty thousand dollars was left to the President and Fellows of Yale College in the city of New Haven, to be held in trust, as a gift from her children, in memory of their beloved and honored mother, Mrs. Hepsa Ely Silliman.

On this foundation Yale College was requested and directed to establish an annual course of lectures designed to illustrate the presence and providence, the wisdom and goodness of God, as manifested in the natural and moral world. These were to be designated as the Mrs. Hepsa Ely Silliman Memorial Lectures. It was the belief of the testator that any orderly presentation of the facts of nature or history contributed to the end of this foundation more effectively than any attempt to emphasize the elements of doctrine or of creed; and he therefore provided that lectures on dogmatic or polemical theology should be excluded from the scope of this foundation, and that the subjects should be selected rather from the domains of natural science and history, giving special prominence to astronomy, chemistry, geology, and anatomy.

It was further directed that each annual course should be made the basis of a volume to form part of a series constituting a memorial to Mrs. Silliman. The memorial fund came into the possession of the Corporation of Yale University in the year 1901; and the present volume constitutes the seventh of the series of memorial lectures.

PREFACE

The contents of this book formed the Silliman Lectures in Yale University for the academic year 1909-1910. They were delivered in the period January 24 to February 4, 1910. Numerous modifications of an entirely minor character have been made in the manuscript, in order to bring out points, by means of text and printed illustrations, which were presented in the lectures with the help of lantern slides. All significant changes or additions made subsequent to the delivery of the lectures are duly indicated in the text.

The following paragraph, which concluded the series of eight lectures, has been brought forward to the preface.

"The lecturer is well aware that the great subject of stellar motions, limited as far as practicable to stellar motions spectrographically determined, has been presented incompletely and imperfectly, but he hopes that he has been able to furnish a glimpse into a surprisingly rich field of astronomical investigation. When we recall that stellar radial velocities afford perhaps our best method of determining the scale on which the solar system is constructed (the solar parallax), and when combined with proper-motion data the best method of determining the scale on which the stellar system exists, certainly the most fruitful method of studying the evolution of double star systems, and a most promising method of studying the evolution of stars in general (illustrated by the relation existing between spectral classes and average radial velocities), we are quite prepared to acknowledge their almost unlimited power. It is a safe prophecy that the possession and study of the radial velocities of the brighter stars will but strengthen future demands for a knowledge of the motions of fainter and fainter stars. The spectrographic methods of observation referred to have been developed almost to the point of standardization, though the future is expected to introduce many and important improvements; and there is abundant justification for entering at once upon the observation of extensive programs embracing all stars down to visual magnitudes approximating 6½. The task is, however, too great for any one institution, and too great in each hemisphere for any one institution. It is hoped that a considerable number of observatories equipped with powerful telescopes may soon agree upon coöperative plans for securing the desired observations, following somewhat the ideas of the great organization (*Die astronomische Gesellschaft*) which is rapidly extending over the whole sky the accurate meridian determinations of stellar positions down to the ninth visual magnitude."

For a large share of the observational materials which have been utilized in these lectures I am under obligations to the late Mr. D. O. Mills, to the Carnegie Institution of Washington, and to essentially all of my colleagues on Mount Hamilton and on Cerro San Cristóbal.

W. W. CAMBPELL.

June 1, 1912.

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CHAPTER I

HISTORICAL AND INTRODUCTORY

When we look at the sky on a clear night we see two distinct classes of objects. The early astronomers called one class the planets, or wanderers; and the other class, the fixed stars. The term, "fixed star," is a misnomer and is becoming obsolete. We are learning to speak only of "the planets" and "the stars." There are no stars whose positions are fixed: all are in motion, with reference to any point, line or plane we may define in general terms. The planets of our solar system, in their orbits around the Sun, are travelling from 5 km. per second for *Neptune* up to 55 km. per second in the case of *Mercury;* yet the refined observations and calculations of the past two decades have established that the average star is moving even more rapidly than the average planet. Our Sun, as one of the ordinary stars, is no exception to the rule, in that it has been found to be travelling rapidly through space, carrying its family of planets along with it.[1]

[1] The astronomers of today are so familiar with the ideas of stellar and solar motions that it is difficult to realize how recently these ideas were developed. Giordano Bruno, martyred by the Inquisition in the year 1600 on account of his original views concerning scientific subjects, was perhaps the first to question the immovability of the stars. He said that we had no right to assume the fixity of the stars with reference to each other because, on account of their enormous distances from us, we could not hope to detect changes of position until after the lapse of long ages.

Robert Hooke (1635-1703) was of the opinion that the stars do not occupy fixed positions, but that all the stars, including our Sun, may well be in relative motion.—*Posthumous Works,* p. 506.

An important paper by James Bradley, presented to the Royal Society in 1748, remarked: "When the causes which affect the places of all the stars in general are known; such as the precession, aberration, and nutation; it

The problems of motions within the solar system, and the problems of motions in the stellar system, possess widely variant orders of difficulty. This condition arises from our relative nearness to the Sun and to the other planets, and from the inconceivably greater distances of the stars. The crude instruments and methods of three hundred years ago, in the hands of Tycho Brahe and Kepler, were immeasurably better able to solve the problems of our own system than are the delicate instruments and the refined methods of today to solve the problems of the distant stars. In fact, our ability to make serious headway in the study of the stellar system is solely because the number of stars is so great that we can apply statistical methods and the doctrine of averages to them. The energies of astronomers prior to 1750 were all but exclusively devoted to the investigation of the solar system, in large part because they felt powerless to attack the problems of the distant stars. From that date on to the present, as telescopes became more perfect and more power-

may be of singular use to examine nicely the relative situations of particular stars; and especially of those of the greatest lustre, which it may be presumed lie nearest to us, and may therefore be subject to more sensible changes; either from their own motion, or from that of our system.''—*Phil. Trans.* (Abridged Ed.), **9**, 437-438, 1748.

It was a favorite subject with Thomas Wright of Durham that our Sun and its planets, in common with all the stars, are in motion.—*An Original Theory or New Hypothesis of the Universe,* London, 1750, p. 52.

We are perhaps justified in saying that the first comprehensive discussion of the subject in the literature of astronomy is due to Lalande, in the year 1776. From mathematical considerations he concluded, among other things, that our Sun, having a motion of rotation about its axis, could scarcely avoid having a motion of translation. His thoughtful statement follows:

''Il me reste à dire un mot sur un effet de la rotation solaire, dont les Physiciens n'ont point encore parlé, mais qui sera peut-être un jour un phénomène bien remarquable dans la Cosmologie; c'est le mouvement de translation du Soleil & de tout notre système planétaire.

''Le mouvement de rotation, considéré comme l'effet physique d'une cause quelconque, est produit par une impulsion communiquée hors du centre. Jean Bernoulli calcule pour chaque Planète le point où cette force doit avoir été appliquée, à proportion de la vîtesse de sa rotation (*Opera,* **4**, p. 283); mais une force quelconque imprimée à un corps, & capable de le faire tourner ful, and as the nature of the stellar problems was better comprehended, the energies of a constantly increasing proportion of investigators have been turned to the stars.

Our knowledge of the internal motions of the solar system is marvellously accurate. It has become a matter of almost routine computation to predict the positions of the planets, the times and places of solar eclipses, and other phenomena, a century in advance. Halley's comet, whose last appearance occurred in 1835, was rediscovered a few weeks ago within seven minutes of arc of its predicted place; and it will reach its closest approach to the Sun on April 19, within three days of the predicted time. It is true that there are minute discrepancies in the motions of *Mercury, Venus,* the Earth, and *Mars,* but it is possible that Seeliger's researches[2] on the attracting power of the finely

autour de son centre, ne peut manquer aussi de déplacer le centre, & l'on ne sauroit concevoir l'un sans l'autre. Il paroît donc très-vraisemblable que le Soleil a un mouvement réel dans l'espace absolu; mais comme il entraîne nécessairement la Terre, de même que toutes les Planèts & les Comètes qui tournent autour de lui, nous ne pouvons nous apercevoir de ce mouvement, à moins que par la suite des siècles le Soleil ne soit arrivé sensiblement plus près des Étoiles qui sont vers une région du Ciel, que de celles qui sont opposées; alors les distances apparentes des Étoiles entr'elles auront augmente d'un côté & diminué de l'autre; ce qui nous apprendra de quel côté se fait le mouvement de translation du système solaire; mais il n'y a pour ainsi-dire que quelques instans d'écoulés depuis que l'on observe; & la distance des Étoiles est immense; il est donc assez naturel qu'on n'ait fait jusqu'ici aucune remarque à ce sujet. . . . .

"Si les positions des Étoiles, observés par Hipparque il y a près de deux mille ans, avoient plus de précision, on pourroit commencer à voir si les différences de longitudes sont plus grandes d'un côté & plus petites de l'autre que celles qui avoient lieu de son temps; mais un jour viendra où cette comparaison pourra nous apprendre quelque chose sur la question dont il s'agit."—*Mémoires de l'Académie Royale des Sciences,* Paris, 1776, p. 513.

Herschel strongly suspected, as early as 1783, to quote his own words, "that there is not, in strictness of speaking, one fixed star in the heavens; . . . . there can hardly remain a doubt of the general motion of all the starry systems, and consequently of the solar one amongst the rest."—*Phil. Trans.* (Abridged Ed.), 15, 397, 1783.

[2] *Sitzungsber. der k. Bayer, Akad. Wiss.,* **36** (III), 595, 1906. A clear estimate of the discrepancies, and a most interesting discussion of

divided zodiacal-light material will explain and eventually remove these discrepancies. The only other noteworthy discrepancy in the whole solar system relates to our Moon, for which there exists a considerable difference between prediction and observation. It is under the auspices of Yale University that a distinguished astronomer is placing our knowledge of the Moon's motions on an improved basis. The story of Newton's law of gravitation, as applied to the Sun, to the planets and their moons, and to the comets and meteors, if written by a worthy pen, would constitute the world's real epic: an epic of intellectual struggles, rather than of physical prowess. We scarcely exaggerate in saying that successive generations of astronomers, spanning two centuries, fully expected that Newton's law would ultimately be found to account for the general motions of all celestial bodies. However, the recent discoveries of forces possessing quite different natures have weakened this point of view. For example, radiation pressure, discovered theoretically by Maxwell,[3] and established experimentally by Lebedew,[4] and by Nichols and Hull,[5] may be far-reaching in its consequences upon cosmical motions. It is too early to forecast the propelling effects of magnetic forces, whose existence in the Earth and Sun points strongly to their existence also in other celestial bodies. And we must not assume that other propelling forces, of natures entirely unknown, will not manifest themselves to future investigators. However, we cannot doubt that all the stars, all the nebulæ, all the dark and invisible bodies which must exist in profusion throughout space—in brief, all tangible bodies making

possible explanations, are contained in Newcomb's *Astronomical Constants*, 1895, pp. 109, 123. [Note added March, 1912.—Brown makes interesting comments on Seeliger's results, in *Mon. Not. R. A. S.*, **70**, 342-344, 1910; and de Sitter likewise, in *Mon. Not. R. A. S.*, **71**, 408-409, 1911. Wacker (*Inaug. Dissertation*, Tübingen, 1909), Lorentz (*Phys. Zeitschrift*, **11**, 1239, 1910) and de Sitter (*Mon. Not. R. A. S.*, **71**, 405, 1911) show that the Principle of Relativity may be an important factor in explaining the progression of planetary perihelia.]

3 *A Treatise on Electricity and Magnetism*, Oxford, **2**, 391, 1873.

4 *Ann. der Physik*, **6**, 433, 1901.

5 *Ap. J.*, **17**, 315, 1903.

up our sidereal universe—are moving in accordance with definite laws. Will astronomers ever be able to tell their fellow men how each bright star in turn is moving, and how groups of stars, great groups as well as small ones, are related to each other? Will the starry heavens be reduced to a system, as the Sun, planets, satellites and comets have been fitted into the solar system? The methods of today, truly remarkable in their accuracy, are contributing to this purpose and ambition; but time alone can tell the outcome.

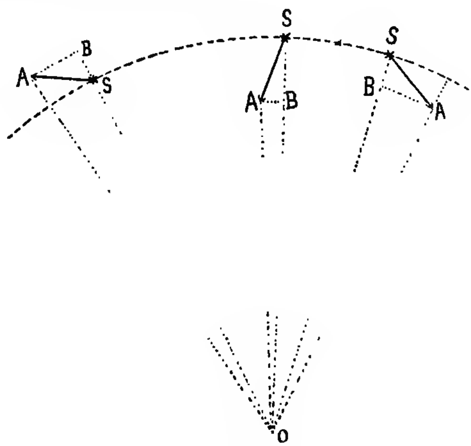


Figure 1

The motion of a star resolves itself naturally into two components: one measured along the line (called the "line of sight") drawn from the star to the observer, and the other at right angles to this line. The former component is known as the star's *radial motion,* or motion in *the line of sight;* and the latter as its *proper motion.* Thus, if S, S, S (Figure 1), represent three stars on the celestial sphere, each moving in a direction and with a speed indicated by its vector SA: then, the observer being in the direction SO, the component BA represents the *linear* value of the proper motion in each case and SB the radial motion or radial velocity. We cannot describe proper motion in linear values except in the cases of the few stars whose distances are known, and it is customary to define proper motions in terms of the angles SOA through which the stars appear to move in the unit of time—a year or a century. It is customary and we have the power to define radial motions in absolute units, as, for example, kilometers per second. In the figure one of the stars has proper motion to the right, and the others to the left. Two of the stars have radial motions of approach toward the observer, and one has radial motion of recession from him.

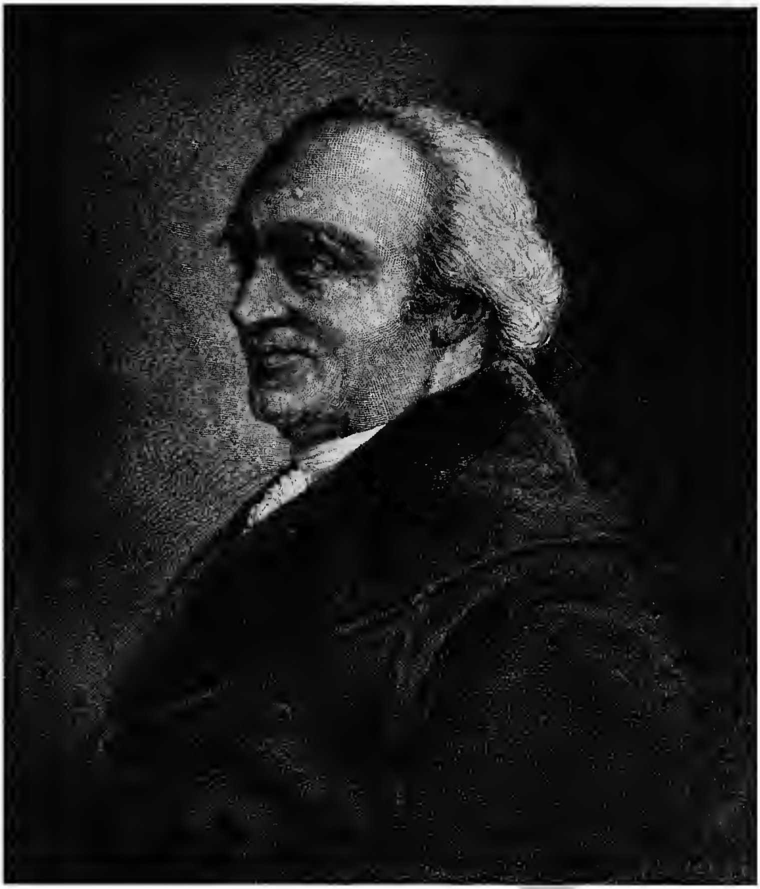
As all stellar bodies are in motion, they are changing both their apparent positions on the celestial sphere and their distances from us. That is, they have both proper motion and radial motion.

Edmund Halley was the first to recognize that certain stars had actually moved. He announced in 1718 that *Sirius, Aldebaran, Betelgeux,* and *Arcturus* were certainly occupying positions appreciably different from those assigned in Ptolemy's *Almagest,* as based upon the observations of Timocharis and Aristyllus about 300 years before Christ, and by Hipparchus about 130 years before Christ. Commenting upon these changes, he said: "These stars being the most conspicuous in Heaven are in all probability the nearest to the earth; and if they have any particular motion of their own, it is most likely to be perceived in them."[6]

When their instruments had reached a considerable degree of perfection, in the latter half of the eighteenth century, astronomers began to determine the proper motions of the brighter stars, in this manner: Their positions on the celestial sphere—their right ascensions and declinations—were measured with great precision; at a later date,—twenty, forty, sixty years later,—the positions of the same stars were remeasured in the same manner; a comparison of the positions at the two epochs showed that some of the stars had moved appreciably in the interval, and at what angular rates; these rates being their so-called proper motions.

As early as 1783, Maskelyne had determined the proper motions of seven of the first-magnitude stars in the northern

[6] *Phil. Trans.* (Abridged Ed.), **6**, 329-330, 1718.



Sir Friedrich Wilhelm Herschel, 1738-1822 sky, and Mayer of twelve of the brighter stars. Six of these were common to both lists, and the motions of thirteen stars were thus known to a fair degree of accuracy. To illustrate at once the great value of proper-motion knowledge, let us recall that in 1783 Sir William Herschel used these thirteen proper motions to support the theory that our Sun and its system of worlds must be travelling rapidly through space in the general direction of the constellation *Hercules*.[7] As the accuracy of instruments and methods of observation improved, and especially as the interval between the old observations and the new grew longer, the power to measure these minute stellar motions increased. The famous observations of star positions made by Bradley at Greenwich between 1750 and 1762 are noteworthy for their value as starting points in determining stellar motions. Today we know the proper motions of several thousand of the brighter stars to a fair degree of accuracy. These have furnished the basis for investigations of great importance as to the structure of the stellar universe. Nevertheless, proper-motion data, standing alone, have serious limitations: they are expressed in angular measurement; and to know that a star is changing its apparent position on the celestial sphere by one-tenth of a second of arc per year is to know next to nothing about the actual motion of that individual star. We need to know three other factors:

First, the component of the star's motion toward or away from us: this component may be negligible, in some cases, or it may be a dozen-fold greater than the proper-motion component, in other cases. Second, the star's distance from us: a given angular motion, that is, proper motion, may mean the rapid motion of a very distant star, or the very slow motion of a close star. Third, the extent to which the star's apparent motion is affected by the observer's motion. To illustrate the last point, let us suppose that the motion of the Earth is carrying the observer due east with a speed of 19 km. per second. If a certain star is in reality also moving due east 19 km. per second, it will seem to us not to move at all; and another star in

[7] See CHAPTER IV.

reality moving due west 19 km. per second will seem to be moving with twice its true speed. In the same manner the observed motion of every star in the sky is not its true motion, for it is affected by the observer's motion. A moment's consideration will make plain that a full knowledge of the direction and speed of our own star's motion is a *sine qua non* to a satisfactory study of the motions of the other stars.

Twenty-two years ago we did not know the radial motion of any star in the heavens. Observed radial velocities for several scores of stars were published, it is true, but they could not be depended upon to be even approximations to the truth. We felt that we knew within 20° the direction of the solar motion, but we knew nothing as to the velocity with which it carried the observer on through space. This was variously estimated at from 10 to 70 km. per second. We knew the distances of not more than two score stars: measures of stellar distances presented difficulties so great that even today we possess reliable knowledge on the approximate distances of not over a hundred stars. At no point in astronomical science is fuller knowledge more desirable, more pressingly urgent, than in the subject of stellar distances; or, speaking technically, of stellar parallaxes.

Sixty-two years ago the human mind had no conception that we should ever be able to measure the radial motions of the stars. Yet in the past twenty-two years the problem has been given practical solution; the radial velocities of more than a thousand stars have been determined. These velocities will enable us, both alone and in combination with proper motions and parallaxes, to solve many of the fundamental problems of stellar astronomy; not suddenly, but by rapid approximations to the truth. The unexpected by-products of the observations are scarcely less important than the foreseen results. In brief, the field of discovery here opened up has proved to be of superlative richness. It is chiefly to this field that these lectures refer.

In his celebrated work, *Cours de philosophie positive* (Paris, 6 vols., 1830-1842), Auguste Comte declared that "We shall never be able to study the chemical composition of the celestial bodies; . . . . Our positive knowledge with regard to them will necessarily be limited to their geometrical and mechanical phenomena. It will be impossible, by any means, to include investigations of their physical, chemical (and other) properties." Fraunhofer had discovered,[8] a quarter of a century earlier, that the visual solar spectrum is not merely a band of variously colored light, passing by insensible gradations from violet at one end to deep red at the other, but that this band is crossed by a multitude of dark lines (at least 600); and that the spectra of the brightest stars are crossed by many similar lines. Laboratory investigations on a few of the chemical elements had shown that their spectra consist character-

[8] Wollaston had indeed observed seven lines in the solar spectrum, in 1802, but they were given a most peculiar interpretation, and the subject was not further pursued. Wollaston described the observations as follows:

"I cannot conclude these observations on dispersion, without remarking that the colours into which a beam of white light is separable by refraction, appear to me to be neither seven, as they usually are seen in the rainbow, nor reducible by any means (that I can find) to three, as some persons have conceived; but that, by employing a very narrow pencil of light, four primary divisions of the prismatic spectrum may be seen, with a degree of distinctness that, I believe, has not been described nor observed before.

"If a beam of daylight be admitted into a dark room by a crevice 1/20 of an inch broad, and received by the eye at the distance of 10 or 12 feet, through a prism of flint glass, *free from veins,* held near the eye, the beam is seen to be separated into the four following colours only, red, yellowish green, blue, and violet; . . . .

"The line A that bounds the red side of the spectrum is somewhat confused, which seems in part owing to want of power in the eye to converge red light. The line B, between red and green, in a certain position of the prism, is perfectly distinct; so also are D and E, the two limits of violet. But C, the limit of green and blue, is not so clearly marked as the rest; and there are also, on each side of this limit, other distinct dark lines, *f* and *g*, either of which, in an imperfect experiment, might be mistaken for the boundary of these colours."—*Phil. Trans.,* 1802, p. 378.

Fraunhofer's remarkable observations (in 1814 and later) were made possible by the use of a narrow slit to admit the light into his spectroscope, as well as of a telescope to collect the light of a star and form its image on the slit.—*Denkschriften der k. Akad. München,* **5**, 1817.

istically of bright lines, certain of which occupy the same positions in these spectra as are occupied by prominent dark lines in the solar and stellar spectra. The significance of these spectral features, since shown to be fundamental and almost unique in their far-reaching power for analysis, were then

GUSTAV ROBERT KIRCHHOFF, 1824-1887

unknown, as may be inferred from Comte's famous dictum. Twenty-four years after the publication of Comte's conclusion, the fundamental principles of spectroscopy were discovered by Kirchhoff,[9] whereupon it became a comparatively simple matter

[9] *Monatsberichte k. Akad. Berlin*, 1859, p. 664. Lack of space prevents

to determine the chemical compositions of the stars, within limits set by their brightness and their physical conditions. But we pass by this great field of astronomy and physics, as not directly concerning our subject. It was soon recognized that the spectroscope supplies, in theory at least, the long-hoped-for method of measuring the components of stellar motions in the line of sight—their radial velocities.

The effect of the approach or recession of a light source, such as a star, upon the spectrum was first considered by Christian Doppler, of the University of Prague, whose conclusions were announced[10] in 1842. He explained the now familiar fact that

a full and painstaking description of the fundamental principles of spectroscopy, but we may state them briefly, as follows:

1. When a solid body, a liquid, or a highly condensed gas is heated to incandescence, its light when passed through a spectroscope forms a *continuous spectrum:* that is, a band of light, red at one end and violet at the other, uninterrupted by either dark or bright lines.

2. The light from the incandescent gas or vapor of a chemical element passed through a spectroscope forms a *bright-line spectrum:* that is, one consisting entirely of isolated bright lines, distributed differently throughout the spectrum for the different elements, or of bright lines superimposed upon a relatively faint continuous spectrum.

3. If radiations from a continuous-spectrum source pass through cooler gases or vapors before entering the spectroscope, a *dark-line spectrum* results: that is, the positions which the bright lines in the spectra of the vapors and gases would have are occupied by *dark* or *absorption* lines. These are frequently spoken of as *Fraunhofer* lines.

To illustrate: The gases and vapors forming the outer strata of the Sun's atmosphere would in themselves produce *bright-line spectra* of the elements involved. If these gases and vapors could in effect be removed, without changing underlying conditions, the remaining condensed body of the Sun should have a *continuous spectrum.* The overlying gases and vapors absorb those radiations which the gases and vapors would themselves emit, and thus form the *dark-line spectrum* of the Sun. The stretches of spectrum between the dark lines are of course continuous-spectrum radiations.

There is an endless variety of radiation and absorption spectra of the elements (see Kayser's *Handbuch der Spectroscopie,* Bände 2 u. 3). There is also a great variety of stellar spectra; for example, there are many stars whose strong continuous spectra are crossed by both bright and dark lines and bands of various widths. (See *Ap. J.*, **2**, 177, 1895.)

10 *Abhandlungen d. k. Böhmischen Gesell. d. Wiss.*, **2**, 467, 1841-1842.

when a source of sound waves, such as the whistle of a locomotive, is moving rapidly away from or toward the listener, the pitch of the sound perceived is not the normal pitch: it is lowered in the case of recession and raised in the case of approach. The sound waves are, in effect, lengthened and the pitch lowered for a receding locomotive; the waves are, in effect, shortened and the pitch raised for an approaching locomotive. Reasoning analogously upon light as a phenomenon of waves in the ether, he correctly concluded that these waves, as perceived by an observer, would in effect be shortened if the light source is approaching him and lengthened if receding from him. It is practically immaterial with velocities thus far observed whether the light source is approaching the observer, or the observer approaching the light source—the observed wave lengths would be shortened in either case; and similarly for the recession of the light source from the observer or of the observer from the light source. As the red rays of the spectrum are the visible result of long waves, speaking popularly, and the violet rays at the other end of the spectrum the result of short waves, Doppler concluded, erroneously, that a star moving very rapidly toward us would be changed in color to a more violet tinge, and one moving rapidly away to a redder tinge. He overlooked the fact that there are stretches of invisible spectrum to the red and to the violet of the visible spectrum, which would be drawn upon to compensate for any loss from the cause described, and thus leave the color sensibly unchanged. Practically, the stellar motions of approach and recession are so small in comparison with the velocity of light that no change of color would be perceptible to the eye, even aside from the compensating principle.

It appears to have been Fizeau, in 1848, who first enunciated the principle, correctly, that motions of approach and recession must cause corresponding shiftings of the entire spectrum, including the dark lines of Fraunhofer, toward the violet and toward the red, respectively, but without change of color. He outlined methods for applying the principle to measuring the motions of celestial bodies toward and away from the observer.

While these methods were sound theoretically, they were unpractical. All matters spectroscopic were still mysterious, and Fizeau's statements attracted no serious attention. In fact, his lecture on the subject, in 1848, before a minor society, in Paris, was not published[11] until 1870. Following the impetus

CHRISTIAN DOPPLER, 1803-1853

given to spectroscopic investigations by Kirchhoff in 1859, Dr. (now Sir William) Huggins and Professor Miller, jointly engaged in observing the spectra of stars in 1862-1863, realized[12]

[11] *Ann. de Chimie et de Physique,* **19**, 217-220, 1870.

[12] *Phil. Trans.,* **158**, 529, 1868.

that stellar motions to and from the observer should displace the lines in the spectra, and unsuccessful efforts to measure the displacements were made in 1866 by Dr. Huggins; but Clerk Maxwell was the first to present the subject in definite form.[13] None of these eminent investigators realized the tremendous importance which the Doppler-Fizeau principle was later to attain in practical astronomy, if we may judge from the tardiness of publication characteristic of all.

It may not be definitely known what causes the phenomenon called light; but, speaking popularly, and according to the mechanical theory, we may say that waves of energy, of an infinite variety of lengths, travel outward in every direction from the light source. In the ordinary image of a star, whether formed by the eye alone, or by an achromatic telescope and the eye combined, the light waves of all lengths fall in a confused heap upon the same minute point, and the observer is unable to say that rays corresponding to any given wave lengths are present or absent. When the star's light has been passed through the prisms or diffracted from the grating of a spectroscope, these rays are separated, one from another, and arranged, side by side, in perfect order, ready for the observer to survey them, and to determine which ones are present in superabundance, and which ones are lacking wholly or in part. The following comparison is a fair one: The ordinary *point image* of a star is as if all the books in the University library were thrown together in a disorderly but compact pile in the centre of the reading room: we could say little concerning the contents and characteristics of that library. The *spectrum* of a star is as the same library when the books are arranged on the shelves in complete perfection and simplicity, so that he who looks may appraise its contents at any or all points. The retina of the human eye is affected only by those waves whose lengths lie between approximately 0.00078 and 0.00038 mm. Waves of the former length, at the extreme red end of the visible spectrum, are 1300 to the mm.; and of the latter length, at the extreme violet end of the visible spectrum, are 2700 to the mm. The wave

[13] *Phil. Trans.*, **158**, 532, 1868.

length varies continuously from one end of the spectrum to the other, and every shade of pure color—more conveniently, every point in the spectrum—has its own definite wave length. Thus, a point can be selected in the yellow whose wave length is $\frac{1}{2000}$ of a mm.; a point in the orange whose wave length is $\frac{1}{1700}$ of a mm.; and so on for every wave length between the limits noted for the retina. Extending apparently indefinitely into the red of the spectrum of the Sun are rays which, as carriers of heat, have been investigated by Langley and Abbot as far as to wave length 0.00534 mm.; and extending into the violet, apparently indefinitely, is the ultra-violet region, invisible, but investigated photographically by Lyman for the element hydrogen as far as to wave length 0.000103 mm.

It has been found convenient to define a given point in the spectrum in terms of the number of ten-millionths of a millimeter which measure the wave length of the radiations reaching that point. The unit, formerly called the *tenth-meter,* is now known as the *Ångström.* Thus the hydrogen line in the green-blue, whose position corresponds to wave length .00048615 mm., is said to have a wave length of 4861.5 Ångströms (formerly 4861.5 tenth-meters), or 4861.5 A, or, simply, the line is said to be at 4861.5 A; and similarly for all other lines throughout the spectrum.

For a monochromatic ray emitted by a light source whose distance from the observer is not changing, let λ denote the number of Ångströms in the wave length, and n the number of waves received by the observer in a mean solar second. Let λ' and n' be the changed values of λ and n resulting from a velocity of the light source with reference to the observer, or of the observer with reference to the light source, amounting to $\pm V$ km. per second; $+V$ for motions which further separate the light source and observer, and $-V$ for motions which bring them nearer together. Theoretically, it makes an extremely slight difference whether the light source or the observer is moving, but practically the difference is easily negligible for all known cosmical motions, as only the higher orders of minuteness are involved. Recalling that the velocity of light through interstellar space is very nearly 299,860 km. per second, we shall have:

$$n' = n \frac{299{,}860}{299{,}860 \pm V}; \qquad (1)$$

the sign of the V term being + for recessions and — for approaches. Since the effective wave length is inversely proportional to the number of waves received per second, or in any given unit of time, we may write:

$$\lambda' = \lambda \frac{299{,}860 \pm V}{299{,}860} = \lambda \left(1 \pm \frac{V}{299{,}860}\right), \qquad (2)$$

or

$$\lambda' - \lambda = \Delta\lambda = \pm \frac{\lambda V}{299{,}860}. \qquad (3)$$

In other words, the change, $\Delta\lambda$ (in Ångström units), corresponding to a relative velocity of recession or approach of $\pm V$ km. per second, is equal to the normal value of the wave length, multiplied by the ratio of the relative velocity of the light source and observer to the velocity of light.

To find the velocity V_1 required to alter the apparent wave length by 1 A, it is but necessary to let $\Delta\lambda = 1$, and we have:

$$V_1 = \pm \frac{299{,}860}{\lambda}. \qquad (4)$$

Thus at 3000 A, a velocity of ± 100 km. per second will change the apparent wave length to 3001 A, and 2999 A, respectively; at 7500 A, a velocity of only ± 40 km. is required to change the value of λ to 7501 A and 7499 A, respectively. In practice V_1 is tabulated for uniformly distributed points throughout the spectrum. Transforming equation (3) into

$$V = \pm \frac{299{,}860}{\lambda} \Delta\lambda, \qquad (5)$$

expressing $\Delta\lambda$ in terms of 1 A, and replacing the fraction by its value from (4), we obtain:

$$V = \pm V_1 . \Delta\lambda; \qquad (6)$$

that is, a velocity, V, of approach or recession, will change any λ by amount $\Delta\lambda$, expressed as Ångströms, to the extent V/V_1. Conversely, if it be found from observation that a wave length has been changed by the amount $\Delta\lambda$, we may compute from (6) the radial velocity which produced this change. The problem of determining the radial velocities of the stars consists, in outline, in measuring the displacement $\Delta\lambda$ by means of a suitable spectroscope; or, if photographic methods are employed, by a spectrograph. In practice the displacement is measured with a micrometer, and it is expressed directly in terms of one revolution, r, of the micrometer screw. Equation (6) is arranged thus:

$$V = \pm rV_1 \cdot \frac{\Delta\lambda}{r}. \tag{7}$$

rV_1 is tabulated for definite points in the spectrum, and $\frac{\Delta\lambda}{r}$ is the linear displacement as read directly with the micrometer.[14]

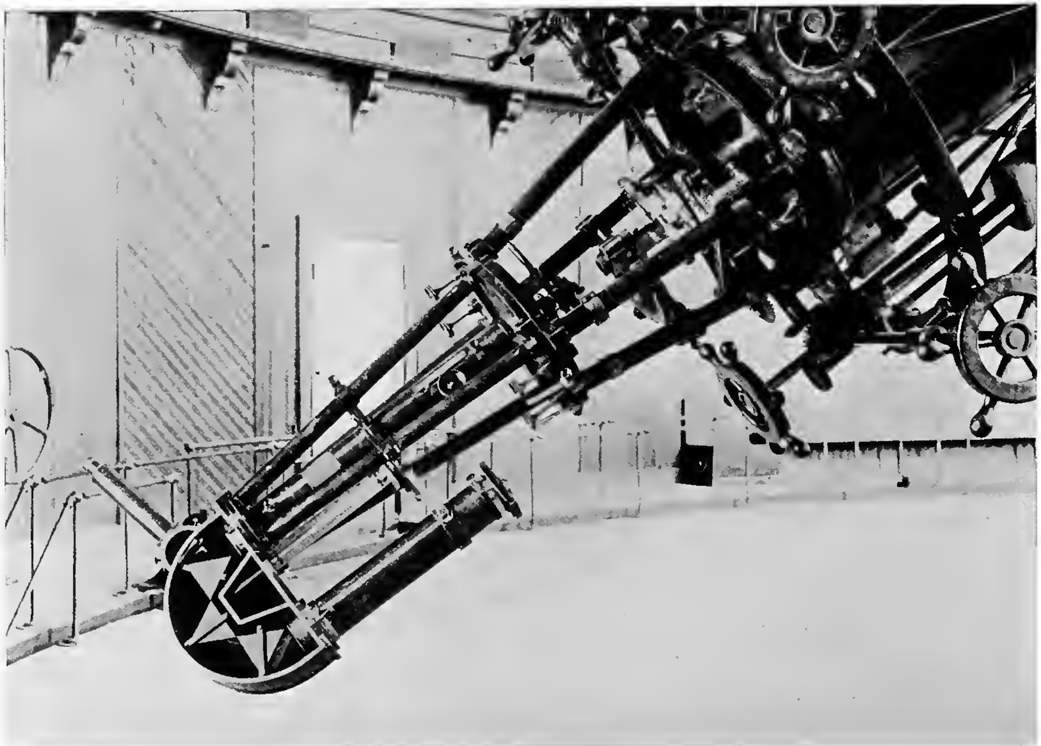
[14] The derivation of radial velocities from stellar spectrograms is most conveniently made through the use of standard reduction tables constructed for each spectrograph concerned. If a solar spectrogram be secured, under good conditions, on a fine-grained plate, and a considerable number of the best lines in the field of good definition be measured with a micrometer microscope, an equation expressing the relationship between the assumed wave lengths of the lines and the micrometer readings on the lines can be determined empirically by means of the simple method first suggested by Cornu and later discovered independently and developed by Hartmann. (*Publ. Astroph. Obs. Potsdam,* **12** (*Anhang*), 3-25, 1898; *Ap. J.*, **8**, 218, 1898.) If the region of spectrum does not cover too large a range of wave-length values, the deduced equation will probably reproduce the wave lengths of micrometer readings within the limits of unavoidable error. By means of the equation we may compute the standard micrometer readings for all the lines of definitely assigned wave lengths which we desire to use. Further, it is a simple matter to compute the radial velocity values, or factors, rV_s, corresponding to displacements of the separate lines through one revolution, r, of the micrometer screw. Here are extracts from the standard table used in reducing the spectrograms obtained with the original Mills spectrograph. This table, constructed before Hartmann's method was available, is based upon an equation determined empirically from a Mills spectrogram of the Sun, as described in *Ap. J.*, **8**, 142-144, 1898.

The spectroscope as applied to stars is used in connection with a telescope. The latter serves to collect a great quantity of the star's light and to deliver this light properly to the spectroscope. The human eye sees a star by virtue of the rays which enter the pupil, estimated to be 5 mm. in diameter at night. The telescope collects more light than the eye does, in proportion as the area of the object-glass is greater than that of the pupil, and delivers this light, neglecting great losses by reflection at the surfaces of the lenses and by absorption within the lenses, to the spectroscope. Even for first-magnitude stars, as observed with the largest telescopes[15] existing, this is none

TABLE I

λ	Micrometer Reading ⊙	rV_s
4238.188A	0.033	188.6 km.
38.970	0.326	188.8
......	.....	.....
......	.....	.....
......	.....	.....
......	.....	.....
4337.216	34.363	215.9
37.414	34.427	216.0
37.725	34.528	216.0
38.084	34.642	216.1
......	.....	.....
......	.....	.....
......	.....	.....
......	.....	.....
4441.881	65.399	245.5
42.510	65.572	245.6

[15] The prospects are not favorable for the erection of refracting telescopes larger than the 36-inch Lick and 40-inch Yerkes telescopes, mounted according to the same system as they are, chiefly because there is reason to believe that a lens greater than 40 inches in diameter, supported at the edge, would bend sufficiently under its own weight to make imperfect images. Again, it must not be overlooked that lenses larger in diameter mean thicker lenses, and greater thicknesses of glass mean increased loss of light by absorption. On the contrary, there appears to be no important reason, save that of cost, why reflecting telescopes should not be increased in diameter several fold.



The Original Mills Spectrograph

too great a quantity in visual observations of their spectra; for the light is no longer condensed in point images, but is spread over the large areas of their spectra, and is correspondingly weakened.

To illustrate the many points involved, let us refer to the photograph of the eye end of the 36-inch refractor and of the original Mills spectrograph attached to its eye end. The upper frame of converging steel rods constitutes the supporting truss for the spectrograph and does not form a vital part of it. It is, in fact, a part of the telescope rather than of the spectrograph. Three main features of the spectrograph are: first, the collimator, or collimating telescope, running centrally down through the lower and smaller supporting truss; secondly, the semicircular prism box containing three dense glass prisms; and thirdly, the camera tube running up to the right from the lower part of the prism box.[16]

At the extreme upper end of the collimator is a minute opening, technically called the slit, through which the light to be analyzed is admitted to the instrument. In photographing the spectrum of a fairly bright star, the slit is usually 1/40 of a mm. (1/1000 inch) wide, and 1/2 mm. long. It is placed exactly in the focus of the great lenses of the telescope, and the combined instrument is moved by clockwork in such a manner that during an exposure, sometimes lasting three hours or more, the star's image will fall constantly in this slender opening. After entering the slit the rays diverge until they enter a small lens at the lower end of the collimator, which renders them parallel. The telescope and collimator collect a great quantity of light—a beam 36 inches in diameter—and condense it into a beam only 38 mm. (1½ inches) in diameter; condensing it in effect more than 500-fold. This beam falls upon and passes through the three prisms. They separate the rays of different colors, by bending them through large angles, in this case, through 180 degrees approximately. The rays pass through a lens at the lower end of the view telescope or camera tube, which forms an image of the

[16] A detailed description of this instrument, which is named for the donor, the late Mr. D. O. Mills, is contained in *Ap. J.*, 8, 123, 1898.

spectrum in the observer's eyepiece or on a photographic plate at the upper end of the camera.

A refracting telescope (designed for visual observations) and a reflecting telescope have each some advantages and disadvantages in radial velocity determinations. In the former, the chromatic aberration of the objective for the photographically active rays is on a large scale, and troublesome. For example, in the 36-inch refractor the Hγ rays reach a focus at a point 49 mm. further from the objective than do the yellow rays, and the H and K rays of calcium come to a focus more than 70 mm. beyond that for the Hγ rays.[17]

To overcome as far as possible the limitations imposed by the chromatic aberration upon photographic observations, Newall placed a small "correcting lens" in the axis of the Cambridge telescope, five feet above the visual focus, which brought the photographically active rays substantially to a common focus.[18] An independent study of the same problem by Keeler led him to recommend the placing of a similar lens at a distance of one or two meters above the visual focus.[19]

In accordance with Keeler's solution of the problem, essentially all visual refractors used in connection with spectro-

[17] Keeler's results for the chromatic aberration of the 36-inch objective are as follows:

Position in Spectrum	Reading of Scale (Scale unit = $\frac{1}{20}$ inch)
B	76.6
Hα	82.7
D	88.0
Minimum Focus	88.5
b	85.2
5007 A	81.4
Hβ	76.6
Hγ	39.8
Hδ	6.5

The scale readings decrease as we pass from the minimum focus (88.5) away from the objective to reach the foci for B, Hγ, etc.—*Publ. Lick Obs.*, **3**, 174, 1894.

[18] *Mon. Not. R. A. S.*, **54**, 378, 1894.

[19] *Ap. J.*, **1**, 101, 1895.

graphic researches are equipped with correcting lenses which, in effect, convert them into photographic refractors. The 36-inch refractor has such a lens placed one meter above the normal Hγ focus.

It is well known that the reflecting telescope has the great advantage of bringing all the rays in the spectrum to the same focus. However, the silver-on-glass parabolic mirror is sensitive to changing temperature, and if the change is rapid the definition of the stellar image on the slit-plate is usually poor, involving a serious loss of light for spectrographic researches. A location with small diurnal range of temperature is therefore desirable; and means are sometimes adopted to control the temperatures of the mirrors. Thus, at the D. O. Mills Observatory, a refrigerating plant is used in the afternoons to hold the temperature of the parabolic mirror at the temperature which it is *estimated* the outside atmosphere will have at dark. It is the practice of the Mount Wilson Solar Observatory to envelop the 60-inch reflector in a heavy blanket covering, from dawn to early evening, which prevents the temperature of the instrument from rising more than a few degrees above the temperature at dawn.

Both forms of telescopes have focal lengths variable with temperature, and it is necessary that the position of the slit with reference to the objective or mirror be changed to correspond. The focal length of the 36-inch refractor is 23 mm. greater at the summer temperature of $+25°$ C. than at the winter temperature of $-5°$ C.[20]

My colleague, Wright, found that the focal position for the D. O. Mills reflector varied an inch or more in the course of a few hours, if the temperature of the atmosphere was falling rapidly.[21]

Before we can determine that the lines in the spectrum have been displaced either to the violet or to the red, and how much, it is necessary that the normal positions of the lines—that is, the positions corresponding to zero radial velocity—be known

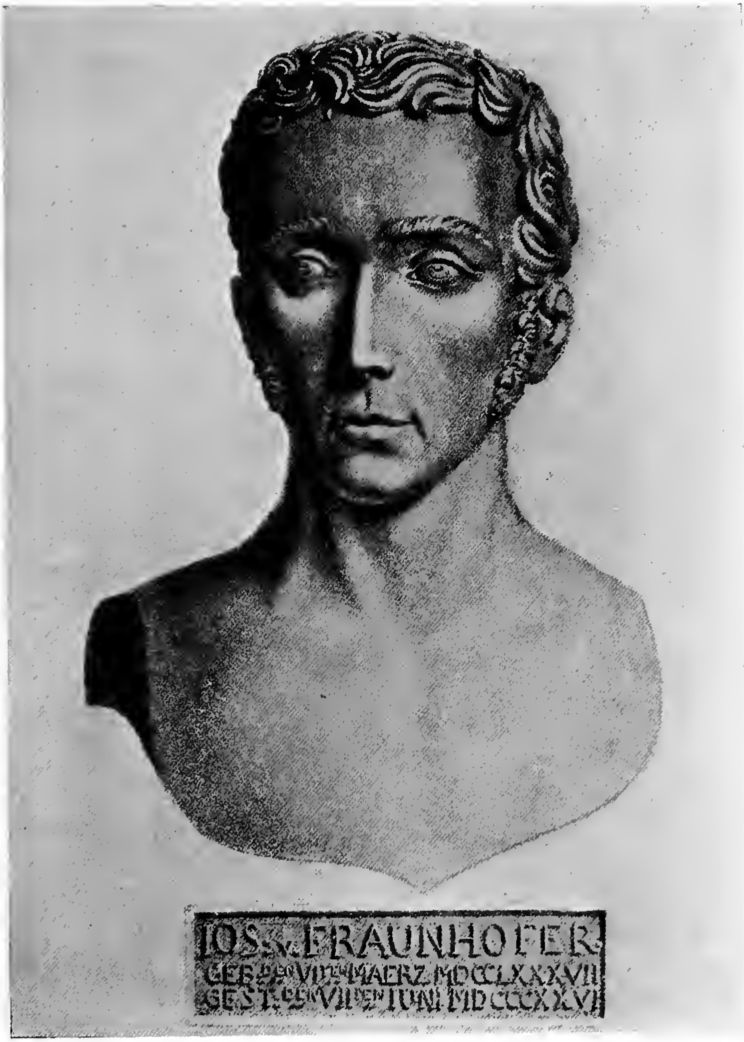
[20] *Ap. J.*, **8**, 138, 1898.

[21] *Publ. Lick Obs.*, **9**, 36-39, 1907.

for each plate. This is accomplished by observing or by photographing the comparison spectrum on either side of the star spectrum. Between the lower end of the telescope and the slit is a device for holding two electrodes—two small pieces of iron, titanium, or other metal—in such a way that their adjacent points will be separated by an interval of a millimeter or more. An electric current, intensified by Ruhmkorff coil and Leyden jar, is caused to pass across the interval in such a way that it burns the points of the metal, converting them into intensely hot vapors,[22] and producing in this manner the light of burning iron, titanium, or other substance. This light, passing through the spectrograph so that it falls on either side of the star's spectrum, forms the *spark spectrum* of the element in the electrodes; and since the burning metal is neither approaching nor receding from the instrument, its characteristic lines should occupy their normal or zero-velocity positions. If the star's lines, due to the same elements in the star's atmosphere, are clearly visible, one can see at a glance whether they are displaced toward the red or toward the violet, due to recession in the one case or to approach in the other; or, if the observations are photographic, an examination of the plate by means of a suitable microscope will indicate a velocity of recession or approach. In either case, the micrometer will determine the speed. If it is desired that the comparison spectrum be that of hydrogen or other gas, the current from a Ruhmkorff coil passed through a Plücker tube containing the gas may form the light source. The *arc spectrum* of iron, titanium, or other metal, which does not utilize the coil and jar, has its advantages, and is preferred by many observers.

We have thus far described in outline the instruments—the tools—which are used in the problems before us, and likewise the general principles underlying radial velocity determinations. We have referred to one or two of the larger problems of the stellar system which we hope soon to solve from radial velocity data. Before planning extensive structures, such as

[22] It should not be inferred that the light from an electric arc or spark is merely a temperature effect.



JOSEPH VON FRAUNHOFER, 1787-1826

the solutions referred to, it is wise to take stock of building materials at hand. There concerns us the question of what celestial objects are available for radial velocity observation. We shall pass in hasty review the chief types of stellar and other spectra, explaining briefly the possibilities and limitations of each type, for our present purposes.

The number of bodies visible in our great telescopes is of the order of one hundred million; and many more, perhaps twice this number, can be recorded photographically by existing instruments. Further, the dark or invisible bodies indicated by several considerations—by the analogy of the planets in the solar system, by the eclipsing variable stars, by the great numbers of close double stars discovered spectrographically, by the flashing out of "new stars," and especially by the apparent gravitational power of the universe—may outnumber the visible bodies several fold. It is a thesis of great value that all celestial bodies—the nebulæ, the bright stars, and the invisible bodies—are related products of a system of sidereal evolution. The general course of the evolutionary process, as applied to the principal classes of celestial bodies, is thought to be fairly well known. We think we are able to group these classes, with little chance of serious error, in the order of their effective ages.

Fraunhofer's discovery of the dark lines in the solar spectrum has already been referred to. To his genius also we owe the first studies of stellar spectra. He recognized clearly, in 1823, that there are different types of stellar spectra.[23]

[23] Die Spectra vom Lichte des *Mars* und dem der *Venus* enthalten dieselben fixen Linien, wie das vom Sonnenlicht, und genau an demselben Orte, wenigstens was die Linien D, E, b und F betrifft deren relative Lage genau bestimmt werden konnte. Im Spectrum vom Lichte des *Sirius* vermochte ich nicht, in dem Orange und in der gelben Farbe fixe Linien wahrzunehmen; im Grünen dagegen ist ein sehr starker Streifen zu erkennen, und zwei andere ungemein starke Streifen sind im Blauen, die keiner der Linien vom Planetenlichte ähnlich zu seyn scheinen; wir haben ihren Ort mit dem Mikrometer bestimmt. *Castor* giebt ein Spectrum, welches dem des *Sirius* gleicht; der Streifen im Grünen hat, des schwachen Lichtes ungeachtet, Intensität genug, dass ich ihn messen konnte, und ich fand ihn genau an demselben Orte wie beim *Sirius*. Die Streifen im Blauen konnte ich zwar erkennen, doch war das Licht nicht stark genug, um ihren Ort zu bestimmen.

Following Kirchhoff's discovery of the significance of the spectral lines, several observers took up the serious study of stellar spectra, independently and almost simultaneously. The principal pioneers, and the dates of the papers containing their first results were: Rutherfurd in New York (December 4, 1862),[24] Secchi in Rome (February 18, 1863),[25] and Huggins and Miller in London (April, 1864).[26]

Rutherfurd and Secchi had noted that stars of different colors have very different spectra. Secchi's spectroscopic survey of the sky, about 1866-1867, which included an examination of the spectra of many hundreds of stars down to the sixth magnitude, led him to a remarkable classification of stellar spectra, which was exceedingly useful to students of the stars during the succeeding third of a century. The footnote[27] quotes the principal con-

Im Spectrum von *Pollux* erkannte ich viele aber schwache fixe Linien, welche wie die der *Venus* aussehen. Ich sah die Linie D sehr gut; sie ist genau an dem Orte wie bei Planetenlicht. *Capella* giebt ein Spectrum, in welchem sich an den Orten D und b dieselben fixen Linien zeigen als in dem aus Sonnenlicht. Das Spectrum von *Beteigeuze* enthält zahlreiche fixe Linien, die bei guter Luft scharf begränzt sind, und wenn es gleich beim ersten Anblick keine Aehnlichkeit mit dem Spectrum der *Venus* zu haben scheint, so finden sich doch genau an den Orten, wo bei Sonnenlicht D und b sind, auch in dem Spectrum dieses Fixsternes ähnliche Linien. Im Spectrum von *Procyon* erkennt man mit Mühe einige Linien, und nicht so deutlich, dass man mit Sicherheit ihren Ort bestimmen könnte. Ich glaube im Orange an dem Ort D eine Linie gesehen zu haben.—*Gesammelte Schriften* (Lommel, Münchеn, 1888), p. 143.

[24] *Amer. Jour. Sci.*, **35**, 71, 1863.

[25] *Astr. Nach.*, **59**, 193, 1863.

[26] *Phil. Trans.*, **154**, 413, 1864.

[27] The principal results at which Secchi arrived are these:

"1. All the stars in relation to their spectrum can be divided into four groups, for each of which the type of spectrum is quite different.

"TYPE I. The first type is represented by the stars *Sirius*, and *Vega* or α *Lyræ*, and by all the *white* stars, as α *Aquilæ*, *Regulus*, *Castor*, the large stars in the *Great Bear*, α excepted, etc. The spectra of all these stars consist of an almost uniform prismatic series of colours, interrupted only by four very strong black lines. Of these black lines the one in the red is coincident with the solar line C of Fraunhofer; another, in the blue, coincides with the line F; the other two are also in the Sun's spectrum, but they have no prominent place. These lines all belong to hydrogen gas; and the tents of Secchi's paper on the subject. For the purposes of these lectures, we may describe Secchi's system very briefly, as follows:

TYPE I. This type includes the so-called white stars, whose spectra are relatively rich in blue and violet light, such as *Sirius, Vega, Regulus* and the stars in the *Big Dipper* (*α Ursæ Majoris* excepted). This type of spectrum is characterized by strong absorption lines of hydrogen and the essential absence or scarcity of metallic lines. Half of the bright stars, more or less, are of TYPE I.

TYPE II. This type embraces the so-called yellow stars, such as the Sun, *Capella, Arcturus,* and *α Ursæ Majoris*, in whose spectra the calcium bands H and K are very strong, the metallic lines are numerous and prominent, and the hydrogen lines are

coincidence of these four black lines with those of the gas has been, by careful experiments, already proved by Mr. Huggins, and also lately by myself. .

"Stars of this first type are very numerous, and embrace almost one-half of the visible stars of the heavens. We observe, however, some difference in individual stars; so that in some the lines are broader, and in others narrower; this may be due to the thickness of the stratum which has been traversed by the luminous rays. The more vivid stars have other very fine lines occasionally visible, but which are not characteristic of the type-form. In this type the red rays are very faint in proportion to the blue, violet and green, so that the colour of the star tends to the blue hue, and occasionally to the green. Of this last kind is the group of the large constellation *Orion* and its neighborhood.

"TYPE II. The second type is that of the yellow stars, as *Capella, Pollux, Arcturus, Aldebaran, α Ursæ Majoris,* etc. These stars have a spectrum exactly like that of our Sun—that is, distinguished by very fine and numerous lines. . . . A fuller description is unnecessary, since the spectrum of the Sun is very well known. The only thing which deserves particular attention is that in this class occasionally the magnesium lines are very strong, so as to produce very strong bands, and the iron lines in the green are in some very distinct. These stars can be distinguished even without the prism by the difference of colour, a rich yellow, which contrasts strongly with that of the first type. Stars of this second type are very numerous, and embrace almost the other half of the stars.

"TYPE III. The third and very remarkable type is that of orange or reddish stars. These have as a prototype the stars *α Herculis, α Orionis, Antares, ο Ceti, β Pegasi.* The spectra of these stars show a row of columns at least eight in number, which are formed by strong luminous bands no longer predominant. TYPE II includes nearly all the stars not embraced in TYPE I.

TYPE III. This type includes the orange or reddish stars, such as α *Herculis*, α *Orionis*, and *Antares*. Here the hydrogen lines are inconspicuous, the metallic lines are even more strongly developed than in TYPE II stars, and superimposed upon the spectrum of fine lines there are broad absorption bands, strong on the violet edges and less strong as we proceed toward the red edges. Secchi catalogued twenty-five spectra of TYPE III.

TYPE IV. This type comprises the extremely red stars, of which Secchi catalogued seventeen examples, all fainter than the sixth magnitude. This type of spectrum is not only rich in the

alternating with darker ones, so arranged as to represent apparently a series of round pillars, closely resembling a colonnade. . . . .

"All the pillars are generally resolved more or less completely in different stars into smaller and finer lines, very sharp and clear. . . . . In these stars some of the divisions of the pillars correspond to some principal lines of Fraunhofer, as D and b; but others, although very near, do not coincide with them, as C and F. The presence of hydrogen, however, is certain, the lines C and F having been found in the principal of them.

"The divisions of the pillars after many measurements have been found to agree perfectly in all these stars; so that this type is very constant and well marked. In my catalogue twenty-five of these most interesting objects are registered; and I do not imagine that I have exhausted the number.

"A very interesting feature connects this type with the preceding one. Here I must remark that we have to distinguish between lines and bands of shadow. The lines are strips narrow and sharp, the bands are shaded; although perhaps each band may be composed of very small lines, the aspect with our instruments (as at present constructed) is that of a more or less continuous shade. . . . .

"It is to be remarked also that all the pillars have their luminous sides toward the red, while the shadowed sides are towards the violet; this difference is very substantial as we shall see presently.

"TYPE IV. The fourth type is not less remarkable. This is the result of a laborious research on the telescopic stars of a red colour. Some of these are very small; and none of them exceed the sixth magnitude. . . . . The spectrum of this type consists of three large bands of light, which alternate with dark spaces so distributed as to have the most luminous side towards the violet. . . . .

"A great part of the red stars of the catalogue of Lalande, and of that of M. Schjellerup, belong to this or the preceding type; of this last class absorption lines of the metals, but is specially characterized by broad superimposed absorption bands, intense on the red edges and less strong as we proceed toward the violet edges.

Secchi commented significantly that these observed distinctions in stellar spectra constitute a grand fact which opens a field for many important cosmological speculations; a most

I have found seventeen remarkable examples. The characteristic colour here also may be a guide in the research, since some of these are like drops of blood in the field of the telescope. It is to be noticed that the line of magnesium b falls almost exactly at the end of the second luminous band in the green; but the full aspect of the spectrum does not justify the presence of such metal, but rather of a gas like carbon, which has luminous bands corresponding almost to the dark ones of the star, but not exactly.

''I do not attempt, however, to fix the nature of the substances, since I have not yet made a sufficient number of comparative measurements; but it seems to me that we are authorized in supposing these stars to be still in a different condition from others, perhaps partly in the gaseous state, or at least surrounded by a very large atmosphere different certainly from that of the others. . . . .

''The most striking object for its singularity which I have met in this examination of the heavens, and which is quite unique, with the exception of a very faint companion, is γ *Cassiopeiæ*. This star showed to me for the first time the lines of hydrogen in a luminous state, exactly the reverse of the dark lines of the stars of the first type. The star β *Lyræ* has the same feature but in a very faint degree.

''We have therefore, without doubt, in the heavens a grand fact, the fundamental distinction between the stars according to a small number of types; this opens a field for very many important cosmological speculations.

''2. Another grand fact which was brought out from these researches was, that the stars of the same type are occasionally crowded in the same space of the heavens. Thus the white stars are thickly gathered in *Leo*, in *Ursa Major*, in *Lyra*, *Pleiades*, etc., while the yellow ones are very frequent in *Cetus*, in *Eridanus*, *Hydra*, etc. The region of *Orion* is very remarkable for having all over and in its neighborhood green stars of the first type, but with very narrow lines and with scarcely any red colour. It seems that this particular kind of star is seen through the great mass which constitutes the great nebula of *Orion*, whose spectrum may contrast with the primitive spectrum of the stars. *Sirius* is perhaps too near us to be affected by this influence.

''This distribution of stars seems to indicate in space a particular distribution of matter or of temperature in different regions.''—*Report B. A. A. S.*, 1868, page 165.

attractive subject, which, however, does not directly concern us on this occasion.

Pickering suggested[28] the addition to Secchi's classification of a Type V, to include the bright-line nebulæ and the large number of stars whose spectra, subsequent to Secchi's work, were observed to contain bright lines.

For a long time astronomers have felt the need of a classification containing more subdivisions than Secchi's system. Vogel[29] proposed such a system, in 1874, which was supposed to be based upon the effective stellar temperatures represented by the spectra. However, as this system rested upon visual observations only, and these not much more extensive than Secchi's observations, it was not widely adopted.

Lockyer has made extensive studies of stellar spectra in comparison with laboratory spectra of those elements whose radiation and absorption lines are most prominent in the stars. His conclusions are in part represented by a proposed "Chemical Classification of the Stars,"[30] which is based exclusively upon estimates of the relative temperatures existing in the radiating and absorbing strata of the stellar atmospheres. The principles involved in Lockyer's system are extremely interesting and no doubt important, but this system has not been extensively used.

The exceedingly extensive photographic survey of stellar spectra made by the Harvard College Observatory, at Cambridge, Mass., and at Arequipa, Peru, on the basis of the Henry Draper Memorial, enabled Professor Pickering, Miss Maury, Mrs. Fleming, and Miss Cannon, to formulate a system which is now utilized as a working basis by nearly all stellar spectroscopists. Inasmuch as a condensed description of the system covers twenty-seven quarto pages,[31] we cannot undertake here to reproduce more than an imperfect skeleton of it.

Starting with the bright-line spectra of the nebulæ and passing successively through the bright-line stars, the so-called white,

[28] *Astr. Nach.*, **127**, 1, 1891.
[29] *Astr. Nach.*, **84**, 113, 1874.
[30] *Astr. Nach.*, **149**, 387, 1899; and *Nature*, **70**, 611, 1904.
[31] *Annals H. C. O.*, **28**, 135-161, 1901.

yellow, red and extremely red stars, thirty-four subdivisions or compartments are utilized. Class P represents the bright-line spectrum of those nebulæ which are called "gaseous." The system of bright lines, in some and perhaps in all cases, is accompanied by a faint continuous spectrum. Succeeding classes refer to stellar spectra, in which the continuous spectrum is always the principal feature. Class Q designates certain peculiar spectra having bright lines. Classes Oa, Ob and Oc are utilized to mark a variety of spectra containing bright lines, principally those of the primary and secondary[32] hydrogen series, and less strongly some of the lines of helium. Classes Od and Oe contain bright bands at 4633A and 4688A, but the primary and secondary hydrogen series of lines are dark. Class Oe contains numerous dark lines, especially those of helium. The spectra of the extremely interesting Wolf-Rayet stars are distributed, taking note of their differences, amongst Classes Q to Oe inclusive.

Classes P, Q, Oa, Ob, Oc, Od and Oe may be considered as forming Pickering's Type V addition to Secchi's system.

Class Oe5 refers to spectra, containing dark lines only, which are intermediate between Class Oe and an interesting group of stars sometimes called *Orion* stars, or helium stars, which are assigned according to their characteristics to Classes B, B1, B2, B3, B5, B8 and B9. The term "*Orion lines*" is used to designate all the dark lines except those due to hydrogen and calcium in Classes Oe, Oe5, B, B1, B2, B3 and B5. The helium lines[33] are the most prominent of the *Orion* lines. Others of these lines have been identified as due to nitrogen, silicon, magnesium, oxygen, and carbon. The helium lines appear to reach their maximum intensities in Class B2. The *Orion* lines fall off in intensity as Classes B8 and B9 are approached. The so-called solar lines are first seen in Class B8 spectra. The secondary hydrogen series of dark lines is visible in Class B, but does not appear in later classes. The primary hydrogen lines become more prominent with the advancing classes.

[32] *H. C. O. Circular,* No. 55, 1901.
[33] *Ap. J.,* 3, 4, 1896.

Spectra of Classes A, A2, A3 and A5 are characterized by the great intensity of the primary hydrogen lines, which attain their maximum intensity in Classes A and A2. The lines of helium are relatively faint or do not exist in Class A and following classes. The magnesium line 4481A is very prominent. The calcium bands H and K assume greater prominence with progress through these subdivisions, and, in general, the same may be said for the metallic lines.

Classes F, F2, F5 and F8 represent subdivisions in which the hydrogen lines diminish and the metallic lines increase in prominence. The stars in these classes may be said to lie between the so-called white and yellow stars.

The fourteen subdivisions, Oe5 to F2 inclusive, may be considered as comprised in Secchi's TYPE I, but F and F2 stars could be grouped in TYPE II.

Class G relates to spectra closely resembling that of the Sun, in which the H and K bands of calcium are the most conspicuous features, and in which the hydrogen lines are scarcely more prominent than large numbers of metallic lines. Class G5 represents a stage slightly more advanced, as shown by the diminished intensity of the hydrogen lines, and by the increased absorption of the blue end of the spectrum.

Classes K and K2 represent spectra of stars which may be said to fall between the yellow and red stars. The relative intensities of the blue and violet regions decrease appreciably as we pass through these classes, the hydrogen lines become less and less conspicuous, and the spectra are more and more weakened through selective absorption in the Fraunhofer lines.

We may say that the six classes, F5 to K2 inclusive, are comprised in Secchi's TYPE II.

Classes K5, Ma, Mb, Mc and Md are comprised in Secchi's TYPE III. The broad superimposed absorption bands, which vary from great intensity on their more refrangible edges to low intensity on their less refrangible edges, are first visible in Class K5[34] spectra, and increase in prominence up to Class Md.

[34] The bands are so inconspicuous in this subdivision that Secchi probably included Harvard Class K5 stars in his TYPE II.

The relative intensity of the continuous spectrum in the blue and violet regions falls off rapidly with progress through these classes. The "long-period" variable stars, in whose spectra some or many of the hydrogen lines are bright, may be said to comprise Class Md; o *Ceti* being the most notable example.

Class N, exactly equivalent to Secchi's Type IV, is characterized by the presence of wide absorption bands, intense on their less refrangible edges and decreasing in intensity as the more refrangible edges are approached, together with exceedingly low relative intensities in the blue and violet regions.

Apparent exceptions to the logic of the Harvard classification exist here and there; for example, in the case of bright lines detected in a few stars of Classes B1, B2, B3, B5 and B8; and extensive use of the system should be based upon the full exposition contained in *H. C. O. Annals,* **28** (II). In 1908 was added another compartment, Class R,[35] to include some fifty stars whose spectra contain a few of the broad absorption bands characteristic of Class N, and whose blue and violet regions are relatively as strong as in spectra of Class K.

The lack of sequence in the alphabetical labels of the thirty-four subdivisions is but evidence of the expansion and development of the system, as a result of advancing knowledge, and occasions no inconvenience.

The large irregular nebulæ, such as the *Great Nebula in Orion* and the *Trifid Nebula* in *Sagittarius,* are thought to represent the earliest form of material life known to us; and to the condensed planetary nebulæ, such as G. C. 4390 and N. G. C. 7027, are usually ascribed but slightly greater effective ages. All such nebulæ appear to have spectra consisting chiefly of sharply defined, strictly monochromatic, bright lines, constituting Class P. The accuracy with which we may hope to determine the radial velocities of such nebulæ seems to depend entirely upon their intrinsic brilliancy. The spiral nebulæ appear to have a great variety of spectra,[36] varying from those which consist principally of bright lines to those closely resembling Class G

35 *H. C. O. Circular,* No. 145, 1908.

36 Fath, *Lick Obs. Bull.,* **5**, 71, 1909.

Rev. Angelo Secchi, 1818-1878

or Class K spectra. As all these bodies without exception are intrinsically faint—exceedingly faint, in fact—there is little chance that we shall be able to determine their motions of approach and recession, by means now available or in sight, to a useful degree of accuracy. It is interesting to note, parenthetically, the tendency of recent evidence to the effect that unresolved spiral nebulæ are exceedingly distant and isolated sidereal systems. The photographs of small representative areas of the sky, made by Keeler and Perrine with the Crossley reflector, established with reasonable certainty that the number of nebulæ easily discoverable with that instrument, whenever we should care to undertake the labor, is of the order of half a million. Keeler further found that more than half of the nebulæ recorded on his plates are spirals. However, the number available for line-of-sight measurement would probably not be perceptibly increased if all the half million (more or less) were discovered and catalogued.

The star-like points occupying the positions of nebular nuclei appear to have closely approached, or reached, the first stages of stellar life. These nuclei are faint in all cases, and it is not certain that present or future means will enable us to measure their radial velocities accurately.

It is thought that the fairly long lists of stars with spectra of Classes Q, Oa, Ob, Oc, Od and Oe, containing a variety of bright lines, cannot be far removed from nebular conditions. Most of the bright stars in these classes have lines, whether bright or dark, of considerable width, whose edges are not sharply defined; and the accuracy with which the radial velocities of the stars can be determined is limited accordingly. The star D. M. $+30^\circ.3639$, is the only one of the Wolf-Rayet stars known at present to have *sharply defined* bright lines in its spectrum. This star is actually surrounded by an atmosphere of hydrogen[37] some $5''$ of arc in diameter, and the monochromatic hydrogen lines are capable of accurate measurement. Again, the fairly extensive list of stars whose spectra contain both bright and dark hydrogen

[37] *Astr. and Astroph.*, **13**, 461, 1894; and *Lick Obs. Bull.*, **6**, 59, 1910.

lines (nearly all of these lines broad and ill-defined[38])—caught apparently in the act of changing from bright-line to dark-line stars—are amenable simply to roughly approximate measurement by present methods.

The stars of Classes Oe to B9 inclusive, known as the *Orion* or helium stars, are characterized by lines varying from those exceedingly narrow and well defined, capable of very accurate measurement, to those which are so broad and ill defined as to be incapable of useful measurement.

The Class A stars are characterized by as great a variety of lines, as to width and definition, as the Class B stars.

With the lapse of time, through the radiation of stellar heat into space and the inevitable contraction in volume, stellar life appears to pass through the Class B and Class A ages into the Class F age. The existence of numerous strong metallic lines in the latter age makes a much larger proportion of the Class F stars amenable to accurate radial observation than of the stars in the preceding classes.

The solar spectrum, Class G, seems to indicate the summit of stellar life. With advancing time the visual radiations pass to the prevailingly yellow color of Class K stars. There is scarcely room for doubt that the stars of Secchi's TYPES III and IV, or Harvard Classes K5, M and N, represent the successive last stages of stellar development. Surface temperatures have lowered to the point of permitting more complicated chemical combinations than exist in the Sun. In the *extremely* red stars, a large proportion of which are variable in brightness, we may be observing the first struggles to form crusts over the surfaces; for example, the several hundred long-period variable stars, whose spectra at maximum brilliancy show bright lines of hydrogen and other elements. The hot gases and vapors responsible for these lines seem to be alternately imprisoned and released. It is further significant that the dull red stars are very faint, all fainter than the sixth magnitude. Their radiating powers seem to be extremely low. All stars of Classes G, K, M and N contain numerous lines so sharply defined that our ability to make accu-

[38] Some of the lines in ϕ *Persei* and β *Monocerotis* are fairly well defined.

rate determinations of their radial velocities is merely a question of their brilliancy; neglecting the fact, for the moment, that other factors than velocity may affect apparent wave lengths.

The period of existence succeeding that of Secchi's Type IV stars has illustrations near at hand, apparently, in *Jupiter* and the other planets of the solar system, invisible save by borrowed light; and the radial velocities of all such (in our system) are apparently obtained more readily and accurately by means of the spectrograph, whose results must here be based upon reflected light, than through the indirect methods based upon solar parallax (see Chapter VIII, p. 313). When the internal heat of a body shall have become impotent, which is apparently the case with the planets, the future promises nothing save the slow levelling influences of its own gravitational and meteorological elements. It is true that a collision may occur to transform a dark body's energy of motion into heat sufficient in quantity to convert the body into a glowing nebula, and start it once more over the path of evolution. This is a beautiful theory, but the facts of observation do not give it extensive support. There is little doubt that the principal new stars of recent years have been the results of collisions or of close approaches, either of two dark bodies, or of a dark massive body and invisible resisting materials. The suddenness with which intense brilliancy is generated would seem to call for the former, but the latter is vastly more probable, in view of many facts. The typical nova spectrum, of very broad bright and dark bands, gives way to a nebular spectrum of broad bright bands in a few months; but in every case thus far observed the bright nebular bands grow faint very rapidly, and in the course of a few years leave a continuous spectrum, apparently that of an ordinary faint star. Either the masses involved in the phenomena are extremely small, for stars, or the disturbances are but skin deep. In any case, the novæ have afforded little evidence as to the complete renebularization of dark bodies. The spectra of novæ, in their brighter stages, have contained exceedingly broad and diffuse bright and dark bands, with only one partial exception—that of *Nova Persei,* in which, for a time, there were well-defined narrow lines of calcium and sodium. This was the only nova out of six or eight, in the last two decades, whose radial velocity[39] could be measured at all.

The comets, which remain the most mysterious of all sidereal objects, have composite spectra. The denser materials forming the nucleus send us a reflected solar spectrum, in which the Fraunhofer lines have been photographically recorded with great success. The most prominent features of comet spectra, however, are the bright bands due to carbon and cyanogen, which have their source chiefly in the more or less condensed head, the carbon monoxide bands which have been observed to proceed chiefly from the tails of the brighter recent comets, and the orange sodium lines which have been seen in the spectra of a few comets whose orbits carried them relatively near to the Sun. There is little doubt that the radial velocities of the brightest comets could be observed, with limited accuracy, by means of the spectrograph, but velocities obtained by computation from orbital data would undoubtedly possess higher orders of accuracy.

We have described the principal classification systems of stellar spectra, and have sketched with exceeding brevity the more generally accepted order of stellar evolution, referring at the same time to the adaptability of the various spectral types for radial velocity determinations. In the following chapter we shall consider the methods which have enabled us to measure the radial velocities of more than one thousand objects with a satisfactory degree of accuracy.

[39] *Lick Obs. Bull.*, 1, 50-51, 1901.

CHAPTER II

DEVELOPMENT OF THE PHOTOGRAPHIC METHOD

The problem of determining stellar radial velocities in the manner described, so exceedingly simple in theory, has been in practice one of the most difficult in the history of astronomy. Using visual methods only, the best efforts of the most experienced observers met with signal failure[1] for more than twenty years, and doubts even as to ultimate success were generally felt. The lines, so distinct and capable of accurate measurement in the brilliant solar spectrum, appear indistinct and are difficult to measure in the spectra of the brightest and most favorable stars. Again, the displacements of the lines, even for high velocities, are really very minute. With the average dispersion employed, a speed of 10 km. per second (6 miles) caused a displacement of the order of 0.01 mm. (0.0004 inch), and speeds much smaller than this were to be observed. The imperfections of instruments and methods, incident to a subject so delicate, introduced errors many fold larger than the quantities to be measured. The efforts of several observers of great skill, in England and in Germany, between 1863 and 1887, some of these efforts continuing through many years, were unproductive of a single trustworthy result for the velocity of any star. The probable error of a single observed velocity was much greater than the average velocities of the stars. As Vogel, the ablest of observers, has remarked, it was often necessary to observe for hours, even under the best conditions, before an estimate of a displacement, and still less a

[1] It is furthest from my purpose to convey the meaning that these efforts were useless; in fact, I should like to be among the first to express respect for and appreciation of the early struggles to measure radial velocity by visual methods. Every effort of the pioneers, whether a success or a failure, is an index pointing the way of success to the observers who follow them.

measurement, could be attempted. Visual methods failed completely until the year 1890, when the combination of the 36-inch Lick refractor, an efficient spectroscope, good atmospheric conditions, and the comprehending observer Keeler, gave us our first reliable visually determined stellar velocity. In that year Keeler found for *Arcturus* an *approach* of 6 km. per second; for *Aldebaran* a recession of 55 km. per second; and for *Betelgeux* a recession of 16 km. per second;[2] results shown by many later photographic observations to be substantially correct.

Illustrating the untrustworthiness of earlier visual observations, we may say that two observers published the speed of *Arcturus*, probably the star most favorable for this purpose in the entire northern sky, as 73 and 89 km. per second, *recession*, respectively. The speeds of only three stars altogether were measured by Keeler. So difficult were the observations, even with the unequalled combination of favorable circumstances, that the number of stars capable of passably accurate visual measurement was seen to be very limited.

At the same time Keeler made a magnificent contribution to the general subject by measuring the velocities of approach and recession of thirteen planetary nebulæ and the *Great Nebula in Orion*. Although these objects are very faint as compared with stars,—only one of the fourteen being visible without a telescope,—they are capable of observation in this manner because nearly all their light is condensed into a few monochromatic bright lines, which are not widened under increased dispersion; whereas a star's light is distributed thinly over a large area of spectrum. Drawings of three of the planetary nebulæ and of their visual spectra (with the slit of the spectroscope passing through the nebular nuclei) are reproduced in Figure 2. Keeler obtained a recession of 17.7 km. per second for the densest part of the *Orion nebula*, and the results for the thirteen planetary nebulæ varied from 50 km. per second recession to 65 km. approach. Keeler's work in 1890-1891 represents the high-water mark in stellar nebular spectroscopy by visual methods. It seemed at the time that perhaps fewer than fifty stars in the

2 *Publ. Lick Obs.*, **3**, 195-196, 1894.

entire sky could be observed satisfactorily by visual spectroscopy, even with the assistance of the most powerful existing telescopes, and it was considered improbable that the number of nebulæ in the whole sky, north and south, whose velocities could be measured in this manner, exceeded forty. Plans were formed to apply visual measurement to as many stars and nebulæ as possible; but before further progress could be made an incomparably better method was discovered and announced. This method is based upon the application of the photographic dry-plate to the problem.

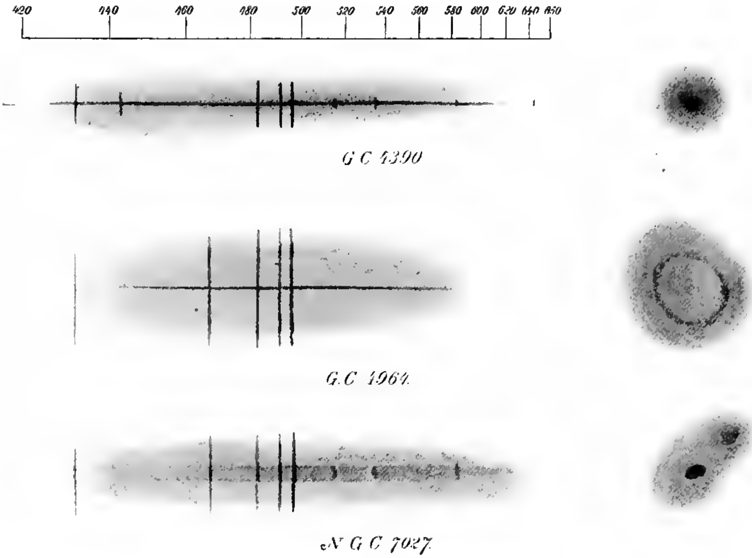


Figure 2

THREE PLANETARY NEBULÆ AND THEIR VISIBLE SPECTRA

An attempt to photograph stellar spectra, on wet plates, was made as early as 1863, by Huggins,[3] to whom we are indebted for pioneer work in so many branches of spectroscopic development. These photographs did not show any lines in the spectra, and

[3] *An Atlas of Representative Stellar Spectra*, London, 1899, p. 109.

they therefore gave no promise of the wonderful possibilities of photography in spectrum analysis, realized twenty-five years later. Dr. Henry Draper's stellar spectrograms of 1872 and following years recorded spectral lines for the first time.[4] The dry-plate became available about 1875; and Huggins's stellar spectra of 1875-1879 and Draper's of 1880 initiated the revolution which the dry-plate was to accomplish in our subject. The untimely death of Draper turned the further development of the photographic method into the capable hands of Director E. C. Pickering of Harvard College Observatory. Late in the 1880's, the Harvard photographs of stellar spectra (stellar spectrograms), obtained on the financial foundation of the Henry Draper Memorial, had made it clear that the dry-plate is able to record, distinctly, hundreds of lines in many spectra where but a few lines, or frequently none at all, can be observed by the eye directly. It remained for Director Vogel of the Potsdam Observatory, assisted by his colleague, Professor Scheiner, to prove, in 1888-1891, that the *positions* of the lines on stellar spectrograms can be measured with considerable ease and accuracy. Photographing with a 30-cm. (12-inch) telescope and a spectrograph attached thereto, Vogel and Scheiner were able to measure the displacements of lines in the spectrograms of *second-* and *third*-magnitude stars, with reference to the lines of comparison spectra impressed upon the same plates, as accurately as the skill of Keeler and the power of the largest existing telescope combined could measure for *first*-magnitude stars. In fact, the photographic method applied to the stars offered advantages so great that visual methods were no longer to be thought of. The speeds of fifty-one of the brighter stars were determined at Potsdam.[5] The results were accurate for their time, with average probable errors of 2.6 km. per second, for single observations, though it later appeared that many of them are affected by constant errors several times as large as this value. The Potsdam work was stopped in 1891, for ten years, awaiting the construction of a more powerful telescope.

[4] *Amer. Jour. Sci.,* (3) **13,** 95, 1877.

[5] *Publ. Astroph. Obs. Potsdam,* **7** (1), 1892.

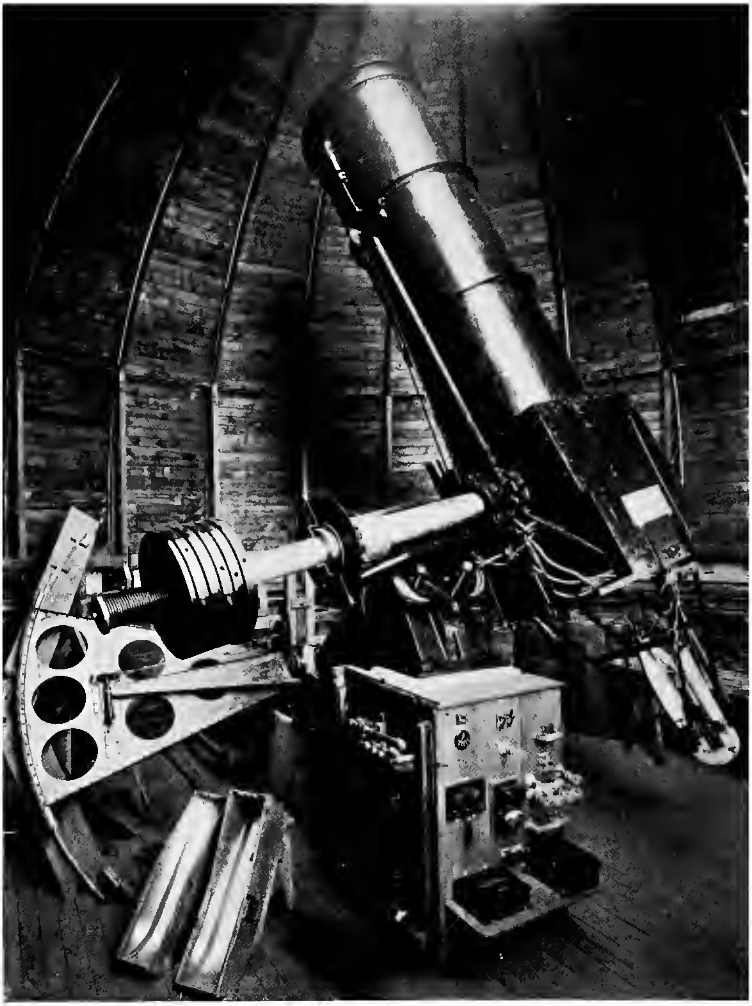
Results by Belopolsky, using a spectrograph of the Potsdam type attached to the 30-inch Pulkowa refracting telescope, in the early and middle 1890's, were considerably more accurate, owing to the larger size of his telescope and in consequence the shorter exposures needed.

There came to me, fifteen years ago, through Professor Holden, then director of the Lick Observatory, the wonderful opportunity of applying the 36-inch Lick telescope three nights per week to the measurement of stellar motions by means of spectrum photography. The spectrograph illustrated in CHAPTER I, planned in all its optical proportions and dimensions and in many of its mechanical features by me, designed in detail and constructed by the John A. Brashear Company, was named for the donor, the late Mr. D. O. Mills, member of James Lick's first board of trustees, and for several decades an efficient patron of educational matters. The instrument was planned to give *maximum efficiency in just one line of research*—the determination of stellar radial velocities—and all concerned were anxious to make it worthy of the donor and of the telescope to which it would be attached. Larger prisms, it is true, would give greater resolving power and offer other advantages, more or less valuable, but they would add serious difficulties in many directions; as examples: the dimensions of the spectrograph mounting would have to be correspondingly enlarged, with danger of increased flexure effects; it is difficult to secure homogeneous blocks of glass large enough for prisms beyond a certain small moderate size, and the demands for a high degree of homogeneity in this case are extremely severe; very little light would be able to pass through the base sections of the prisms; the lenses would be larger and thicker; and the camera lens, for the same focal length, would give satisfactory definition to a decreased length of spectrum. A successful design is a bundle of fortunate compromises between the great number of conflicting interests. When the new instrument was attached to the telescope and submitted to exacting tests, most discouraging defects were encountered, especially in some of the optical parts. The spectrum was very imperfectly defined, and the greater part of a year was devoted to locating the difficulties, one by one, and to removing them. When the changes which seemed to be required had been incorporated, in 1896, and the spectrograph was put into commission, the spectra were beautifully defined, and the accuracy attained indeed surpassed expectations. Probable errors were reduced at once from 2.6 km. to half a km. per second for the brighter stars containing good lines, and down to nearly a quarter of a km. for bright stars containing the best quality of lines. What was still more important, systematic errors appeared to be of minute or vanishing size. One photograph with this instrument gave greater accuracy, depending upon the character of spectrum, than could be obtained from ten to fifty spectrograms made with its predecessors.

There were obvious reasons for this advance in accuracy, which I take the time to describe, in outline, as accuracy in measuring exceedingly minute quantities is the very essence of success in the problems before us. The spectrograph was of more rigid design, to resist bending during a long exposure, for differential flexure introduces error. Changing temperatures of the prisms, lenses and metal parts during the exposures introduce error, and fortunately the night temperatures on Mount Hamilton are remarkably constant;[6] further, the parts of the spectrograph most affected by temperature changes were heavily wrapped with several thicknesses of insulating woolen blankets. The large telescope made the exposures relatively short, thus keeping flexure and temperature errors down to small and almost inappreciable dimensions. The lenses finally secured, and the prisms, were better in quality, I believe, and certainly better in design[7] for the problem in hand, than those used in earlier spectrographs. A number of important instrumental and observing precautions, more advantageously described later, were taken.

[6] It frequently happens that the temperature variation in the 36-inch dome from one hour after sunset to sunrise does not exceed one or two degrees Centigrade. The excellent atmospheric drainage existing at the summit of the mountain is no doubt the effective cause.

[7] Simple 60° prisms, following Keeler's adoption of this form in the Allegheny spectrograph, instead of compound prisms.

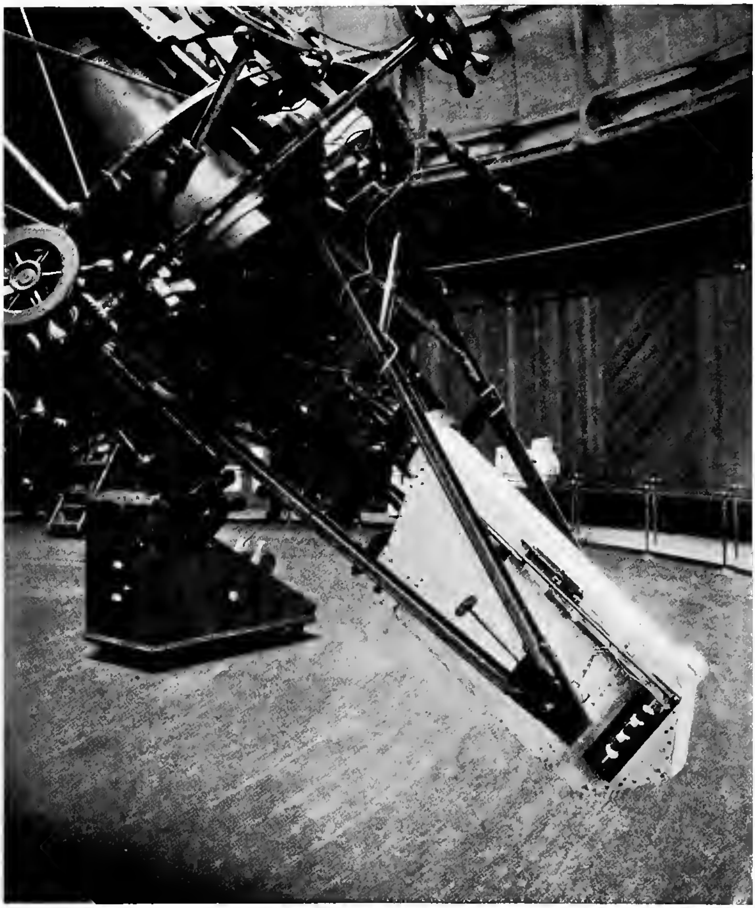


37-inch Reflecting Telescope and 3-Prism Spectrograph,
D. O. Mills Observatory, Santiago, Chile

Finally, a system of measurement and reduction was devised and employed whereby the *best lines* throughout the spectrum were utilized, whether they had comparison lines to match them or not. Earlier observers had in effect limited their measures to those stellar lines which had corresponding comparison lines, and frequently those lines were of poor quality, or unreliable, because in reality not single lines, but blends of two or more close lines. It was vitally important to be able to select the best stellar lines, whatever their positions in the spectrum. The spectrograms of 1896-1898 have not been improved upon, for the brighter stars, in the twelve succeeding years; but there have been appreciable improvements in the spectrograms of the fainter stars, which require long exposures, through means adopted for the more complete elimination of differential-flexure and temperature-variation effects.

The conventional spectrographs up to the present decade—and this included all astronomical spectrographs—were poorly designed to resist flexure. They were attached to the telescopes by, and supported entirely from, their upper ends; vital parts projected several feet beyond their supports, and differential flexures under the varying components of gravity, as the telescopes moved by clockwork to follow the stars, were almost literally invited. Flexure in the Mills spectrograph was nearly a negligible quantity, but in most spectrographs this effect was large and serious.

My assistant and colleague, Wright, suggested that such an instrument should be supported *near its two ends,* like a bridge truss or beam, in order to give minimum flexure. Acting upon this suggestion I designed the supports of the spectrograph for the D. O. Mills Expedition to Chile, in 1901, as shown in the illustration. To guard against further flexure effects, an extremely rigid but relatively light single steel casting forms the mounting to which the slit apparatus, the prism box, and the camera are directly attached. The new Mills spectrograph at Mount Hamilton, constructed in 1902, and in continuous use since May, 1903, is similarly supported, but the immediate mounting of the spectrograph may be described as a steel box constructed of saw-steel plates, and of three light steel castings which connect the slit mechanism, the prism box, and the plate holder, respectively, to the steel box. The inclined steel bars running down from the telescopes in either case form merely a supporting truss, and are not a part of the spectrograph. They form rather an extension of the telescope. A bar passing through



THE NEW MILLS SPECTROGRAPH

an opening of rectangular cross section in the casting at the base of the prism box is pivoted at the centre of the casting in such a way that it is free to rotate about the pivot through an angle of several degrees. The ends of this bar are attached to the supporting trusses. A cylindrical ring bearing near the upper end of the spectrograph receives a spherical flange of the spectrograph casting in such a way that the spherical flange has universal freedom of motion within the cylindrical ring. This form of support ensures that any strains generated in the truss system cannot induce corresponding strains in the spectrograph. Neither of the two spectrographs shows any trace of differential flexure.[8] In the original Mills spectrograph, as in similar instruments elsewhere, the collimator section alone was moved with reference to the remainder of the spectrograph, in order to place the slit in the focus of the telescope objective. Attention was called in CHAPTER I to the large variations of focal positions at different or with changing temperatures. In the Mills spectrograph at Santiago and in the new Mills spectrograph at Mount Hamilton, the ring bearings near the slits and the sliding supports for the pivoted bar bearings near the lower ends enable the spectrographs as a whole to be moved parallel to the axes of their collimators easily and quickly, in order to place the slits in the focal plane.

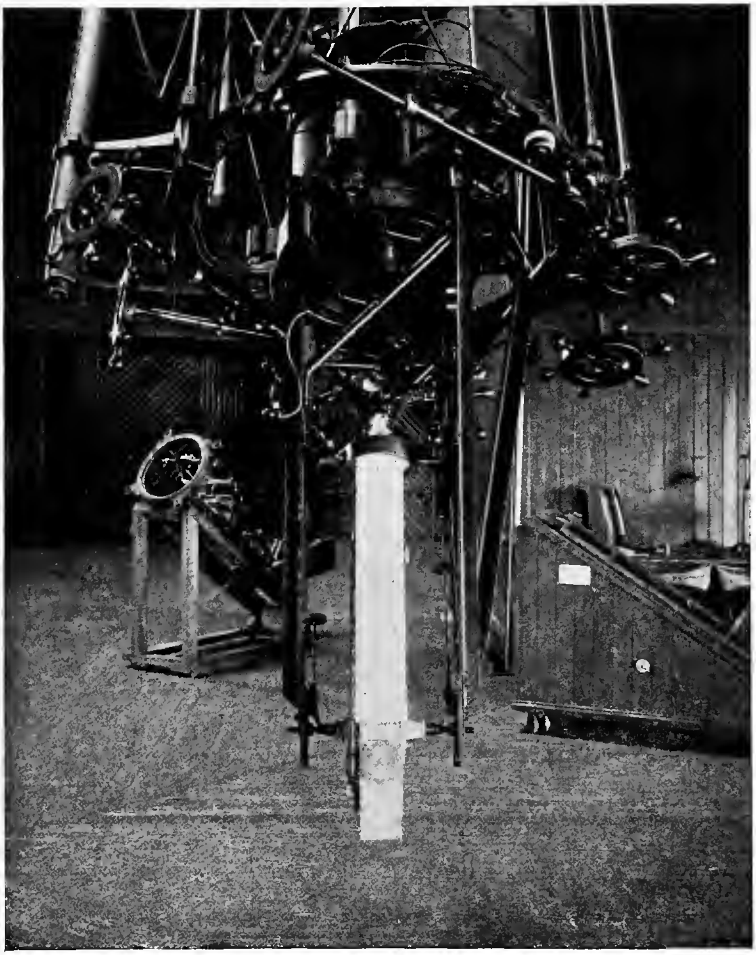
[Note added in 1911.—The simple devices described above, for supporting spectrographs at points near their two ends, and for *moving the instruments as a whole* into the proper focal positions, as well as the construction of the immediate spectrograph mountings in box form from thin plates, have been quite widely adopted in other instruments, apparently with entire success; for example, in the 1-prism spectrographs of the Allegheny,[9] Ottawa and Detroit observatories.]

The method of impressing the comparison spectrum on either side of the stellar spectrum is an important matter. If temperature changes or flexure effects occur during the time of an

[8] A more detailed description of the supporting system, with illustrative photographs, may be found in *Publ. Lick Obs.*, 9, 50-53, 1907.

[9] *Publ. Allegheny Obs.*, 2, 3, 7, 1911.

exposure to the stellar spectrum, the spectral lines will be more or less blurred, and be slightly displaced, as many observers have pointed out. It is essential that the comparison spectrum be permitted to fall upon the plate several times at stated intervals throughout the exposure, if the exposure is of more than fifteen



THE NEW MILLS SPECTROGRAPH

or twenty minutes' duration. Wright's device[10] for sending the light from two comparison-spectrum sources through two minute totally reflecting prisms, whose adjoining edges define the length of the slit, permits the throwing in of the comparison spectrum as often as is desired without interrupting the exposure on the star spectrum, and does not require dangerous handling of the delicate mechanism in the vicinity of the slit-plates. The device has been used on all the spectrographs of the Lick Observatory since 1900. It was adopted in designing the 1-prism spectrograph of the Allegheny Observatory, and is probably used at a few other places; but that it has not been generally adopted by stellar spectroscopists is to me a surprising fact.

The errors due to variations of temperature during exposures of considerable length, which were reduced in amount by wrapping the more vital parts of the spectrograph with heavy insulating materials, were later almost entirely eliminated by the adoption of devices for maintaining the spectrograph at a sensibly constant temperature. It was well known that the increase and decrease in the indices of refraction of the prism glass with rising and falling temperatures were the principal sources of error. The expansion and contraction of the metallic mounting with varying temperature, though less effective in displacing the spectrum, were nevertheless far from negligible. Deslandres[11] was the first observer who attempted to eliminate this source of disturbance. He mounted long strips of metal around and near the prisms and lenses of the Paris Observatory spectrograph, connecting them with a metallic thermometer in such a way that a slight fall in the temperature would produce an electric contact, and pass a current over the metallic strips. This would heat the air in the spectrograph, which in turn would raise the temperature of the thermometer, and break the contact at the proper point. In this manner the temperature of the air around the prisms and lenses would oscillate between two close limits. Deslandres made more extensive use of another very ingenious device. The outer walls of the spectrograph were

[10] *Ap. J.*, **12**, 274, 1900.
[11] *Bull. Astr.*, **15**, 57-61, 1898.

hollow, and so designed that water from the city supply could be circulated continuously through the channels in the walls. In this manner temperature variations within the spectrograph were decreased to about one-third their natural amount. Deslandres further suggested that it would theoretically be possible to render temperature variations essentially ineffective, by constructing the mounting of Guillaume's nickel-iron composition known as *invar,* whose temperature coefficient of expansion is about one-fiftieth that of ordinary steel, and by selecting glass for the prisms whose indices of refraction would be independent of temperature.

About the same time Lord[12] attempted to maintain the prisms of the Columbus spectrograph at a fairly constant temperature by placing coils of resistance wire outside of the prism box, the whole being wrapped in a layer of felt. A thermometer, whose bulb was within the prism box, was read from time to time, and, following its indications, the heating current was turned on for short intervals by hand.

Wright[13] made the useful suggestion that the entire spectrograph should be enclosed in a tight-fitting box on whose inner surfaces resistance wires would be disposed. This idea was adopted for the Mills spectrograph and applied in 1900, as shown in the illustration. One-half of the constant-temperature case is shown in position on the instrument, and the other half is on the floor. It is lined with thick hair-felt. The two halves fit together by a tight tongued-and-grooved joint, and the metal clamps which bind them together serve also to conduct the electric current between the wire systems in the two halves. An open-ended thermometer, forming a delicate thermostat, mounted on the inner surface of the case, operates a relay which is connected with the heating circuit. The sensitiveness of the system is such that when the temperature of the air *in the case* falls 0°.05 C. the heating current is turned on. When the heat thus generated has brought the temperature of the air in the case up to the level selected for the night, the current is automatically

[12] *Ap. J.*, **8**, 66, 1898.
[13] *Ap. J.*, **11**, 259, 1900.

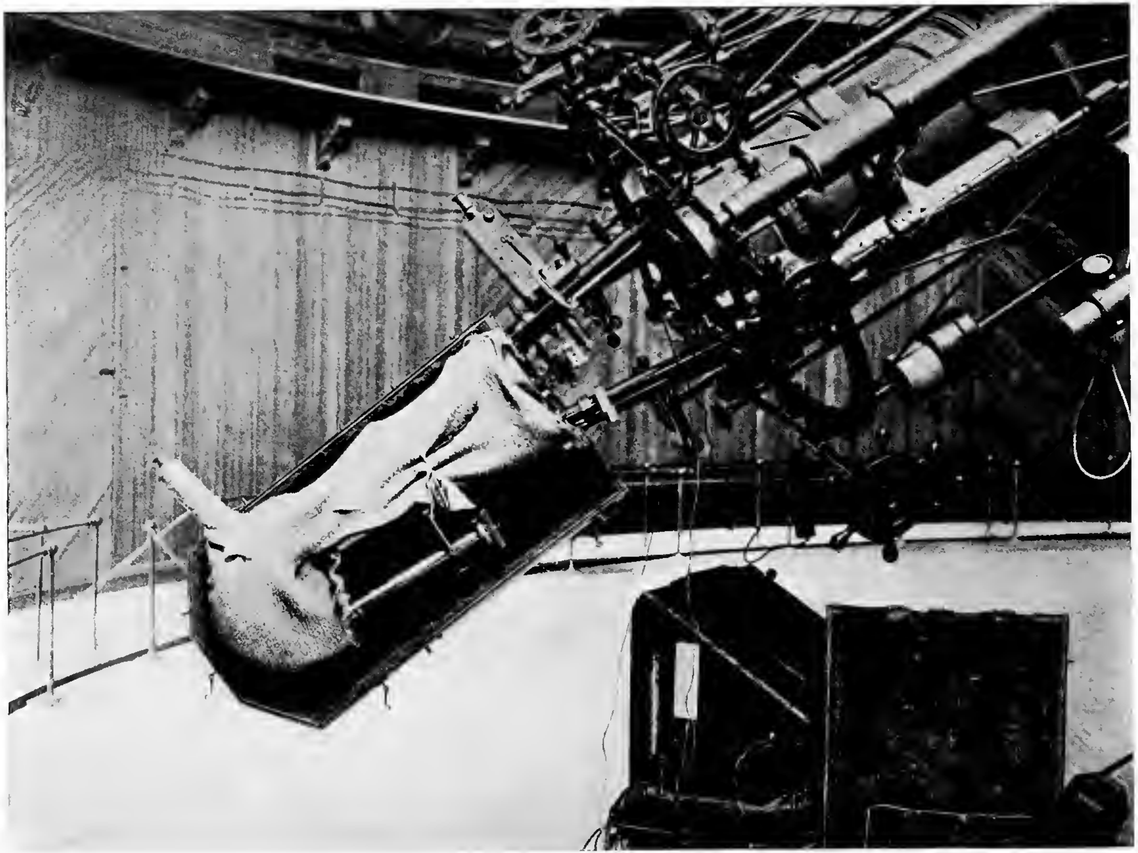
turned off. An electric fan within the case prevents stratification of the air. The heating current is turned on and off, on and off, every few minutes automatically throughout the night, keeping the temperature in the case nearly constant without attention on the part of the observer. The superiority of this system of control lies in the fact that the air *outside* of the prism box is kept nearly constant, giving the assurance that the temperatures within the prism box, and especially those prevailing in the slowly conducting glass prisms, are still more nearly constant. (To be accurate, it should be said that the heating current was turned on and off by hand during about one year, according to the readings of a sensitive thermometer within the wooden case, before the adoption of the automatic thermostat control.)

A comprehensive treatment of this subject, including a description of the elaborate constant-temperature control of the Potsdam 3-prism spectrograph, has been given by Hartmann.[14]

With the flexure and temperature factors essentially eliminated, as described in the preceding paragraphs, there is no apparent reason why the radial velocities of fifth-magnitude stars cannot be observed as accurately with exposures of eighty minutes as first-magnitude stars with exposures of two minutes, using the same spectrograph and sensitive plates; especially as any slight residual disturbances from these sources are in practice reduced almost to the vanishing point by inserting the comparison spectrum, not all at once, but at previously determined intervals throughout the exposures.

To guard against misunderstanding, it is well to repeat that we cannot measure the speeds of all stars, even of all bright stars, equally well. As explained in Chapter I, there are some stars with which we can do nothing because their spectra do not contain measurable lines. Other spectra have only broad and ill-defined lines which cannot be measured accurately; and so on, through a great variety of spectra measurable with increasing accuracy, up to the stars whose spectra contain a few sharply

[14] *Ap. J.*, 15, 172, 1902.



The Original Mills Spectrograph, with Temperature Control defined lines in some cases, and hundreds of excellent lines in other cases.

It is a fair and frequent question: How do we know that our instruments give accurate and dependable results? We shall consider some of the more important tests which have given satisfactory assurances of accuracy, and later refer to conditions which may lead to systematic errors more or less unavoidable.

In the year 1900, Professor Belopolsky was successful in securing laboratory measurements[15] of Doppler-Fizeau effects, which, within the limits of unavoidable error, confirmed the correctness of the principle. He attached systems of mirrors on the peripheries of two strong wheels which were so mounted that they could be rotated in opposite directions at very high speed. A ray of sunlight, falling upon a mirror on one of the wheels, would be reflected back and forth a controllable number of times between the mirrors of the two wheels. By virtue of these multiple reflections from mirrors in effect approaching or receding from the 3-prism spectrograph which finally received the light from the mirror systems, the spectrum as photographed in the usual way should exhibit an appreciable shift of the Fraunhofer lines. Measured displacements on six plates equalled 0.78 km. per second in the mean, and the corresponding displacements computed from the known linear speeds of the mirrors averaged 0.65 + km. per second. The difference of approximately 0.1 km. was well within the probable error of an individual observation.

In 1907 Galitzin and Wilip repeated[16] these experiments, using Belopolsky's revolving mirrors but replacing the prism spectrograph by a Michelson echelon spectrograph which possessed much more powerful dispersion. Seven observations with fourfold reflection gave a mean measured displacement of 0.254 km. per second, in comparison with the computed displacement of 0.256 km. Three observations on the basis of sixfold reflection gave a mean measured displacement of 0.364 km., in comparison

[15] *Bull. de l'Acad. des Sci. de St. Petersbourg*, **13**, 461, 1900; *Ap. J.*, **13**, 15, 1901.

[16] *Bull. de l'Acad. des Sci. de St. Petersbourg*, (6) **1**, 213, 1907; *Ap. J.*, **26**, 49, 1907.

with the computed displacement of 0.366 km. The average of the ten individual discrepancies between observed and computed displacements amounted to but 21 meters per second. These results, exhibiting a very high order of experimental skill, must be regarded as satisfactory confirmation of the Doppler-Fizeau principle.

It is preferable to depend upon observed velocities of celestial objects whose motions are known for proving by observation the correctness of the Doppler-Fizeau principle and the freedom of results for stars, nebulæ, etc., from serious error.

The first test of the Doppler-Fizeau principle was secured by Professor Vogel in 1871. It is known from observations of sun spots that the equatorial region of the Sun rotates once in approximately twenty-five days. From this fact, and the known diameter of the Sun, we find by simple computation that the east point of the solar equator is approaching the Earth approximately 2 km. per second more rapidly than the centre of the Sun's disk, and that the west point is receding from the Earth at the same relative rate. Because of the great intensity of sunlight, Vogel was able to observe the displacements of the Fraunhofer lines at the east and west equatorial points of the Sun, visually, with high dispersion. The displacements of the lines toward the violet for the east limb and toward the red for the west limb were estimated to be of the magnitude required by theory.[17]

Several spectroscopists, following Professor Vogel, while engaged in observing solar rotation, on the basis of the Doppler-Fizeau principle, have made important improvements in spectroscopic methods. Hastings,[18] in 1873, designed two small

[17] *Astr. Nach.*, **78**, 250, 1871.

Observations of sun spots by various observers, notably by Carrington and by Spörer, had established the interesting fact that the apparent rotation period is a function of the solar latitude, the period increasing rapidly for increasing latitudes; for examples, in solar latitude 30° the apparent period from the sun spots is 26½ days, and in latitude 45°, 27½ days or more; but the rotation periods in latitudes higher than 45° could not be determined owing to the paucity of sun spots in those regions.

[18] *Amer. Jour. Sci.*, (3) **5**, 371, 1873.

totally reflecting prisms, which, placed respectively before the two halves of the slit of his instrument, brought the spectra of the two opposite edges of the Sun side by side, and in contact, as viewed in the eyepiece; and the effect of the Sun's rotation, displacing one spectrum to the violet and the other to the red, was clearly shown. In 1877, Langley,[19] employing two prisms similar to Hastings's in front of the slit, noted that the absorption lines introduced in the apparent spectra of the east and west limbs of the Sun by the water vapor and oxygen in the Earth's atmosphere—the so-called telluric lines—were not displaced to violet and red, respectively, but formed straight lines across both spectra, whereas the absorption lines of solar origin were displaced in accordance with the Doppler-Fizeau principle. He called attention to the fact that the use of these telluric lines, whose positions are fixed, would permit the velocity displacements of neighboring solar lines to be measured with extreme accuracy, inasmuch as all errors due to instrumental sources could be absolutely eliminated. He further noted that these characteristics offer an exceedingly simple method for distinguishing telluric lines from solar lines; and this method has since been used by Cornu, Thollon, and others.

Dunér, using telluric lines as bases of reference, measured the rotational displacements of certain Fraunhofer lines at the Sun's limb in solar latitudes 0°, 15°, 30°, 45°, 60° and 75°. His results for rotational velocities were in good accord with the values previously deduced from sun spots, for latitudes in which sun spots are found. However, existing observations of solar phenomena occurring at different depths in the Sun's atmosphere leave little doubt that the apparent rotational periods are functions of the depths[20] to which the observations seem to refer; and it is not at all certain that the absorption lines used by Dunér and others should lead to rotation periods in exact agreement with

[19] *Amer. Jour. Sci.*, (3) 14, 140, 1877.

[20] For example, the stratum of the Sun which is most effective in forming the Ca absorption line at 4227 A is believed to be further from the Sun's centre than the stratum most effective in forming the absorption line of Fe at 4265.4 A.

Hermann Carl Vogel, 1841-1907

those deduced from studies of sun spots. In CHAPTER III we shall refer more extensively to observed radial velocities in the Sun, not as checks upon the displacement principle, but as means to the investigation of conditions existing in the Sun.

Other bodies in the solar system afford better tests than the Sun as to the correctness of results obtained with spectrographs of *moderate dispersive power,* such as now concern us. The orbits of *Venus, Mars,* and the Earth around the Sun are well known as to form and relative dimensions, and comparatively well known as to absolute dimensions. The velocity of *Venus* and *Mars* toward or away from the Earth may be computed with an accuracy dependent upon our knowledge of these absolute dimensions. It is a simple matter to measure the velocities by means of the spectrograph; and it has been found again and again that the observed and computed velocities agree within the limit of unavoidable errors. However, as we are here dealing with sunlight reflected from the planet surfaces, the spectrographic velocity of a planet is the actual velocity of the observed planet-area with reference to the Sun, plus the actual velocity of the same area with reference to the observer on the Earth's surface. Simple methods of computing these Sun-planet and planet-Earth terms are developed and illustrated in the footnote.[21] If the slit of the spectrograph is directed upon the centre of the illuminated area of the planet, the rotational velocity of the planet need not be taken into account; but if the slit is

[21] Campbell, *Ap. J.*, 8, 151-156, 1898. Let D be the distance between the centres of the two bodies whose relative velocity is required. The American Ephemeris tabulates the function

$$f = \log D$$

at regular intervals and stated times, for each planet and the Earth, and for each planet and the Sun. Let T be the date in the Ephemeris nearest the instant for which the velocity is required, and let ω be the tabular interval of time. Then the adjacent dates in the Ephemeris may be represented as in column 1 of the table below, and the corresponding values of the function $f = \log D$ as in column 2. The remaining columns contain the first, second, third, and fourth "differences" of the function f, formed in the usual manner. Lastly, the quantities $a = \frac{1}{2}(a_{,} + a')$ and $c = \frac{1}{2}(c_{,} + c')$ are inserted in the positions indicated.

directed upon the limb or terminator, the spectrographic velocity will be affected by the planet's rotation as a function of the angle at the planet included by lines drawn to the Sun and to

$T-2\,\omega$	$f(T-2\,\omega)$				
		$a_{\prime\prime}$			
$T-\;\omega$	$f(T-\;\omega)$		b'		
		$a_{\prime}$		$c_{\prime}$	
T	$f(T)$	a	b	c	d
		a'		c'	
$T+\;\omega$	$f(T+\;\omega)$		$b_{\prime}$		
		a''			
$T+2\,\omega$	$f(T+2\,\omega)$				

Let the instant for which the velocity is wanted be represented by $T+t$; and let n be the ratio of t and the tabular interval ω; *i.e.*, $n=t/\omega$, or $t=n\,\omega$. The formula for computing $\log D$ at the same time $T+t$ from the above data [adapted from Chauvenet's *Spher. and Prac. Astr.*, Formula (71), Vol. I, 5th Ed.] is

$$\log D = f(T+t) = f(T)$$
$$+\left(a-\frac{c}{6}\right)n+\left(\frac{b}{2}-\frac{d}{24}\right)n^2+\frac{c}{6}n^3+\frac{d}{24}n^4+\;.\;\;.\;\;. \tag{8}$$

The rate of change of $\log D$ at that instant is

$$\frac{\delta}{\delta t}\log D=\frac{\delta}{\delta t}f(T+t)\,;$$

whence, letting m be the modulus of the logarithmic system used,

$$\frac{\delta D}{\delta t}=\frac{D}{m}\left[a-\frac{c}{6}+\left(b-\frac{d}{12}\right)n+\frac{c}{2}n^2+\frac{d}{6}n^3+\;.\;\;.\;\;.\right]. \tag{9}$$

Now $\frac{\delta D}{\delta t}$ is the desired velocity at the time $T+t$, but expressed in terms of the astronomical unit of distance employed in the Ephemeris, and of the tabular unit of time ω. The astronomical unit of distance corresponding to the solar parallax 8″.80 is 149,500,000 kilometers. If ω is expressed in seconds of mean solar time, the desired velocity V will be given in kilometers per second by

$$\frac{\delta D}{\delta t}=V=$$
$$\frac{149{,}500{,}000}{\omega m}\,D\left[a-\frac{c}{6}+\left(b-\frac{d}{12}\right)n+\frac{c}{2}n^2+\frac{d}{6}n^3+\;.\;\;.\right]; \tag{10}$$

or, for logarithmic computation,

$$\log V=\log\frac{149{,}500{,}000}{\omega m}+\log D+\log\,[\;.\;\;.\;\;.\;\;.\;].\tag{11}$$

the Earth. This function is expressed in CHAPTER III, page 94. The spectrographic velocity is always affected by the motion of the observer by virtue of the Earth's rotational velocity, unless

The form [. . . .] is used to express the quantity within the brackets in (10).

The following table contains the values of $\log \frac{149{,}500{,}000}{\omega m}$ in the cases which may arise.

$$\text{If } \omega = 1 \text{ mean solar day, } \log \frac{149{,}500{,}000}{\omega m} = 3.6003$$
$$\text{“ } \omega = 2 \text{ “ “ days, “ } = 3.2993$$
$$\text{“ } \omega = 4 \text{ “ “ “ “ } = 2.9983$$
$$\text{“ } \omega = 8 \text{ “ “ “ “ } = 2.6973$$

Example. Required the velocity of *Venus* with reference to the Earth at Mount Hamilton mean time, 1898, June $8^d\ 7^h\ 58^m$, the slit being directed upon the centre of the illuminated area of the planet.

The Greenwich mean time is June $8^d\ 16^h\ 05^m$. The American Ephemeris gives for

Greenwich M. T.	log D	a	b	c	d
1898, June 5.0......	0.1591081				
		—32651			
“ 7.0......	0.1558430		—818		
		—33469		—12	
“ 9.0......	0.1524961	—33884	—830	—14	—3
		—34299		—15	
“ 11.0......	0.1490662		—845		
		—35144			
“ 13.0......	0.1455518				

In this case the assigned instant precedes T=June 9.0 by $7^h\ 55^m$, and $\omega = 2$ days. Therefore

$$n = -\frac{7^h\ 55^m}{2 \text{ days}} = -0.165.$$

Solving (8) and (11) we have

$$f(T) = +0.1525 \qquad a - \frac{c}{6} = -0.003388$$

$$\left(a - \frac{c}{6}\right)n = + \quad 6 \qquad \left(b - \frac{d}{12}\right)n = + \quad 14$$

$$\log D = +0.1531 \qquad [\,.\ .\ .\ .\ .\,] = -0.003374$$

the planet be observed exactly on the observer's meridian. The correction for this term is explained in the footnote, page 66, and its value may be taken from TABLE IV of this footnote.

$$\begin{aligned} \log \frac{149{,}500{,}000}{\omega m} &= 3.2993 \\ \log D &= 0.1531 \\ \log [\ .\ .\ .\ .\ ] &= 7.5281_n \\ \hline \log V &= 0.9805_n \\ V &= -\ 9.56 \text{ km.} \end{aligned}$$

Example. Required the velocity of *Venus* with reference to the Sun at Mount Hamilton mean time, 1898, June $8^d\ 7^h\ 58^m$.

As in the preceding example, the Greenwich mean time is 1898, June $8^d\ 16^h\ 05^m$, and we have from the Ephemeris for

Greenwich M. T.	log R	a	b	c	d
1898, June 1.0......	9.8563710				
		+ 568			
" 5.0......	9.8564278		+371		
		+ 939		—14	
" 9.0......	9.8565217	+1118	+357	—16	—4
		+1296		—18	
" 13.0......	9.8566513		+339		
		+1635			
" 17.0......	9.8568148				

In this case $T =$ June 9.0 and $n = -0.082$.

The solutions of (8) and (11) are

$$\begin{aligned} f(T) &= 9.8565 \\ \left(a - \frac{c}{6}\right) n &= \qquad 0 \\ \hline \log R &= 9.8565 \end{aligned} \qquad \begin{aligned} a - \frac{c}{6} &= +\ 0.000112 \\ \left(b - \frac{d}{12}\right) n &= -\qquad 3 \\ \hline [\ .\ .\ .\ .\ ] &= +\ 0.000109 \end{aligned}$$

$$\begin{aligned} \log \frac{149{,}500{,}000}{\omega m} &= \quad 2.9983 \\ \log R &= \quad 9.8565 \\ \log [\ .\ .\ .\ .\ .\ ] &= \quad 6.0374 \\ \hline \log V &= \quad 8.8922 \\ V &= +\ 0.08 \text{ km.} \end{aligned}$$

Finally, the most satisfactory test of the displacement principle is afforded by the motion of the Earth itself. The speed of the Earth in its annual orbit varies between 29.3 km. and 30.3 km. per second.[22] It follows, therefore, for a star situated in the plane of the Earth's orbit, *i.e.*, in the ecliptic, that the Earth is approaching this star at a certain definite time in the year at a rate of about 30 km. per second, and six months later is receding from the same star about 30 km. per second. It is found by observation that the velocities of a star in the ecliptic, observed under these two conditions, do differ about 60 km. per second; agreeing in fact with the computed difference within the unavoidable errors of observation. It is clear that the observed velocities of all stars are functions of their apparent positions on the celestial sphere, and of the Earth's position in its orbit, as well as of their own motions with reference to the solar system.

The observed velocities of all celestial bodies are likewise affected, to a very small degree, by the motion of the Earth around the centre of mass of itself and the Moon. In effect, the Moon swings the Earth in a small orbit whose radius is approximately three-quarters the radius of the Earth, for the centre of mass of the Moon and the Earth lies at this distance from the Earth's centre. This causes the observer to be moved away from or toward the star that he is observing, but never to exceed ± 0.014 km. per second.

These results, — 9.56 km. for the relative velocity of *Venus* and the Earth, and + 0.08 km. for the relative velocity of *Venus* and the Sun, refer to the centres of the Sun, *Venus*, and the Earth. The term due to the rotation of *Venus* is negligible, as the effective source of light was the area midway between the limb and the terminator of the planet. The term due to the Earth's rotation, $-v_d$, may be taken from the table for v_d, page 69, as follows:

		h m
Sidereal time of observation	=	13 09
Right Ascension of *Venus*	=	7 14
Hour Angle of *Venus*	=	+5 55
Declination of *Venus*	=	+24°.0
$-v_d$	=	+ 0.34 km.

[22] Assuming a value 8″.80 for the solar parallax.

All such observed velocities are likewise affected by the observer's diurnal motion. Owing to the rotation of the Earth, the observer is constantly approaching the east point of the horizon and receding from the west point. If he is situated at the equator, measuring the velocity of a star at rising in the east and again at setting in the west, he should obtain results in the two cases differing by nearly 1 km. per second. The diurnal effect is clearly a function of the observer's latitude and the star's hour angle and declination.

It is obviously desirable and essential to express the observed velocities of a star according to a system independent of the time of the year, month, day, and minute when the observation is made, and of the latitude and longitude of the observatory making the observation, in order that results secured at different times and places may be compared. This is accomplished by referring all such observations to the centre of the Sun, through the application of corrections for the effects of the observer's motions described above. I shall not take time to develop the equations which permit us to compute these corrections. The footnote[23]

[23] Campbell, *Astr. and Astroph.*, 11, 321, 1892.

Let $e =$ the eccentricity of the Earth's orbit,
$= 0.016751$ for 1900,
$a =$ the semi-major axis of the Earth's orbit,
$= 149{,}500{,}000$ km., or 92,900,000 English miles,
$90° - i =$ the angle which the tangent to the Earth's orbit makes with the radius vector drawn to the point of tangency,
$T =$ the number of mean solar seconds in a sidereal year,
$= 31{,}558{,}149$,
$\Pi =$ the longitude of the Sun at perigee,
$= 281° \, 13'.25$ for 1900,
$\odot =$ the Sun's longitude at the time of observation,
☾ $=$ the Moon's longitude at the time of observation,
$\lambda =$ the longitude of the star observed,
$\beta =$ the latitude of the star observed,
$t =$ the hour angle of the star observed,
$\alpha =$ the right ascension of the star observed,
$\delta =$ the declination of the star observed,
$\phi =$ the latitude of the observer,
$V_a =$ the Earth's velocity in kilometers per second in its orbit,
$v_a =$ the correction to the observed velocity of the star for this annual motion,

describes the methods which have been in use at the Lick Observatory since 1892, and gives references to the more important periodical literature of the subject. It is sufficient to say that the correction for the observer's annual motion, depending upon the time of the year and the position of the star in the sky, lies between 0 and ± 30.3 km. per second; that the lunar correction, depending upon the time, the observer's latitude and longitude, and the star's position in the sky, lies between 0 and ± 0.014 km. per second; and that the diurnal correction, depending upon the time of observation, the observer's location on the Earth, and the star's position in the sky, lies between 0 and ± 0.47 km. per second. After these corrections have been applied, the resulting velocity is said to be *the star's velocity with reference to the Sun*, or *with reference to the solar system.*

It is necessary to refer here to an instrumental term known as the curvature correction. If a straight slit has been used, the

V_m = the Earth's velocity in kilometers per second due to its revolution about the centre of gravity of the Earth and Moon,

v_m = the correction to the observed velocity of the star for this monthly motion,

V_d = the velocity in kilometers per second of a point on the Earth's equator due to the diurnal rotation, and

v_d = the correction to the observed velocity of the star for this daily motion.

The values of i and V_a are given by

$$\tan i = \frac{e \sin(\odot - \Pi)}{1 + e\cos(\odot - \Pi)} \tag{12}$$

and

$$V_a = \frac{a}{\sqrt{1 - e^2}} \cdot \frac{2\pi}{T} \cdot [1 + e\cos(\odot - \Pi)] \sec i. \tag{13}$$

Equations similar to (12) and (13) are derived in Chauvenet's *Spherical and Practical Astronomy*, Vol. I, § 391, 5th Edition.

When the Sun's longitude is $\odot$, the Earth is approaching the point of the ecliptic whose longitude is $\odot + 270° - i$, with a velocity V_a. Projecting this motion upon the line joining the observer and the star (λ, β) we obtain

$$v_a = - V_a \sin(\lambda - \odot + i) \cos\beta. \tag{14}$$

In TABLE III the values of V_a and i are tabulated as functions of $\odot$; so that to find the value of the correction v_a it is only necessary to find $\odot$ in the Ephemeris for the instant of observation, take the values of V_a and i corresponding to this value of $\odot$ from TABLE III, and substitute them in equation (14). The maximum error introduced by neglecting i is 0.50 km.

lines in the stellar and comparison spectra are curved in the sense that they are concave on the side toward the violet. This is due to the fact that the rays from any part of the slit except its optical centre pass through the prisms in planes which are oblique to the edges of the prisms; those rays proceeding from the ends of the slit passing in planes which make the largest angles with the edges of the prisms. In measuring spectrograms taken under these conditions the micrometer settings on the comparison lines, or the resulting velocity displacements, must be corrected to reduce them to the basis of straight lines. These corrections may be determined empirically from a spectrogram obtained with the instrument concerned, or they may be computed from Ditscheiner's formula,[24] using the known constants of the lenses and prisms in the spectrograph. In practice the

per second. If it is desired to use a value of the solar parallax different from that employed here, it is only necessary to multiply the values of V_a given by TABLE III by a constant factor.

The value of the lunar correction v_m can usually be neglected. But its maximum value is about ± 0.014 km. per second, and the degree of precision adopted for these tables requires that it should be considered here. It is not necessary, however, to take into account the ellipticity of the orbit and its inclination to the ecliptic. The average value of V_m is nearly 0.01 km. per second, and the motion is toward the point of the ecliptic whose longitude is ☾ + 270°. Projecting this motion upon the line drawn to the star (λ, β), we have

$$v_m = -V_m \sin(\lambda - \text{☾})\cos\beta = -0.01 \sin(\lambda - \text{☾})\cos\beta. \qquad (15)$$

Owing to the diurnal rotation the observer is constantly approaching the east point of the horizon, with a velocity

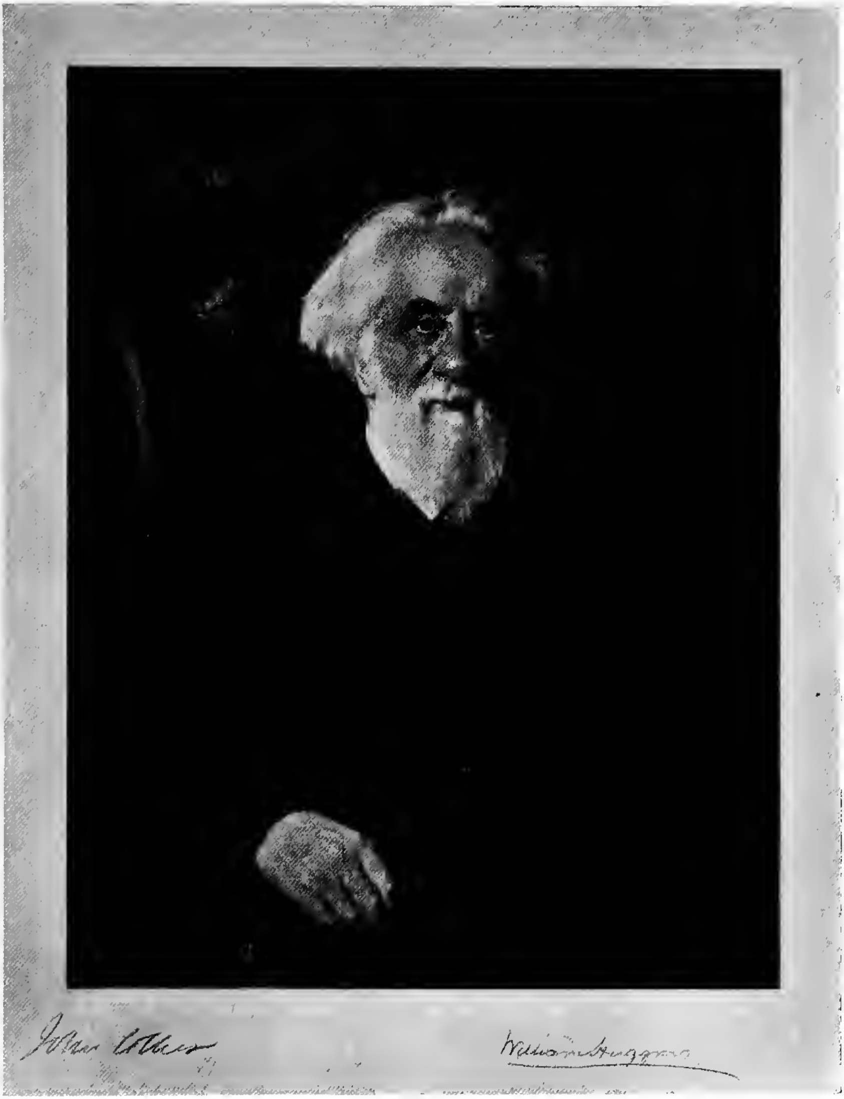
$$V_d \cos\phi = 0.47 \cos\phi.$$

Projecting this motion upon the line drawn to the star (t, δ), we have

$$v_d = -V_d \sin t \cos\delta \cos\phi = -0.47 \sin t \cos\delta \cos\phi. \qquad (16)$$

The values of this correction for the latitude of Mount Hamilton (37° 20′) are tabulated in TABLE IV with the arguments t and δ. The corresponding corrections at any other latitude ϕ' can be obtained from these by multiplying them by $\frac{\cos\phi'}{\cos\phi}$. v_d is negative if the star is observed west of the meridian, positive if the star is observed east of the meridian.

[24] Frost's Scheiner's *Astronomical Spectroscopy*, p. 15.



Sir William Huggins, 1824-1910

TABLE II

Velocities Corresponding to Displacements of One Ångström Unit

Line		V_s	Log V_s	Wave Length	V_s	Log V_s
		km.			km.	
Neb.	3727.	80.46	1.9056	3000A	99.95	1.9998
Hζ	3889.	77.10	1.8871	3200	93.71	1.9718
K	3933.82	76.23	1.8821	3400	88.19	1.9454
H	3968.62	75.56	1.8783	3600	83.29	1.9206
Hε	3970.2	75.53	1.8781	3800	78.91	1.8971
He	4026.4	74.47	1.8720	3900	76.88	1.8858
Hδ	4101.9	73.10	1.8639	4000	74.96	1.8748
Fe	4308.08	69.60	1.8426	4100	73.13	1.8641
Hγ	4340.63	69.08	1.8394	4200	71.38	1.8536
He	4471.7	67.06	1.8264	4300	69.73	1.8434
Mg	4481.4	66.91	1.8255	4400	68.14	1.8334
Neb.	4686.	63.99	1.8061	4500	66.64	1.8237
Hβ	4861.5	61.68	1.7901	4600	65.18	1.8141
Neb.	4959.0	60.47	1.7815	4800	62.46	1.7956
Neb.	5007.0	59.89	1.7773	5000	59.97	1.7779
b_1	5183.79	57.85	1.7623	5200	57.66	1.7609
E_2	5269.72	56.90	1.7551	5400	55.53	1.7445
Cor.	5303.	56.55	1.7524	5600	53.54	1.7287
D_3	5875.9	51.03	1.7078	5800	51.70	1.7135
D_2	5890.19	50.91	1.7068	6000	49.98	1.6988
D_1	5896.35	50.86	1.7063	6500	46.13	1.6640
Hα	6563.04	45.69	1.6598	7000	42.84	1.6318
B	6867.6	43.66	1.6401	7500	39.98	1.6019
A	7594.06	39.49	1.5964	8000	37.48	1.5738

TABLE III

The Earth's Orbital Velocity V_a and the Deviation i when the Sun's Longitude is ⊙

⊙	V_a	Log V_a	i	⊙	V_a	Log V_a	i
0°	29.87km	1.4752	+56′.3	180°	29.68km	1.4724	—56′.7
10	29.78	1.4740	+57 .5	190	29.76	1.4737	—57 .6
20	29.70	1.4727	+57 .0	200	29.85	1.4749	—56 .8
30	29.61	1.4715	+54 .8	210	29.93	1.4762	—54 .2
40	29.53	1.4703	+50 .8	220	30.01	1.4773	—50 .1
50	29.46	1.4692	+45 .4	230	30.08	1.4783	—44 .4
60	29.40	1.4683	+38 .4	240	30.14	1.4792	—37 .5
70	29.34	1.4675	+30 .3	250	30.20	1.4800	—29 .4
80	29.30	1.4669	+21 .2	260	30.24	1.4805	—20 .5
90	29.28	1.4666	+11 .4	270	30.26	1.4808	—11 .0
100	29.27	1.4664	+ 1 .2	280	30.27	1.4810	— 1 .2
110	29.28	1.4665	— 8 .9	290	30.26	1.4809	+ 8 .6
120	29.30	1.4668	—18 .8	300	30.24	1.4806	+18 .2
130	29.33	1.4673	—28 .1	310	30.21	1.4801	+27 .3
140	29.38	1.4681	—36 .5	320	30.16	1.4794	+35 .6
150	29.44	1.4690	—43 .8	330	30.10	1.4786	+42 .8
160	29.51	1.4700	—49 .7	340	30.03	1.4776	+48 .8
170	29.59	1.4712	—54 .0	350	29.95	1.4764	+53 .4
180	29.68	1.4724	—56 .7	360	29.87	1.4752	+56 .3

TABLE IV

$$v_d = -0.47 \sin t \cos \delta \cos (37° 20')$$

Hour Angles t		Declinations δ 0°	±10°	±20°	±30°	±40°	±50°	±60°	±70°	±80°	±90°	Hour Angles t	
h	h											h	h
0.0 or	12.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	12.0 or	24.0
0.5	11.5	.05	.05	.05	.04	.04	.03	.03	.02	.01	.00	12.5	23.5
1.0	11.0	.10	.10	.09	.08	.08	.06	.05	.03	.02	.00	13.0	23.0
1.5	10.5	.14	.14	.13	.12	.11	.09	.07	.05	.02	.00	13.5	22.5
2.0	10.0	.19	.18	.17	.16	.14	.12	.09	.06	.03	.00	14.0	22.0
2.5	9.5	.23	.22	.21	.20	.17	.14	.11	.08	.04	.00	14.5	21.5
3.0	9.0	.26	.26	.25	.23	.20	.17	.13	.09	.05	.00	15.0	21.0
3.5	8.5	.29	.29	.28	.26	.23	.19	.15	.10	.05	.00	15.5	20.5
4.0	8.0	.32	.32	.30	.28	.25	.21	.16	.11	.06	.00	16.0	20.0
4.5	7.5	.34	.34	.32	.30	.26	.22	.17	.12	.06	.00	16.5	19.5
5.0	7.0	.36	.35	.34	.31	.27	.23	.18	.12	.06	.00	17.0	19.0
5.5	6.5	.37	.36	.35	.32	.28	.24	.19	.13	.06	.00	17.5	18.5
6.0	6.0	.37	.37	.35	.32	.28	.24	.19	.13	.06	.00	18.0	18.0

NOTE.—Schlesinger has developed two valuable methods for reducing observed radial velocities to their values with reference to the solar system. One of these (*Ap. J.*, **9**, 159, 1899) combines in one process not only the term due to the elliptic motion of the Earth but also the disturbing effects of the Moon's and planets' attractions. The other method (*Ap. J.*, **10**, 1, 1899) computes the correction for the Earth's elliptic motion separately, and is especially advantageous in reducing a large number of spectrograms of the same star.

corrections are taken from a simple table as functions of the distances that the selected points of observation on the comparison lines are from the central line of the star spectrum.[25]

By using a suitably curved slit, instead of a straight slit, the spectral lines are straight, and the so-called curvature correction is completely eliminated. The Mills spectrographs at Mount Hamilton and in Chile have been provided with such curved slits since about 1900, but nearly all spectroscopic observers continue to use straight slits, requiring the application of curvature corrections to the results.

To be accurate, it should be said that the curvature of the slit is computed to give straight lines in the middle of the photographed spectrum. To the red of the middle the stellar and comparison lines will be very slightly concave toward the red, and to the violet of the middle the lines will be very slightly concave toward the violet. The departures from straight lines in the regions of good definition will be so slight as not to be troublesome in making the measures, and the slight errors due to curvatures in the two directions will usually balance each other.

To fix in mind the foregoing principles, and as an introduction to another phase of the subject, we shall examine some of the actual results of observation.

Here are three check observations of *Mars* and three of *Venus*. The spectrographic ''observed velocities,'' after correcting for curvature of the lines, are given in the fifth column. The next three columns contain, respectively, the computed velocities of these planets with reference to the Earth's centre, of the planets with reference to the Sun as the source of light, and of the observer's motion to or from the planets due to the diurnal rotation of the Earth. The sum of these three terms is given in the column ''computed velocity''; and with this the ''observed velocity'' should agree. The last column quotes the discrepancies between Observation and Computation, and these we call the (at present) unavoidable errors of observation. I was especially gratified that the *mean* of these errors is so small, only a

[25] See *Ap. J.*, **8**, 145, 1898.

MARS

Mount Hamilton M. T.	No. of Lines Measured	Measured Velocity, V_s	Corr. for Curv.	Observed Velocity	Rel. Velocity *Mars*—Earth	Rel. Velocity *Mars*—Sun	Diurnal Velocity, $-V_d$	Computed Velocity	Residuals O—C
1896 Sept. 15[d] 16[h] 13[m]	33	— 8.66 km	—0.39 km	— 9.05 km	—10.38 km	+1.94 km	—0.11 km	— 8.55 km	—0.50 km
Oct. 3 16 43	30	— 7.86	—0.39	— 8.25	—10.35	+2.12	+0.01	— 8.22	—0.03
1897 Jan. 14 08 05	18	+14.79	—0.39	+14.40	+12.71	+1.90	—0.08	+14.53	—0.13

Average residual, — 0.22 km.

VENUS

Mount Hamilton M. T.	No. of Lines Measured	Measured Velocity, V_s	Corr. for Curv.	Observed Velocity	Rel. Velocity *Venus*—Earth	Rel. Velocity *Venus*—Sun	Diurnal Velocity, $-V_d$	Computed Velocity	Residuals O—C
1897 July 22[d] 16[h] 09[m]	37	+13.92 km	—0.82 km	+13.10 km	+13.56 km	—0.17 km	—0.33 km	+13.06 km	+0.04 km
* July 22 16 09	32	+14.80	—0.81	+13.99				+13.06	+0.93
Aug. 26 16 52	31	+12.14	—0.62	+11.52	+12.06	—0.24	—0.31	+11.51	+0.01
* Aug. 26 16 52	27	+12.36	—0.70	+11.66				+11.51	+0.15
1898 June 8 07 58	31	— 9.04	—0.62	— 9.66	— 9.56	+0.08	+0.34	— 9.14**	—0.52
* June 8 07 58	22	— 8.71	—0.90	— 9.61				— 9.14	—0.47

Average residual, ±0.35 km.
Mean residual, 6 plates, —0.10 km.

*Second measures of the same plates by other observers.

**The terms due to the relative velocity of *Venus* and the Earth, to the relative velocity of *Venus* and the Sun, and to the diurnal rotation of the Earth, which enter into this computed velocity, are calculated in detail in the footnote on pages 61-63.

tenth of a kilometer per second, for this indicates the practical absence of systematic error.

The results for nine Mills spectrograms of the first-magnitude star *Aldebaran* are contained in the following table. The displacements obtained from the immediate measures of the plates as recorded in column two vary from + 26.2 km. to + 79.3 km. per second; but when the annual, lunar, diurnal, and curvature corrections are applied, as quoted in their respective columns, it is seen that the nine resulting values of the star's velocities, with reference to the solar system, are in remarkably good accord.

α TAURI (ALDEBARAN)

Date Gr. M. T.	V_s	V_a	V_m	V_d	Curv. Corr.	V	Observer
1896 Aug. 24.04	+26.77	+29.12	—0.01	+0.13	—0.50	+55.51 km	Campbell
Sept. 16.98	+26.70	+28.11	—0.01	+0.12	—0.30	+54.62	"
Dec. 8.79	+59.90	— 5.12	—0.01	+0.05	—0.30	+54.52	"
Dec. 17.73	+64.17	— 9.68	0.00	+0.11	—0.20	+54.40	"
1897 Jan. 20.66	+79.17	—23.96	+0.01	+0.06	—0.35	+54.93[26]	"
1898 Jan. 19.71	+79.30	—23.60	0.00	—0.04	—0.50	+55.16	Wright
1899 Sept. 25.98	+29.32	+26.64	0.00	+0.07	—0.34	+55.69	Burns
1901 Dec. 5.91	+58.28	— 3.07	0.00	—0.17	......	+55.04	Allen
1907 Sept. 16.02	+26.20	+28.47	—0.01	+0.03	......	+54.69	Plummer
					Mean V =	+54.95	

Probable error of mean $V = \pm 0.10$ km.
Probable error of one observation $= \pm 0.30$ km.

[26] In order to illustrate the methods employed in determining the several terms which make up this result I insert the full reduction sheet for plate No. 252, *Aldebaran*, obtained 1897, January 20, Mount Hamilton sidereal time $3^h\ 51^m$. The first column of the table contains Rowland's wave lengths of all the lines, both stellar and comparison, that were measured on this plate. The next column contains the micrometer readings for these lines as given in the standard reduction, of which a part is quoted in the footnote on page 19. It should be said that the standard solar plate was secured with a triple camera lens, whereas this plate of *Aldebaran* was obtained with a double camera lens later discarded. The reduction curves for the two lenses are slightly different, but their second differences are practically identical, so that either curve will answer all requirements. The third column contains the micrometer readings on the star lines, and the fourth column, the readings on the comparison lines of iron and the one comparison

ALDEBARAN. PLATE 252 B

λ	⊙	*	Fe and H	Zero Lines	Displacement	$_rV_s$	v_s
4282.565	16r.092		15r.859				
87.566	17 .831	17r.987		17r.591	+0r.396	202.7 km	+80.3 km
94.301	20 .150		19 .897	19 .899			
99.410	21 .893		21 .637	21 .635			
4300.211	22 .165	22 .287		21 .907	.380	206.1	78.3
00.732	22 .342	22 .469		22 .084	.385	206.3	79.4
02.692	23 .006	23 .136		22 .746	.390	206.9	80.7
08.081	24 .819		24 .553				
15.262	27 .213		26 .942				
16.962	27 .775	27 .892		27 .502	.390	210.5	82.1
25.939	30 .721		30 .441				
26.520	30 .911	31 .016		30 .631	.385	213.3	82.1
27.274	31 .156	31 .225		30 .876	.349	213.5	74.5
28.080	31 .418	31 .501		31 .137	.364	213.7	77.8
33.925	33 .308	33 .394		33 .026	.368	215.3	79.2
37.216	34 .363	34 .443	34 .080		.363	216.3	78.5
37.725	34 .528	34 .603		34 .244	.359	216.4	77.7
38.084	34 .642	34 .708		34 .358	.350	216.5	75.8
38.430	34 .753	34 .840		34 .469	.371	216.6	80.3
38.854	34 .888	34 .973		34 .603	.370	216.7	80.2
40.634	35 .456	35 .535	35 .169		.366	217.1	79.5
41.167	35 .626	35 .711		35 .339	.362	217.1	78.6
41.530	35 .741	35 .811		35 .454	.357	217.2	77.6
43.861	36 .481	36 .564		36 .194	.370	217.9	80.6
44.670	36 .738	36 .811		36 .451	.360	218.1	78.5
47.403	37 .601	37 .678		37 .313	.365	218.9	79.9
49.107	38 .137	38 .212		37 .849	.363	219.4	79.7
55.257	40 .062	40 .141		39 .773	.368	221.1	81.4
59.784	41 .467	41 .543		41 .178	.365	222.4	81.2
69.941	44 .586	44 .645		44 .295	.350	225.3	78.9
76.107	46 .456	46 .515		46 .165	.350	227.1	79.5
79.396	47 .446	47 .495		47 .155	.340	228.0	77.5
83.720	48 .741		48 .449				
89.413	50 .433	50 .487		50 .141	.346	230.9	79.9
4404.927	54 .975		54 .683				
06.810	55 .519	55 .554		55 .227	+0.327	235.8	+77.1

Mt. Ham. Sid. T. 1897 Jan. 20 $3^h\ 51^m$

α 1900.0 = 4 30

$t = -$ 0 39

δ 1900.0 = + 16° 18′

Greenwich M. T. 1897 Jan. 20 $15^h\ 55^m$

β 1900.0	= −	5° 28′ .5
λ 1900.0	=	68 23 .5
⊙ 1900.0	=	301 28 .6
i	= +	20 .0
λ − ⊙ + i	=	127 14 .9
log V_a	=	1.4805
sin (λ − ⊙ + i)	=	9.9009
cos β	=	9.9980
log v_a	=	1.3794n
v_a	=	− 23 96 km.

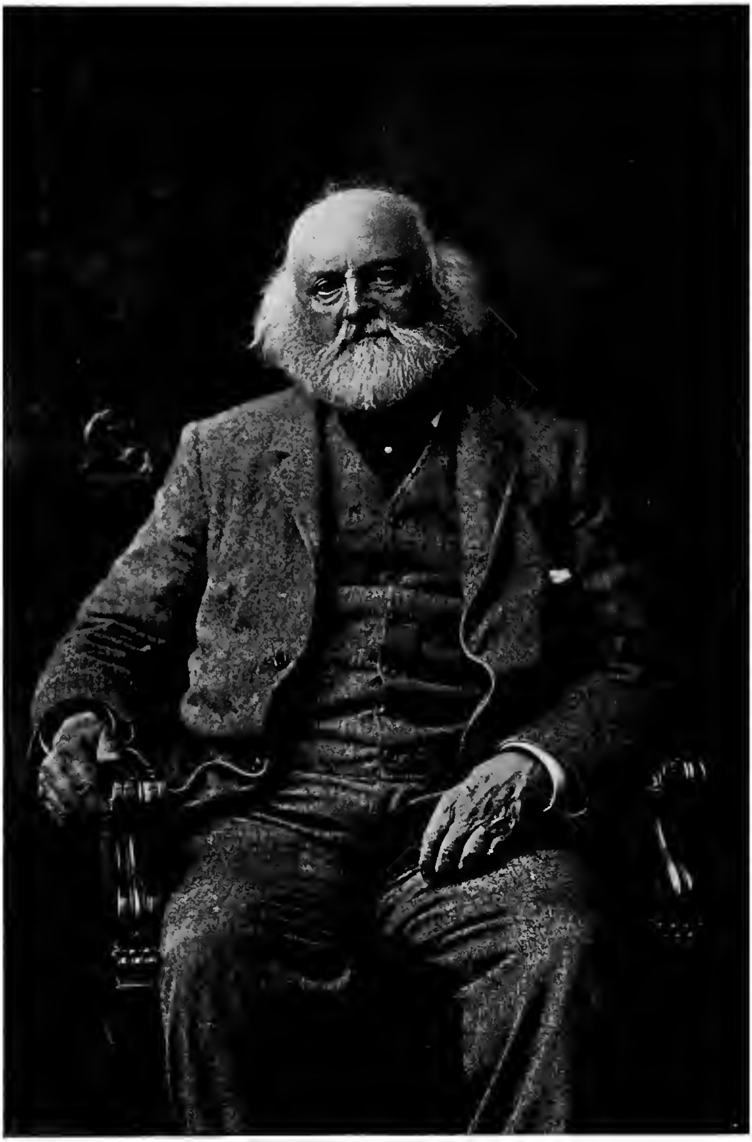
λ	=	68°
◐	=	137
λ − ◐	=	291
sin (λ − ◐)	=	9.97
log V_m	=	8.00
log v_m	=	7.97n
v_m	=	− 0.01 km.
Mean v_s	=	+79.17 km.
Corr. for Curvature	=	− 0.35
v_a	=	−23.96
v_m	=	+ 0.01
v_d	=	+ 0.06
V	=	+54.93

They show a range of only 1.3 km. We have no reason to suppose that the velocity of *Aldebaran* is variable to this extent: the star seems to be travelling on and on through space with an unvarying speed of approximately 55 km. per second, *with reference to the solar system.* The small differences in the last column are believed, as with *Mars* and *Venus,* to be the unavoidable errors of observation. The mean of the nine results is + 54.95 km. per second, with a probable error of ± 0.10 km. The probable error is simply a measure of the accordance of the individual results. It does not guarantee the results to be free from systematic errors, which are generally more insistent and more important than accidental errors. We reassure ourselves, every few weeks, that the velocities are free from perceptible systematic error by measuring the velocity of a planet, as explained above, using exactly the same methods of observation and measurement as for the stars. If the observed planet velocity agrees satisfactorily with the computed velocity, we proceed. If it differs 1 km. per second there are anxious repetitions of the process until the source of the discrepancy comes to light. The remark-

line of hydrogen at 4340.634 A. The next column, "Zero lines," contains the micrometer readings which comparison lines, or *lines of zero velocity,* would have if there were such lines having the wave lengths given in the first column. In other words, they are the readings of the corresponding solar lines reduced to the curve on which the Fe and H comparison lines lie. These values are readily supplied. The micrometer readings on the comparison lines are assumed to be correct. A short curve similar in all respects to the corresponding section of the solar reduction curve is analytically passed through each adjacent pair of comparison lines, and the readings on this curve corresponding to the wave lengths of the lines observed in the star spectrum are obtained by interpolation. Sections of the solar curve are passed through the comparison lines, pair by pair, throughout the spectrum. (The iron comparison lines at 4294 and 4299 are usually treated as one line in their mean position 4296.9.) The micrometer readings supplied for the zero lines are fully as accurate as those obtained by actual measurement from the comparison lines. The micrometer readings on the star lines minus those on the comparison and zero lines are the observed displacements. The value of a revolution of the micrometer screw expressed in kilometers per second, which is taken from the reduction table, varies for different plates, owing to changes of temperature, etc. The readings on the comparison lines of the two plates enable us to determine the relative able accuracy with which the photographic plates can be measured is in a sense indicated by the smallness of the probable error of the velocity obtained from one plate. The linear value on the plate corresponding to 0.30 km. is but 1/2900 mm., or less than .000,014 inch. This does not mean that each line in a spectrum can be measured to such a high degree of accuracy, but only that if 20 to 40 lines on a plate are measured, the mean of all the measures will be trustworthy to approximately this degree of probable error. To attain such accuracy, a long list of precautions must be held in mind. I have already spoken of the necessity for eliminating flexure- and temperature-changes in the spectrograph. The parts of the spectrograph must be adjusted perfectly with reference to each other, and the instrument as a whole with reference to the telescope which pours the light into the slit. The comparison light from the electric arc, spark, or other source, must be made to follow the same path through the spectrograph that the star's light does. The guiding on the star must be skillfully done, and the exposure times on the star and comparison spectra must be right. Granting that the exposure has been made so carefully that the spectrogram is free from appreciable error, there still remains the equally important task of measuring the plate with circumspection: an inexperienced or careless measurer will obtain poor results from

linear scales of the stellar plate and the standard solar plate, so that the correct values of rV_s, corresponding to the scale of the plate under reduction, are readily supplied. Thus, for the plate in question, the corrected values of a revolution of the micrometer screw in kilometers per second are 0.2 per cent greater than those quoted in TABLE I. The measured velocities v_s for the individual lines are quickly found by means of Crelle's tables. The mean of the velocities from twenty-eight star lines is + 79.17 km. per second. The readings on the comparison lines were made at points 0.85 revolution of the micrometer screw from the central line of the star spectrum. Hence, the correction for curvature taken from a suitable table (*Ap. J.*, **8**, 145, 1898) is — 0.35 km.

Following the methods described in the footnote, pp. 64-69, the correction for the Earth's annual motion is — 23.96 km., the correction for the lunar term is + 0.01 km., and the correction for the diurnal term is + 0.06 km. The finally deduced radial velocity of *Aldebaran*, with reference to the solar system, is + 54.93 km. per second.



Lewis Morris Rutherfurd, 1816-1892

perfect plates. The plate to be measured must be illuminated uniformly; no more strongly from one direction than from another, or the effect will be to displace the star lines and comparison lines differently. It must be measured with the violet end to the right and with the red end to the right, for the two sets of measurements will, in general, not agree.[27] Constant judgment is called for in selecting lines suitable for measurement. One side of a given line, the other side of another line, or one end of a third line, may be made irregular by the presence of silver grains in the film that are especially sensitive or insensitive, and such lines must be avoided. A good line in one spectrum may not be usable in another, or it may have an appreciably different position in a third, due to the coming in of a close companion line in the latter.

My assistant and colleague, Albrecht, has shown[28] that certain lines, as observed with 3-prism dispersion, systematically increase in wave length as we pass from the white stars through the yellow to the red stars; that other lines decrease their wave lengths in a similar manner; whereas still other lines change their positions in one direction in passing through certain types of spectra and in the other direction in passing through the remaining types. It is thought that this shifting of a line with reference to neighboring lines is due to the fact that the line in question, single with 3-prism dispersion, would in reality be seen double or multiple under much higher dispersive power; and that the relative intensities of the two or more components vary in different spectral types. The use of a fixed wave length for such a composite line will frequently give an erroneous velocity for that line, amounting to 2, 3 or even 5 kms. per second, though the velocity for all the measured lines of a spectrogram will be made erroneous by but a small fraction of this amount; and it will usually happen that errors due to this source are present with opposite signs, thus tending to correct one another. Such lines need not necessarily be avoided, but should be allowed

27 See articles by Lord, *Ap. J.*, **6**, 425-426, 1897; Reese, *L. O. Bulletin*, **1**, 126, 1901; Hasselberg, *Ap. J.*, **15**, 208, 1902.

28 *L. O. Bulletin*, **4**, 90, 1906.

for, on the basis of an investigation into their behavior in the various spectral types.

For the wave lengths of stellar lines, it has been customary to employ Rowland's values of the corresponding lines in the Sun's spectrum,[29] as observed with the high dispersion of a powerful grating spectrograph, in the case of all stellar lines which appear in the solar spectrum. The wave lengths of several lines, in Classes B and A, which do not appear in the solar spectrum, have been adopted as determined in the laboratory from arc and spark sources; as examples, the principal lines in the spectra of helium, magnesium, carbon and oxygen. Rowland's determinations of wave lengths, incomparably better than previous results, were originally thought to form standards so accurate that they would remain sufficient and satisfactory for the spectroscopic work of several generations. However, it has been found, by Fabry and Perot,[30] and by Kayser,[31] that Rowland's wave lengths contain errors, in long stretches of the spectrum, so large as to limit their usefulness. Similar conditions prevail as to Rowland's and others' wave lengths of the laboratory lines of iron, titanium, etc., used as comparison spectra in the line-of-sight problem. Some observers use Kayser's more accurate wave lengths for the iron comparison lines, and Rowland's wave lengths for the stellar lines; but the apparent slightly greater accuracy of resulting velocities is in my opinion more than counterbalanced by the confusion which results from the mixing of two systems. This part of the subject is at present in an unsatisfactory state, though the matter is not now vital, as it is not the absolute error which enters, but only a differential; and it is a question, in nearly all cases, of fractions of 1 km. in resulting velocities. Michelson's invention of the interferometer and the applications of it by himself,[32] Fabry and

[29] *A Preliminary Table of Solar Spectrum Wave-Lengths,* in *Ap. J.*, Vols. I to VI, 1895-1897.

[30] *Ap. J.,* **15**, 272, 1902.

[31] *Ap. J.,* **19**, 157, 1904, and later articles.

[32] *Tran. et Mem. du Bureau intern. des Poids et Measures,* **11**, 1895; *Zeitschrift für Instrumentenkunde,* **22**, 293, 1902.

Buisson,[33] Pfund[34] and Eversheim,[35] to the very accurate determinations of wave lengths, has led to comprehensive plans, under the auspices of The International Union for Co-operation in Solar Research,[36] for reconstructing wave-length tables to a marvellous degree of accuracy. It will be many years before these tables become available in our problem, through the solar spectrum as a basis. Further, the completion of the tables will not provide the line-of-sight observer with all the wave lengths necessary and sufficient for his purpose. From the standard tables, as a basis, he will probably find it desirable in many, or most, cases to construct tables of wave lengths in stellar spectra to conform to his own special requirements. To illustrate, the high dispersion of Rowland's solar spectrum separates two, three, or more lines in a group, whereas, the relatively low dispersion of the Mills 3-prism spectrograph and similar instruments causes these lines in many cases to blend into an apparently single line. A slight uncertainty would exist as to the wave length to be assigned to the composite line, no matter how perfectly the wave lengths of the individual lines might be known. Again, in passing from the solar stars in one direction toward the white stars, or in the other direction toward the red stars, the components of the blend referred to will in many cases change their relative intensities, and thus change the effective wave length of the blended result. The errors arising from these and similar sources will be dependent most largely upon the dispersive power of the spectrograph employed. The ideal method of procedure in the future will, I believe, be this: Let *observers* in this field select for use as *comparison* lines a list of carefully determined *single* lines in the spectra of iron, titanium, etc., as may be required, in the region of spectrum to be observed. With these as a basis of wave lengths, let all observers using instruments of substantially equal dispersive power, by coöperation, determine the effective solar and stellar wave lengths corresponding to that

[33] *Ap. J.*, **28**, 169, 1908.

[34] *Ap. J.*, **28**, 197, 1908.

[35] *Ann. der Physik*, **30**, 815, 1909.

[36] *Transactions of the Intern. Union*, **1**, 153, 1906.

power; one set of wave lengths for 3-prism instruments, possibly another set for 2-prism instruments, and a third set for 1-prism instruments. The two or three systems will differ appreciably. Following this suggestion, the methods of constructing such tables, beginning with the spectrum of the Sun, planet, or other body whose velocity is known, and with the isolated lines of hydrogen, helium, magnesium, etc., in the simpler stellar spectra, will readily appear to those concerned.

Spectroscopists have held in mind the question of eliminating the influence of errors in assumed wave lengths; and this, in effect, has been accomplished for spectra of purely solar type. Vogel placed a spectrogram of the Sun and one of a solar-type star obtained with the same instrument, side by side, and film to film, in the measuring microscope, making the lines in the two spectra, in effect, to coincide. The reference lines on the star spectrogram were then compared micrometrically with the corresponding absorption lines in the solar spectrum. This method[37] eliminated wave-length values, theoretically, but there was nothing gained, practically; for flexure effects and temperature effects existing in the star spectrogram were larger than errors in assigned wave lengths; and, in fact, the solar lines finally measured—those corresponding to the comparison lines—were in nearly all cases the lines least sharply defined in the spectrum.

One of my students, Curtiss, made an advance[38] on Vogel's method by constructing reduction tables, from micrometer measures of Sun or sky spectrograms, arbitrarily giving such values to the wave lengths of the absorption lines that each line would reproduce the correct velocity of the Sun with reference to the observer, as computed from the Earth's orbital[39] and diurnal motions. These values of the wave lengths were assumed, then, to be the same in all strictly solar types. Similar reduction

[37] *Publ. Astroph. Obs. Potsdam,* **7** (I), 36, 1892.

[38] *L. O. Bulletin,* **3**, 22, 1904.

[39] The radial velocity of the Earth's centre with reference to the Sun varies between + 0.51 km. per second, about April 8, and — 0.50 km. about October 13.

tables could be constructed for any type of spectrum, but such tables would be liable to yield results systematically in error because the radial velocities of the celestial objects upon which they were based would be unknown. The degree of systematic error to which such results would be liable would depend upon the number of well-defined lines in the basal spectra, such as those of helium, hydrogen, magnesium, etc., whose normal wave lengths could be assumed as known.

Tables such as these would, of course, be useful in measuring differences in the velocities of the same object, such as a star revolving in an elliptic orbit.

The greatest contribution to this subject is Hartmann's spectro-comparator,[40] an instrument which compares, in one eyepiece, a standard solar spectrogram with the stellar spectrogram of solar type; each spectrogram having the usual reference spectrum on either side of it. The micrometer screw moves one of the plates until the two reference spectra of bright lines are in coincidence; and again until the solar and stellar lines coincide. The micrometer difference of the two positions of coincidence gives at once, by simple computation, the difference of the radial velocities of Sun and star. That of the Sun being known, the velocity of the star becomes known. The result is free from wave-length error provided the star's spectrum is a duplicate of the Sun's. The method is not applicable to other spectral types; but within this type the instrument is of great utility.

The Hartmann comparator offers a splendid method of determining the differences in the velocities of the same object, by selecting one spectrogram of the series to serve as standard of reference in the measurement of all the other spectrograms of that object. Repeated measurement of the spectrogram selected as standard, by several observers, using the original form of measuring microscope, will give the basis for converting the measures of all the plates from relative to absolute velocities. The same considerations enable a selected spectrogram of any spectral class, such as Class A, B, or M, to serve as a basis of comparison for all other stellar spectra of the same class.

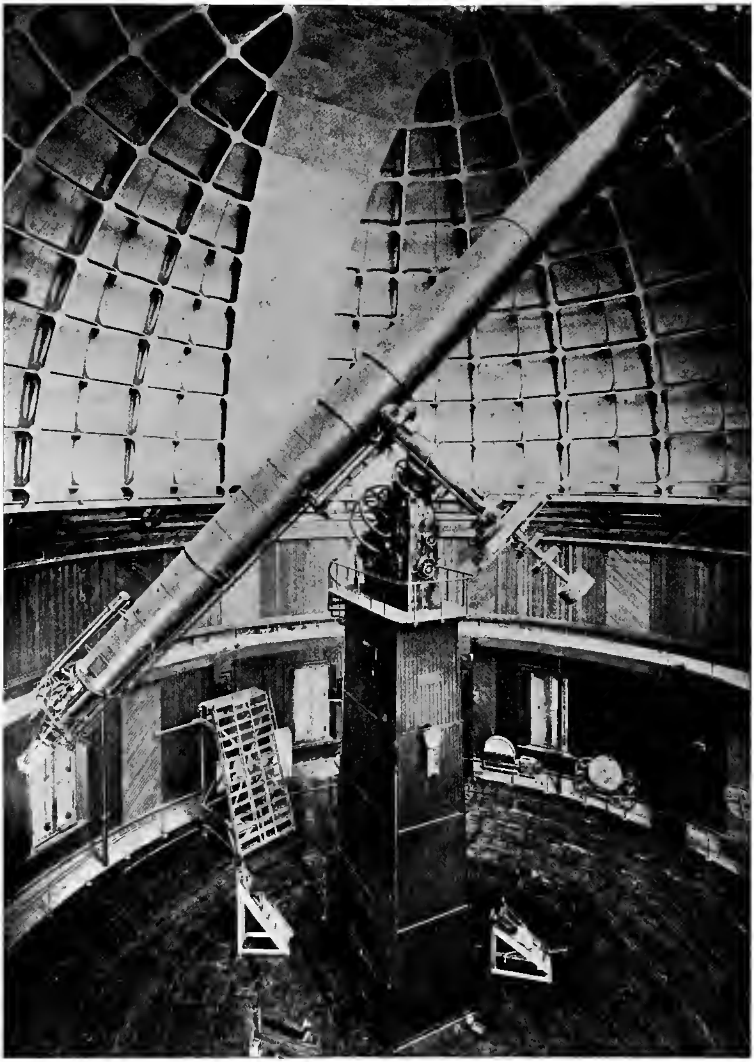
[40] *Publ. Astroph. Obs. Potsdam,* **18**, No. 53, 1906.

Of a different nature are questions relating to changes of wave lengths of *individual* lines relatively to neighboring lines, due to changed conditions in the source of the light radiations. It was noticed by Jewell, of Johns Hopkins University, that certain solar lines were shifted toward the red, in comparison with laboratory standards, not by the same amount for lines of different elements nor for the different lines of the same element, and therefore, clearly not as Doppler-Fizeau effects. A little later Humphreys and Mohler, of Rowland's laboratory, discovered that the wave lengths of lines in the arc spectra of the elements are functions, to a small but easily measurable degree, of the pressures of the atmosphere in which the arc is burning. This effect has been investigated under varying pressures from nearly zero up to 101 atmospheres, by Humphreys, Duffield, and many others; and several simple laws governing the displacement have been formulated.[41] In brief, an increase of pressure in the light source has the result that all the isolated lines—that is, lines not appearing in banded spectra—are displaced toward the red. For simple lines of the same element, these displacements are proportional to the increase of pressure and to the wave lengths of the lines. Further, they are functions of the temperature coefficients of expansion and of the atomic weights of the elements to which the lines belong; and, in many cases, of the melting points of the metals. Such line displacements in the Sun are small, due to pressures, according to Jewell, up to only two or three atmospheres, as a maximum, except in the case of the very broad lines. The situation is of concern in our line-of-sight problems. If lines in the solar spectrum are displaced in this manner, we cannot doubt that lines in other stars are similarly affected. Our Sun is believed to be an average-sized star. It may readily occur, in the stars vastly larger than our Sun, that the pressures under which the lines are formed are greater than in the case of our Sun; that the wave lengths are greater, in consequence, than we assume them to be; and that resulting radial velocities based upon the assumed wave lengths will be estimated slightly greater than they really are. For stars

[41] Humphreys, *Jahrbuch der Radioaktivität und Elecktronik*, **5**, 324, 1908.

less massive than our Sun, on the contrary, the observed velocities may be smaller than they really are. However, it seems equally probable that the lines of the same element, in stars of widely varying masses but of the same spectral type, may be formed under essentially equal pressures in all, at the various depths in their atmospheres which equalize the pressures. It has not been found possible to test stellar spectra for this pressure effect by *direct* methods; our telescopes have had too little power to let us use the high dispersion required. It is hoped that the large reflecting telescopes recently completed may be successfully applied to the problem. Unfortunately, while it is the pressure displacements which will affect our results, it is only the *differences* of these displacements for different closely related lines throughout the spectrum which will be observable; and from these very minute differences we should have to work back to the greater quantities wanted. Knowledge is lacking, but the displacements for all stars approximating the solar type are believed to be small. Systematic errors from this source cannot be considered as wholly absent from line-of-sight results. As to pressure effects in spectra very different from the Sun's spectrum, such as Classes B and A in the one direction, and Classes M and N in the other, nothing is known; but it would be surprising if accurate knowledge when finally obtained would not decidedly require us to take these effects into account. Radial velocities assigned to the stars in general may be appreciably in error from the pressure effect alone. We shall refer again to this important subject in the discussion of recent observations, as I think it quite probable we have strong evidence that the observed radial velocities for certain classes of stars are systematically too large on this account.

The velocity measurements on the sources of canal rays in vacuum tubes made by Stark and others afford an interesting example of the pressure effect, and at the same time illustrate the wide field of application of the Doppler-Fizeau principle. With one end of the tube pointed toward the slit of the spectrograph, the lines were observed strongly displaced toward the violet; and with the tube reversed in direction, the lines were correspond-



THE 36-INCH REFRACTOR WITH NEW MILLS SPECTROGRAPH ATTACHED ingly displaced toward the red. With the axis of the tube placed at right angles to the axis of the collimator, the lines were observed in their normal positions. Taking space for only one result, the velocities of the canal-ray light sources in a helium tube were observed to be 399 km. per second for the 4471.7 A line and 343 km. per second for the D_3 line.[42] These velocities correspond only to the conditions under which the experiment was made, as the observed displacements appear to be sensitive functions of the gas pressure within the tube.

It was thought for a time that the positions of the laboratory lines of the elements vary slightly with the voltage, amperage, and other constants of the electric current used in forming the spark or arc source. Researches in the Johns Hopkins physical laboratory[43] have recently shown that this is not the case, at least within the limits of accuracy of existing instruments to detect.

The studies of spark spectra by Schuster and Hemsalech, by Schenck, and by others indicate that there are Doppler effects within the spark structure,[44] due probably to the diffusion of generated vapors in the general direction from the cathode toward the anode; but as the line joining cathode and anode is always parallel to the slit-plate of the spectrograph, in radial velocity observations, any Doppler effects from this source may be considered negligible.

Whether such changes from normal positions of the lines as accompany the Zeeman effect will have to be taken into account when dealing with stellar spectra is a question for the future. Gmelin[45] has shown that the central component of a bright line trebled in a magnetic field is shifted toward the red, minutely, in proportion to the square of the magnetic force. Hale has

[42] A condensed *résumé* of the results obtained by many investigators on eleven of the principal elements is given in *Ann. der Physik,* **26,** 829, 1908.

[43] Kilby, *Ap. J.,* **30,** 263-266, 1909. References are given in Kilby's paper to the more important literature of the subject.

[44] *Phil. Trans.,* **193,** 189, 1900; *Ap. J.,* **14,** 116, 1901; *Conduction of Electricity through Gases,* 2d Ed., 520, 1906.

[45] *Phys. Zeitschrift,* **9,** 212-214, 1908.

made the brilliant discovery of Zeeman effects in the spectra of sun spots;[46] but in the case of a star for which an entire hemisphere is integrated into a point image, we now have no reason to believe that this effect would be appreciable.

If our Sun is situated in a strong magnetic field—an attractive subject which has been carefully considered by many physicists—it is quite possible that the Fraunhofer lines should show appreciable Zeeman effects; but as no such effects have yet been observed it seems hopeless to expect that they could be observed in the distant stars with the low dispersive power which must be used upon the point images. The bright hydrogen lines in the spectrum of the well-known variable *o Ceti* are sometimes trebled, and it was suggested by Miss Clerke more than a decade ago that this might possibly be a Zeeman effect. At the recent maximum of this star, my colleague, Wright, photographed the triple $H\gamma$ bright line through a variety of analyzing optical pieces, but no traces of polarization effects could be detected in any one of the three components.

The most recent question calling for consideration in connection with radial velocity determinations is that of a possible dispersion of light in its passage through interstellar space, announced independently and almost simultaneously by Nordmann and by Tikhoff. Observing certain variable stars, they were convinced that the recorded minima of brightness were more and more retarded in point of time as they observed these stars in light of shorter and shorter wave lengths.

Belopolsky and Tikhoff[47] report that the minima of *β Aurigæ* occur 0.015 day earlier in blue light than in violet light. Albrecht has observed for T *Vulpeculæ* that "Wilkens's[48] light curve, determined by a photographic method, gives the epoch of maximum 0.4 day earlier than the curves which were determined by visual methods." The minimum in Albrecht's radial velocity curve for this star falls between the times of visual and photographic maximum light, but nearer to the photographic maxi-

46 *Ap. J.*, **28**, 315, 1908.

47 *Publ. School of Mines Jekatermoslaw*, 32-33, 1905.

48 *L. O. Bulletin*, 4, 137, 1907.

mum. Nordmann[49] estimates that in *β Persei* the λ6800 minimum precedes the λ4300 minimum by 16 minutes, and that the λ5400 minimum precedes the λ4300 minimum by 9 minutes. Schlesinger and Curtiss[50] observed that the photometric minimum of *β Persei* in visual rays lags from one and a half to two hours behind the time required by the velocity determinations based upon blue rays; and Schlesinger finds that Belopolsky's[51] observations of the same star give a result in accord with his own. Schlesinger[52] has just reported a similar effect amounting to at least an hour in the system of δ *Libræ*. The evidence quoted seems to be somewhat contradictory.

Nordmann and Tikhoff reached the same conclusion, that the velocity of stellar light is a function of its wave length, owing to the retardation of an interplanetary medium. Such an effect, if existent, would enter seriously into the observed velocities of many and perhaps all distant celestial objects. The question has been discussed at length, and current opinion inclines strongly to the view that the thesis has not been maintained. It is not certain even that the observed differences of phase have not their explanation in purely photographic or allied causes, or in the conditions existing in the variable stars themselves.

Going directly to Nordmann's results for one variable star, and to Tikhoff's results for another variable star, we find, as Lebedew[53] pointed out, that one set of observations yields 30 times as much retardation as the other. Under ordinary circumstances this 30-fold discrepancy would justify dropping the subject; but here the uncertainties in the distances of the two stars enter directly; and if there is any real lag of the minima of variable stars, with decreasing wave lengths of light used, the general subject remains important and should be investigated. Lebedew recalls that the electro-magnetic theory of

[49] *Comptes Rendus*, **146**, 266 and 383, 1908.

[50] *Publ. Allegheny Obs.*, **1**, 32, 1908.

[51] *Mitt. Pulk. Stern.*, **1**, 103, 1906.

[52] *Publ. Allegheny Obs.*, **1**, 127-129, 1909.

[53] *Ap. J.*, **29**, 105, 1909.

dispersion, or, in fact, any known theory of dispersion, demands also absorption; and he deduces the result, unquestionably correct, that the minimum dispersion in space reported to exist by Nordmann and by Tikhoff would also require so heavy an absorption of light in space that not only would the stars be invisible, but the Sun itself would be snuffed out.

Frost has given negative evidence on the same question. Measuring the radial velocities of the spectroscopic double star, μ *Orionis*, which revolves at high speed in a short-period orbit,—only 0.77 day,—he was unable to detect any differences between the velocities afforded by the separate lines distributed throughout a long range of spectrum, as should have been the case if interstellar dispersive effects were large; though, if Lebedew is correct, a dispersive effect of the magnitude which could have been detected by this method would have left no unabsorbed light to form the spectrum. The subject is in need of accurate and experienced observations on variable star minima, as given by widely different parts of the spectrum.

W. Michelson[54] has shown that the spectral lines of a light source, even absolutely at rest in reference to the observer, must be changed from their normal positions if an absorbing medium lying between the light source and the observer is changing its thickness or its indices of refraction during the progress of observations. This fact has been favorably considered by some students of the Sun in explanation of apparently high or rapidly changing velocities[55] observed for certain solar details of structure. It would seem that the integrated spectra of point-image stars would not be subject to frequent disturbances from this source.

Julius has developed very extensively the possible bearing of anomalous dispersion upon the interpretation of astronomical phenomena. He is of the opinion,[56] for example, that anomalous dispersion may cause the lines in the spectrum of the Sun's edge to be displaced slightly toward the red with reference to their

[54] *Ap. J.*, **13**, 192, 1901.
[55] Fényi, *Ap. J.*, **19**, 70, 1904.
[56] *Le Radium*, **7**, 281, 1910.

positions in the spectrum of the Sun's centre. Whether this effect, if real, would be appreciable in the integrated spectrum of a distant star is doubtful.

While the pressure and other effects which we have described in the last few pages may be troublesome in the problems immediately before us, we should not regard their existence as causing unfortunate complications, for in them we have promise of future means of analysis of great power and value in studies on conditions existing in the stars. Going back over the history of science, we recall that apparent complications, finally reduced to law, were the source of much of our present power to interpret conditions existing in the stars.

CHAPTER III

ROTATIONAL VELOCITIES IN THE SOLAR SYSTEM

RADIAL VELOCITIES OBTAINED FOR INDIVIDUAL STARS

Before passing on to considerations of the motions of the distant stars, let us note several interesting applications of the Doppler-Fizeau principle to studies on members of the solar system.

In CHAPTER II (pp. 56-57) reference was made to radial velocity measures of the Sun's rotation. Of unusual merit is Dunér's[1] solution of the problem, based upon observations made between 1887-1901, inclusive. From the relative displacements of a few lines in the spectra of the two limbs of the Sun, with reference to neighboring telluric lines of oxygen, and with the slit of his instrument directed to latitudes differing fifteen degrees successively, he observed average velocities of approach and recession of the east and west limbs, respectively, as quoted in the second column of the accompanying table.

ϕ	v	O-C	P
0°.4	2.09 km.	0.00 km.	24.2 days
15 .0	1.97	—0.01	24.7
30 .0	1.70	+0.02	26.1
44 .9	1.27	+0.01	28.3
60 .0	0.80	—0.01	31.1
75 .0	0.39	0.00	33.4
(90 .0)	——	——	(34.3)

Faye had earlier succeeded in representing the law of solar rotation, as based upon sun-spot observations, by an equation of the form:

$$\xi \cos \phi = a \cos \phi + b \cos^3 \phi,$$

[1] *Nova Acta Soc. Sci. Upsaliensis*, 1 (IV), 1907.

in which ξ is the angular rotational velocity per mean solar day at any latitude ϕ, and a and b are constants. Determining the values of a and b by the method of least squares, from the six mean spectrographic velocities, Dunér deduced the following equation:

$$\xi \cos \phi = 10^\circ.491 \cos \phi + 4^\circ.410 \cos^3 \phi. \qquad (17)$$

This equation represents the observed radial velocities within the residual values assigned in column three of the table. Corresponding rotational periods, expressed in mean solar days, as computed from the formula, are for the different latitudes as in the last column of the table.

The results of similar measures of Fraunhofer lines made in recent years at Edinburgh by Halm,[2] and at Mount Wilson, principally by Adams,[3] with the help of powerful apparatus, are in remarkably good accord with Dunér's. However, when Adams's measures were based upon the Fraunhofer lines whose origins are thought to lie high in the solar atmosphere, quite different laws of rotation were deduced. A few of the results are quoted in the following table.

ϕ	From Sun Spots	Dunér Fraunhofer Lines	Adams Many Fraun. Lines	Adams Calcium 4227A	Adams Hydrogen Hα
	d	d	d	d	d
0°	25.0	24.2	24.6	23.9	24.0
15	25.4	24.7	25.2	23.2	24.5
30	26.3	26.1	26.4	25.3	25.5
45	27½	28.3	28.1	26.5	26.3
60	——	31.1	31.2	28.8	27.8
75	——	33.4	33.2	27.4	26.4
(90)	——	(34.3)	——	——	——

The second column contains the average periods in mean solar days, as derived from very extensive sun-spot observations. Dunér's spectroscopic periods, based upon a few Fraunhofer lines, are given in column three; Adams's periods, depending upon many Fraunhofer lines, are quoted in column four; upon

[2] *Trans. Royal Soc. Edinburgh*, 41.

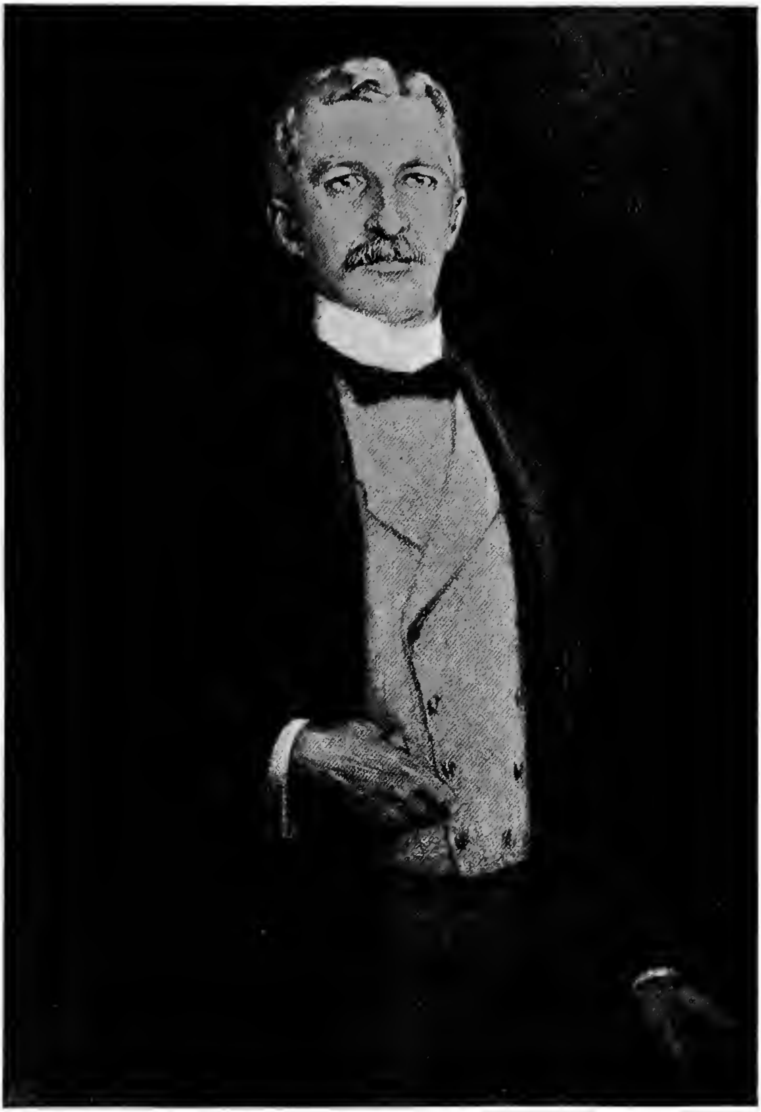
[3] *Ap. J.*, **29**, 110, 1909.

the calcium 4227 A line, in the next to the last column, and upon the hydrogen Hα line, not including observations of this line at the extreme limb of the planet, in the last column. The fact that the solar rotation period, derived from the sun spots and the ordinary Fraunhofer lines, increases with latitude is surprising enough, but that origins of observation in the higher atmospheric strata should have shorter and more constant periods than are yielded by the deeper strata is a mysterious fact, indeed. We can admire the exceedingly accurate measures upon which these rotation periods depend, but it is a question to what extent the apparent periods are real. It seems possible that factors at present unknown may be seriously influencing the spectrographic measures and preventing their correct interpretation.

The Doppler-Fizeau principle lends itself to a great variety of observations upon apparent motions within the solar structure: illustrated at one point by Evershed's[4] success in detecting motions outward from sun-spot centres, as if gases and vapors rise through the core of a sun spot, so to speak, and then spread in all directions over the Sun's surface; at another point, by the efforts of Deslandres,[5] Campbell,[6] and Newall[7] to measure the rate of rotation of the corona; and again by the displacements, amounting to more or less violent distortions, of the bright lines in the spectra of the chromosphere and prominences observed by Young and many others. Lack of space prevents adequate treatment of these and analogous subjects, inasmuch as our present purpose relates more directly to motions of celestial bodies as a whole.

It has long been known that the spectroscope affords a valuable means for observing the rotational velocities of the planets. If the slit of the spectroscope be directed upon the planet in such a way that the projected axis of rotation coincides with the slit, the lines in the planet's spectrum will of course show no rota-

4 *Mem. Kodaikanal Obs.*, 1, 49, 1909.
5 At the eclipse of 1893, in Senegal.
6 *Ap. J.*, 10, 186, 1899.
7 *Proc. Royal Soc.*, 64, 56, 1898.



James Edward Keeler, 1857-1900

tion effects; but if the slit be placed across the planet's image in any other direction, the spectrum lines will be inclined to their normal positions, for one end of each line will be displaced to the violet corresponding to the velocity of approach of one limb of the planet, and the other ends of the lines will be displaced to the red, owing to the velocity of recession of the corresponding limb.

Maunder observed the displacements of the Fraunhofer lines in *Jupiter's* spectrum, using visual methods,[8] taking note of Niven's[9] demonstration that, in the case of a rotating planet shining by reflected sunlight, the displacement of the lines is the sum of the geometrical rotational effects both with reference to the Sun as the source of light and with reference to the observer. Deslandres's treatment[10] of the subject brought out several important points; for example, that Fraunhofer lines displaced by planetary rotations remain straight lines, and that rotational velocities involved can be determined most accurately and conveniently by measuring the angles between the comparison lines and the inclined planetary lines. Deslandres was apparently the first to establish by spectrographic methods, in 1893-1895, that the rotational velocity of *Jupiter* is in satisfactory agreement with the velocity computed from the known diameter and rotation period of the planet. Deslandres's observed equatorial velocity of *Jupiter* was 12.1 and the computed velocity 12.4 km. per second. Similar observations of *Jupiter* have been made by Belopolsky[11] and others; all of the observations reproducing the computed velocity within the limits of unavoidable error. In a footnote are developed the equations which connect a planet's equatorial and spectrographic velocities.[12]

[8] *Observatory*, **8**, 118, 1885.

[9] *Mon. Not.*, **34**, 345, 1874.

[10] *Comptes Rendus*, **120**, 417, 1895.

[11] *Ast. Nach.*, **139**, 209, 1895.

[12] Let A B C be the planet's equator; and let

i = the angle at the planet between the Sun and Earth (tabulated in the Ephemeris for *Mercury, Venus, Mars*, and *Jupiter*),

AC = the section of the equator visible to the observer on the Earth,

I_s = the angle that the Sun is above or below the planet's equator, Perhaps our most interesting application of the Doppler-Fizeau principle was that made by Keeler to the system of *Saturn* in 1895, with a 13-inch telescope in the smoky atmosphere of Allegheny. The investigations of Clerke Maxwell, in 1867, had shown that the rings could not long exist if they were solid, liquid, or gaseous; but that to remain permanently in position they must be a collection of small bodies, each body an independent tiny moon revolving around the planet in its own orbit. Accordingly, the inner edge of the ring system must revolve more rapidly than the outer edge. Keeler placed the slit of his spectrograph across the system of *Saturn*, upon the long axis of the projected rings, as indicated by the parallel lines drawn across the image of the ball and rings in Figure 3. After an exposure of two hours on *Saturn's* spectrum, the spectrum of the Moon was photographed on each side of the spectrum of *Saturn* and his rings, and nearly in contact with it. Quoting Keeler's description:[13]

I_e=the angle that the Earth is above or below the planet's equator,
V_o=the equatorial rotational velocity of the planet in km. per second,
v_a=the spectrographic velocity of A,
v_c=the spectrographic velocity of C,
v_s=the excess of v_c over v_a,

then

$$v_c = V_o \cos I_s + V_o \cos i \cos I_e,$$
$$v_a = -V_o \cos i \cos I_s - V_o \cos I_e,$$

and

$$v_s = V_o (1 + \cos i)(\cos I_s + \cos I_e). \tag{18}$$

If the Sun and Earth are in the plane of the planet's equator equation (18) reduces to

$$v_s = 2\, V_o (1 + \cos i). \tag{19}$$

If, further, the planet is in opposition or in superior conjunction the equation reduces to

$$v_s = 4\, V_o. \tag{20}$$

As the differential velocity is then a maximum, and is therefore most easily measurable, a superior planet should be observed at opposition. The inferior planets, *Venus* and *Mercury*, should be observed as near superior conjunction as the atmospheric and other conditions will permit, when the Sun is in or just below the horizon.

[13] *Ap. J.*, 1, 417, 1895.

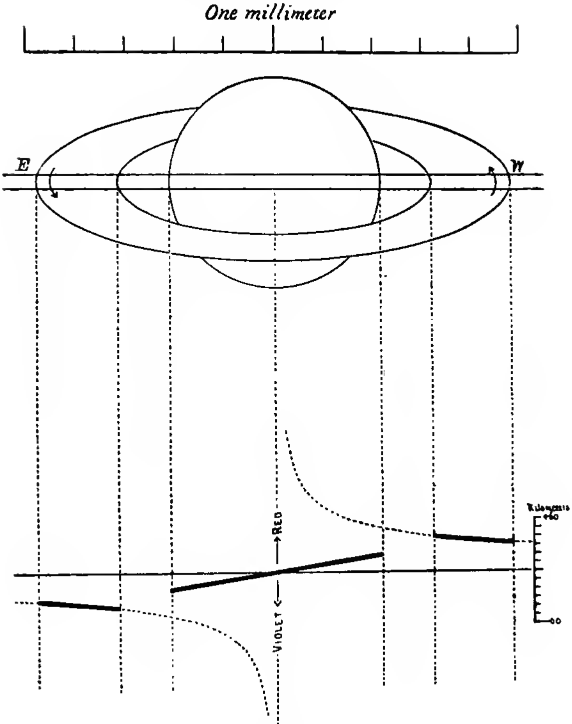


Figure 3

"The planetary lines are strongly inclined, in consequence of the rotation of the ball, but the lines in the spectra of the ansæ do not follow the direction of the lines in the central spectrum; they are nearly parallel to the lines of the comparison spectrum, and, in fact, as compared with the lines of the ball, have a slight tendency to incline in the opposite direction. Hence the outer ends of these lines are less displaced than the inner ends. Now it is evident that if the ring rotated as a whole the velocity of the outer edge would exceed that of the inner edge, and the lines of the ansæ would be inclined in the same direction as those of the ball of the planet. If, on the other hand, the ring is an aggregation of satellites revolving around *Saturn*, the velocity would be greatest at the inner edge, and the inclination of lines in the spectra of the ansæ would be reversed. The photographs are, therefore, a direct proof of the approximate correctness of the latter supposition.''[14]

[14] Keeler, *Ap. J.*, 1, 418, 1895.—''To apply more precise reasoning to the subject under consideration, let us determine the form of a line in the spectrum of *Saturn* when the slit is in the major axis of the ring, on the assumption that the planet rotates as a solid body and the ring is a swarm of particles revolving in circular orbits according to Kepler's third law. . . . . The upper part of Fig. 3 represents the image of *Saturn* on the slit of the spectroscope (the scale above it applies to the instrument used at Allegheny), and the narrow horizontal line in the lower part of the figure represents an undisplaced line in the spectrum, or solar line. (The curvature of the line in a prismatic spectrum need not be considered.) Let this line be taken as the axis of x, and the perpendicular line through its centre as the axis of y. The red end of the spectrum is supposed to be in the direction of the positive axis of y, and the camera and collimator of the spectroscope are assumed to have the same focal length, so that the breadth of the spectrum is equal to the length of the illuminated part of the slit. Corresponding points in the slit and spectral line will then have the same value of x.

''Now let x, y, be the coördinates of a point on the displaced line,

v = velocity of a point corresponding to x, y in the line of sight,
V_o = velocity of a point on the equator of *Saturn*,
α = angle between the line of sight and the radius of *Saturn* which passes through the point corresponding to x, y,
2ρ = width of spectrum,
β = elevation of Earth (and Sun) above the plane of the ring. (The slight error introduced by the assumption that the Earth and Sun are in the same direction from *Saturn* is inappreciable, when *Saturn* is anywhere near opposition.)

''The displacement of y is proportional to the velocity in the line of sight. Then we have

$$x = \rho \sin \alpha,$$
$$y = av = aV_o \sin \alpha \cos \beta,$$
$$\frac{y}{x} = \frac{aV_o}{\rho} \cos \beta = \text{constant}. \qquad (21)$$

''Hence the planetary line is straight, but inclined to the solar line at an angle

$$\phi = \tan^{-1} \frac{aV_o}{\rho} \cos \beta. \qquad (22)$$

''To determine the form of a line in the spectrum of the ring, regarded as a collection of satellites, we have, by Kepler's third law,

$$P^2 = cR^3,$$

or, since $P V = 2 \pi R$,

$$V^2 = \frac{4 \pi^2}{cR}.$$

"Since x is proportional to R and y to v (where v = velocity in the line of sight = $V \cos \beta$), we may write

$$xy^2 = b, \qquad (23)$$

which is the equation to the curve of which the lines in the spectrum of the ring are a part. The curve is represented by the dotted line in the figure; it is symmetrical with respect to the axis of x, but only the upper branch has a physical meaning, and the curve corresponding to the other half of the image is obtained by taking both x and y with negative values.

"In the equation $V = \frac{k}{\sqrt{R}}$, $\log k = 3.7992$ for the Saturnian system, R being expressed in kilometers and V in kilometers per second. The computed motions of different parts of the system are given in the following table. The gauze ring is not considered, as its spectrum does not appear on the photographs; the rings known as A and B are not separately distinguishable.

Object	R	Period of a Satellite at Distance R	Velocity in Equatorial Plane	Velocity in Line of Sight Apr. 10, 1895
	km.	h	km.	km.
Outer edge of ring	135,100	13.77	17.14	16.35
Middle of ring	112,500	10.46	18.78	17.91
Inner edge of ring	89,870	7.47	21.01	20.04
Limb of planet	60,340	4.11	25.64	24.46
		Rotation		
Limb of planet	60,340	$10^h.23$ (A. Hall)	10.29	9.82

"With the values given in the above table, and others which do not correspond to actual points in the system, the dotted curves were platted. For the ordinates, however, twice the values in the last column were taken, since the displacement of a line, due to motion in the line of sight, is doubled in a case of a body which shines by reflected and not by inherent light, provided (as in this case) the Sun and the Earth are in sensibly the same direction from the body. . .

"The most accurate method (due to Deslandres) of measuring the relative displacement of the opposite ends of a line in the spectrum of the planet is to measure the angle ϕ.

"The value of ϕ depends upon the dispersion and other constants of the spectroscope employed, as well as upon quantities which are independent of the instrument. If we let L = the velocity of light in kilometers per second = 299,860; λ = the wave length of the measured line in tenth-meters; D = the linear dispersion of the photographed spectrum at the position of the same line, expressed in tenth-meters per millimeter; ρ = half the width of the spectrum in millimeters;—we have by Doppler's principle, allowing for the double effect already mentioned,

$$y = x \tan \phi = \frac{2\,v\,\lambda}{D\,L},$$

or,

$$v = x \tan \phi\, D\, \frac{L}{2\,\lambda},$$

from which we obtain the velocity in the line of sight at the limb ($V_o \cos \beta$) by placing $x = \rho$. That is,

$$V_o = \frac{\rho\, D\, L \tan \phi}{2\,\lambda \cos \beta}. \qquad (24)$$

"The value of ρ is computed from the angular semi-diameter of the planet, and the ratio of the focal lengths of the camera and collimator of the spectrograph. . . . .

"The relative displacement of a line in the spectra of the ansæ is measured directly, the micrometer wire having first been placed parallel to the lines of the comparison spectrum. If δ is this measured interval, the mean velocity of the ring is

$$V_o^1 = \frac{D\,L\,\delta}{4\,\lambda \cos \beta}. \qquad (25)$$

"The results of all the measurements on the spectrogram of April 9, 1895, are given in the following tables:

λ	D	ϕ	Velocity of Limb	C — O	δ	Mean Velocity of Ring	C — O
A.	A.		km.	km.	mm.	km.	km.
5324.3	27.55	3° 36′	10.92	—0.63	0.0456	18.54	+0.24
5328.4	27.65	4 24	13.39	—3.10	0.0464	18.92	—0.14
5371.6	28.77	3 11	9.99	+0.30	0.0404	17.01	+1.77
5383.5	29.09	3 20	10.56	—0.27	0.0362	15.37	+3.41
5429.9	30.37	3 8	10.27	+0.02	0.0402	17.67	+1.11
			11.03	—0.74		17.50	+1.28

[The spectrogram of April 10, 1895, yielded 9.58 km. for the velocity of the limb and 18.52 km. for the mean velocity of the ring.]

"The results from both photographs are

Observed velocity of the limb 10.3 ± 0.4 km. per second
Mean velocity of ring 18.0 ± 0.3 km. per second;

the computed values being 10.29 and 18.78 km. per second respectively."

Immediately following the announcement of Keeler's results, they were confirmed by Belopolsky,[15] by Deslandres,[16] and by Campbell.[17]

The scale of Keeler's photographs was so small on account of the small telescope employed that he did not attempt to measure the inclinations of the lines in the spectra of the ansæ. He merely determined that the inner edge of the ring system is travelling more rapidly than the outer edge. The large scale of the photographs secured with the 36-inch refractor and Mills spectrograph enabled me to measure the inclination of the ansæ lines in the direction opposite to that of the planet's lines. The excess of the velocity of the inner edge over that of the outer edge was found by measuring the inclination ϕ of the lines in the ring spectrum to the lines in the lunar spectrum, and reducing to kilometers per second by the formula

$$\Delta V_o = \frac{\rho D L \tan \phi}{2 \lambda \cos \beta}, \tag{26}$$

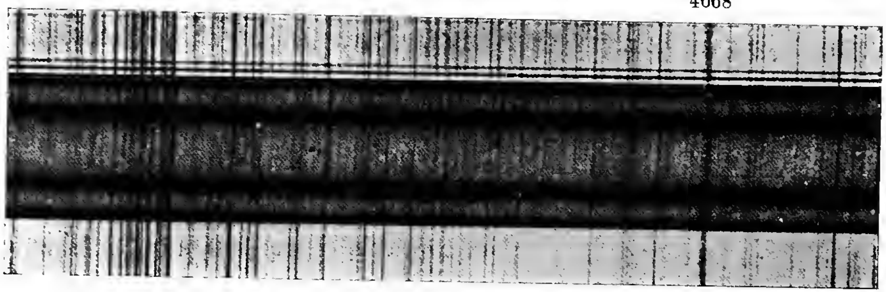
in which ρ is the width of the ansa in millimeters. The inclinations of ten lines in both ansæ and the corresponding excess velocity for the inner edge, as determined from the spectrogram of May 10, 1895, are quoted in the following table:

λ	D	ϕ	Excess for Inner Edge	C — O
A.	A.		km.	km.
4340.6	12.40	1° 8′	3.03	+0.84
4359.8	12.86	1 30	4.14	—0.27
4367.8	13.02	1 29	4.14	—0.27
4369.9	13.05	1 7	3.12	+0.75
4371.3	13.08	0 59	2.75	+1.12
4395.3	13.56	0 50	3.03	+0.84
4404.9	13.76	1 9	3.34	+0.53
4415.3	13.94	0 49	2.41	+1.46
4425.6	14.12	1 12	2.84	+1.03
4427.5	14.16	0 56	2.79	+1.08
			Means 3.16	+0.71

15 *Astr. Nach.,* **139,** 1, 1895.

16 *Comptes Rendus,* **120,** 1155, 1895.

17 *Ap. J.,* **2,** 127, 1895.



Class G. Spectrum of *Saturn* and rings, with the *Moon's* spectrum for comparison
1909, November 28

The computed excess of a supposed satellite at the inner edge of the ring system over that of a supposed satellite at the outer edge was 3.87 km., the value of ρ for the date being 0.3410 mm. The excess of the computed velocity over the observed velocity, as determined from the ten lines, is given in the last column of the table. The excess velocities, computed from three spectrograms of dates May 10, May 14, and May 16, 1895, respectively, were 3.16, 3.06 and 3.17 km. per second. The mean of the three values, 3.13 km., differs 0.75 km. from the computed value.

In the year 1900, Belopolsky[18] applied the spectrographic method to the study of the rotation period of *Venus.* The average equatorial velocity observed by him, 0.5 km. per second, corresponds to a rotation period of approximately twenty-two hours, from west to east. A quite different result was obtained by Slipher[19] in 1903. His extensive observations led with remarkable accordance to a rotational velocity vanishingly small. The corresponding rotation period would exceed several days; it could, in fact, equal the period of revolution around the Sun, 225 days, which Schiaparelli, from his long-continued observations of surface markings on the planet, announced to be the planet's rotation period. It is desirable that the spectrographic rotational velocity of *Venus* be re-observed with higher dispersion, if possible.

As a check upon the accuracy of his results for the rotation of *Venus,* Slipher[20] used the same instrument and methods in measuring the known rotational velocity of *Mars.* The period from the spectrographic observations came out $25^h\ 35^m$, or just one hour longer than the true period.

Attempts have been made to measure spectrographically the rotation periods of *Uranus* and *Neptune,* at present unknown; but practical difficulties, in part due to the intrinsic faintness of these planets, have prevented definite results. Successful applications of the spectrograph to determinations of the rota-

[18] *Astr. Nach.,* **152,** 263, 1900.
[19] *Lowell Obs. Bull.,* **1,** 9, 1903.
[20] *Lowell Obs. Bull.,* **1,** 19, 1903.

tional periods of *Mercury, Uranus,* and *Nepture* remain as future problems.

It is extremely desirable, in current studies of the stellar system, that we know, as soon as practicable, the radial velocities of all stars bright enough for observation with 3-prism spectrographs, such as that described in the first chapter. The *larger* telescopes now engaged in this work can observe satisfactorily down as far as the fifth visual magnitude; that is, in the entire sky, about 1600 stars, of which 1100 are within reach of northern observers. In measuring the radial velocity of a star, as in other departments of physical science, it is advisable and necessary to repeat an observation four or five times, taking the mean of the values obtained on different nights, in order to reduce the effects of unavoidable errors, and to give confidence in the observed result. Now astronomical observation is dependent upon the weather, over which we have no control. Not all clear nights, even, are workable. Freedom of the air from confused currents of unequal temperatures is more important than great transparency of the air. Some nights on which the stars shine with great brilliancy are absolutely useless to observers with telescopes, on account of the non-homogeneous qualities of the air strata. Because of atmospheric imperfections and other vicissitudes not necessary to describe, the securing of four *satisfactory* spectrograms of a star will, on the average, necessitate five or six exposures. The exposure times vary from a few minutes for the first-magnitude stars up to 2½ hours for the fifth visual magnitudes, of the solar type, with average exposure of fully 1½ hours—recalling that the faint stars are much the more numerous. Six plates constitute an average night's work, from sunset to sunrise. If the telescope is available three nights per week to the spectroscopist, he can count on ninety usable nights per year. Four good spectrograms of each of the 1100 stars referred to thus require from ten to twelve years for observation alone. The dark-room work on the plates, the measurement and mathematical reduction of the plates, and the keeping of the apparatus in adjustment, not to mention extensive and frequent experiments for developing new ideas and improving the methods, consume fully threefold as much time as the night work of observation. Further, the unexpected discovery, during the progress of the investigation, that at least one star on the average out of every four is a double star with components so close together as to be invisible, each such double requiring twenty-five or more spectrograms for its proper preliminary study—this unexpected development has more than doubled the labor, both night and day. Fortunately, the pressing need for a knowledge of radial velocities of the stars, and the untold fruitfulness of this field of research, have brought several of the largest telescopes into the problem. In this country, the Allegheny, Lick, Lowell, and Yerkes Observatories; in Canada, the Ottawa Observatory; in Germany, the Bonn and Potsdam Observatories; and in Russia, the Pulkowa Observatory; these eight institutions have been contributing a part of their resources for several years to this labor. The observatories at Columbus, Ohio, at Cambridge, England, and at Paris, France, were valued contributors for a short time, but their energies have been turned in other directions.

In 1892 I called attention to the important fact that we should not be able to make a satisfactory start in the study of the stellar system upon the basis of stellar radial velocities until we should have determined "the velocities of several hundred stars distributed fairly uniformly over the celestial sphere."[21] As no other plans seemed to be forming for the purpose of securing these observations of the southern stars, I looked forward, in 1894, to the organizing and conducting of an expedition to the Southern Hemisphere for this purpose. When the subject was presented to the late Mr. D. O. Mills in 1900, he was pleased to provide funds for the equipment and maintenance of a suitable observing station in the Southern Hemisphere. The D. O. Mills Observatory, located on the summit of Cerro San Cristóbal, in the suburbs of Santiago, Chile, is equipped with a 92 cm. (36¾-inch) reflecting telescope, and with spectrographs of 3-prism, 2-prism and 1-prism dispersions, for investigations exclusively in the line-of-sight field. Since 1903, successively in charge of

[21] *Astr. and Astroph.*, 11, 321, 1892.

my colleagues, Wright, Curtis, and Moore, this expedition has measured the radial velocities of some 500 stars south of Declination —30°. In the past two years the Cape of Good Hope Observatory has become a second southern contributor. Each of these ten observatories—eight in the Northern and two in the Southern Hemisphere—is pushing some phase of this work with energy. It is hoped that all stars down to the fifth visual magnitude, through the efforts of these ten observatories in both hemispheres, will have been observed in two years from date, except in the cases of perhaps 300 binary systems whose investigation must extend over many additional years, partly because of the numerous observations required and partly because it will be necessary to await the completion of revolution periods covering several years.

Let us examine the radial velocity results secured for representative stars, in order to gain an idea of the accuracy attainable for stars of different magnitudes and spectral types, and of the magnitudes of the speeds to be expected. The following are collected for *α Cassiopeiæ*, visual magnitude 2.4, whose spectrum, of Class K, is excellent for measurement. (A negative velocity indicates approach; a positive velocity, recession, from the solar system.)

α CASSIOPEIÆ

Date			Radial Velocity	Observatory
1885			+90 km.	Greenwich (visual)
1887			+58	Greenwich (visual)
1890	Feb.	20	—15.9	Potsdam (photographic)
	Feb.	21	—14.5	Potsdam (photographic)
	Mean		—15.2	
1896	Nov.	12	— 4.1 km.	Lick
	Dec.	8	— 4.1	Lick
	Dec.	17	— 4.9	Lick
	Dec.	24	— 4.2	Lick
1898	Nov.	1	— 3.0	Lick
1900	Sept.	18	— 3.6	Lick
1901	Oct.	22	— 3.0	Lick
1907	Aug.	12	— 4.2	Lick
	Mean		— 3.9 ± 0.15 km.	

The D. O. Mills Observatory, Cerro San Cristóbal, Santiago, Chile

	Date		Radial Velocity	Observatory
1897	Sept.	4	—2.8 km.	Columbus
	Sept.	14	+1.6	Columbus
1898	Nov.	2	—4.0	Columbus
	Nov.	7	—2.3	Columbus
	Nov.	11	+5.7	Columbus
	Nov.	20	—2.1	Columbus
	Dec.	8	+0.6	Columbus
1903	Sept.	24	+0.7	Columbus
	Oct.	12	—3.5	Columbus
	Oct.	18	—2.7	Columbus
	Oct.	24	—2.9	Columbus
	Oct.	25	—4.4	Columbus
	Mean		—1.3	
1904	Oct.	19	—2.4 km.	Bonn
	Oct.	27	—1.9	Bonn
1905	Nov.	30	—3.1	Bonn
1906	Dec.	23	—2.5	Bonn
	Mean		—2.5	
Systematic correction			—1.0	
Corrected mean			—3.5	

The Greenwich velocities were measured visually, and they illustrate the futility of visual methods, using small telescopes, in this delicate field of observation.

The results obtained photographically at the other four observatories differ considerably, but there is no apparent reason to suspect that the velocity changes. The pioneer observations at Potsdam were accordant as to each other, but a systematic error of 11 or 12 km. seems probable. The Columbus observations are discordant amongst themselves, and the mean result is perhaps in error both fortuitously and systematically. The Lick and Bonn series are very accordant within themselves, but their means differ 1.4 km. If we apply the systematic correction — 1.0 km. to all observations of this and other stars made at Bonn, as recommended[22] by the Bonn observers, the Lick and Bonn means, — 3.9 km. and — 3.5 km., are in remarkably close agreement.

[22] *Ap. J.*, 27, 324, 1908.

The next observations are of the second brightest star in the sky, *Canopus* (α *Carinæ*), whose spectrum, of Class F, is probably as favorable for accurate measurement as any we could select.

CANOPUS (α CARINÆ)

1903	Sept.	21	+20.7 km.	Chile, D. O. Mills
1904	Jan.	19	+20.9	
	Sept.	6	+20.1	
	Sept.	11	+20.0	
1905	Jan.	6	+20.3	
	Jan.	6	+20.7	
	Jan.	6	+20.1	
	Jan.	6	+21.2	
	Nov.	9	+20.0	
	Nov.	24	+21.1	
	Mean		+20.6	

No other results are available for this star. The extreme range is 1.2 km., and the probable error of the mean result is ± 0.10 km.

Here are results for the third-magnitude star ε *Pegasi*, whose spectrum, of Class K, is favorable for accurate measurement. The means are taken for each observatory, the number of individual results being indicated by the subscript.

ε PEGASI

Mean Date	Mean Velocity	Observatory
1888.9	$V_2 = +8.0$ km.	Potsdam
1901.5	$V_8 = +7.5$	Columbus
1901.7	$V_8 = +5.0$	Lick
1902.7	$V_3 = +6.2$	Yerkes
1903.4	$V_7 = +5.9$	Pulkowa
1903.8	$V_3 = +3.3$	Cambridge
1905.6	$V_4 = +6.1$	Flagstaff
1905.8	$V_5 = +5.0$*	Bonn
Mean	$V_{38} = +5.8$	

* Corrected by application of — 1.0 km.

Here are the results available for β *Aquilæ*, a fourth-magnitude star of Class K:

β AQUILÆ

1896	Aug.	25	—39.3 km.	Lick
	Aug.	31	—38.5	Lick
1897	May	12	—40.9	Lick
1898	Sept.	9	—39.2	Lick
1905	June	11	—39.4	Lick
1907	June	27	—40.0	Lick
	Mean		—39.6 ± 0.22	
1905	July	26	—37.3	Bonn
	Aug.	17	—37.4	Bonn
1906	July	17	—40.3	Bonn
	Sept.	19	—39.0	Bonn
	Mean		—38.5	
Systematic correction			— 1.0	
Corrected mean			—39.5	

Following are Lick observations of μ *Cygni*, a fifth-magnitude star of favorable spectral type, Class F5:

μ CYGNI

1900	July	31	+18.2 km.
	July	31	+19.1
	Aug.	1	+18.6
1905	Aug.	7	+19.7
1907	July	5	+18.7
	Aug.	1	+18.7
	Mean		+18.8 ± 0.13

The results for the fifth-magnitude star appear to be essentially as accurate as those for *Canopus*, as the range is but 1.5 km.

Selecting some large velocity results, we have for λ *Aurigæ* of the fifth magnitude, Class G:

λ AURIGÆ

1899	Dec.	18	+67.1 km.	Lick
	Dec.	25	+67.5	
1900	Jan.	22	+64.1	
	Oct.	9	+66.5	
1904	Oct.	26	+67.5	
	Mean		+66.5	

The third plate, giving a somewhat discrepant result, is underexposed, owing to poor atmospheric conditions.

For η *Cephei*, of the fourth magnitude, Class K:

η CEPHEI

1897	Sept.	29	—87.4 km.	Lick
1898	July	20	—86.2	Lick
	Aug.	21	—86.9	Lick
	Aug.	25	—86.2	Lick
1901	Aug.	24	—88.8	Lick
1904	July	6	—86.8	Lick
	Mean		—87.0	
1904	Aug.	29	—86.3	Bonn
	Oct.	14	—85.3	Bonn
	Oct.	15	—86.0	Bonn
1906	Aug.	30	—86.4	Bonn
	Mean		—86.0	
Systematic correction			— 1.0	
Corrected mean			—87.0	

Again, for the fourth-magnitude star, δ *Leporis*, of Class K:

δ LEPORIS

1900	Dec.	24	+98.5 km.	Lick
	Dec.	25	+99.1	
	Dec.	30	+99.7	
1904	Dec.	13	+99.8	
1906	Nov.	8	+99.1	
	Mean		+99.2	

It is instructive to compare the velocities of the *Great Nebula in Orion,* Class P, probably the most favorable nebula for such observation in the whole sky, as determined visually by Keeler with the 36-inch refractor, and photographically at Potsdam, Lick, and Yerkes Observatories.

ORION NEBULA

	Keeler *	Lick †	Potsdam ‡	Yerkes §
	Visual 1890-91	Photo. 1901	Photo. 1901-02	Photo. 1903-04
	+17.5 km.	+17.1 km.	+18.0 km.	+19 km.
	19.3	16.1	14.1	23
	15.6	17.0	17.0	18
	6.4	14.8	17.8	21
	13.5		18.1	19
	23.8		18.9	16
	8.5		18.4	19
	34.6		19.8	16
	20.0		16.2	19
	19.0		15.7	19
	14.0			14
	21.7			
	16.4			
Means	+17.7	+16.2	+17.4	+18.5

Mean_4 +17.4

Keeler's individual values differ considerably amongst themselves, showing a total range of 28 kilometers, but he would be a skillful observer indeed who could improve upon them, using visual methods. The photographic results exhibited in the last three columns are very accordant, showing total ranges of but 2.3, 5.7 and 9 km., the latter being somewhat large because the low dispersion of a 1-prism spectrograph was employed, whereas the Lick and Potsdam observers used 3-prism instruments. The means of the four series are remarkably accordant. The advantage of the photographic method is clearly apparent.

Other factors being equal, the accuracy of radial velocity determinations is roughly proportional to the dispersion employed in the spectrograph. In the case of such a bright star

* *Publ. Lick Obs.*, **3**, 197, 1894.

† Wright, *Lick Obs. Bull.*, **1**, 155, 1902.

‡ Vogel and Eberhard, *Ap. J.*, **15**, 303, 1902; and *Sitzungsber. Kgl. Akad. Wiss. Berlin*, p. 260, 1902.

§ Frost & Adams, *Ap. J.*, **19**, 354, 1904.

as *Canopus,* for example, an instrument giving five times as much dispersion as the Mills spectrograph could be designed and applied, thereby reducing the accidental and unavoidable errors of observation to perhaps a third their present dimensions; and similarly for several other first-magnitude stars. A very considerable gain in accuracy may be accomplished by using fine-grained (slow) photographic plates on all stars bright enough to permit their employment. On the other hand, if it is a question of observing the speeds of stars fainter than the fifth visual magnitude, an instrument with 2-prism dispersion can be used to advantage. Decreasing the dispersion by a third decreases the accuracy nearly a third, but this enables us to measure velocities with quite satisfactory accuracy for stars nearly a magnitude fainter than the limit for three prisms, with exposures of the same lengths. Going yet further, it is possible to measure the speeds of stars considerably fainter, certainly down to the seventh visual magnitude, sufficiently accurately for many purposes, with 1-prism instruments. For example, here are 1-prism observations of Lacaille 661 = *R. H. P.* 637, of 6.3 visual magnitude, Class G:

LACAILLE NO. 661

1908	Oct. 7	+51 km.	Chile, D. O. Mills
	Oct. 11	+48	
	Oct. 18	+52	
	Dec. 12	+47	
	Mean	+49	

And again, for Weisse I, $4^h.1189$=*R. H. P.* 1614, magnitude 6.5, Class K.

1908	Oct. 18	+33 km.	Chile, D. O. Mills
	Oct. 21	+31	
	Oct. 24	+31	
	Dec. 12	+28	
	Mean	+31	

Less difficult—easy, in fact—was it to measure the speed of the 9.3 magnitude star B. D. + 30°.3639, of the Wolf-Rayet type, for its light is condensed principally into a few bright lines.

Measures of the positions of the $H\gamma$, $H\delta$ and $H\epsilon$ hydrogen bright lines by Duncan yielded speeds as follows[23]:

B. D. + 30°.3639.

1908	July 7	—25 km.	Lick
	July 17	—32	
	July 21	—33	
	Mean	—30	

My principal purpose in quoting these results for faint stars is to introduce and support the statement that 2000 stars, more or less, fainter than those already observed with 3-prism dispersion, are within reach of 2-prism spectrographs; and that 5000 still fainter stars, more or less, are easily within practicable reach of 1-prism spectrographs. Although the velocities determined with such instruments would not be so accurate as those obtained with 3-prism instruments, they would serve admirably in the study of individual stars in very many cases, and be of tremendous importance in statistical studies as to the distribution and motions of the stars throughout the stellar system. The carrying out of this suggestion, using 2-prism dispersion on stars sufficiently bright, and 1-prism dispersion on the fainter stars, would constitute at least a decade's exceedingly fruitful labor for ten or twelve well-manned observatories, in the two hemispheres; and a beginning on such a coöperative program should not be long delayed. The reasons will appear, forcibly, in the discussion (CHAPTERS VI, VII and VIII) of recent observations.

I have in nowise indicated the limiting possibilities of the 1-prism instrument. Dr. Curtis, when in charge of the D. O. Mills Expedition, measured the radial speed of a 9.2 visual magnitude star, whose spectrum is approximately of the solar type, by making exposures *on the same plate for four consecutive nights,* twenty-nine hours in all. There was special reason for observing this star, *Cordoba Zones* $5^h.243$, as it possesses the largest proper motion of any known star, 8″.7 per year. In 225

23 *Lick Obs. Bull.*, 6, 59, 1910.

years its position on the surface of the sphere changes through an angle equal to the Moon's diameter. Is its motion in the line of sight of unusual magnitude? Here are the Mills observations:

C. Z. $5^h.243$

1908	Dec. 2	(mean of 2 nights)	+240 km.
	Dec. 9	(mean of 4 nights)	+244
	Mean		+242

The foregoing references to the use of 1-prism dispersion have been chiefly in connection with faint stars whose spectra contain sharply defined lines. One-prism spectrographs have in the past eight years done important service at several observatories in measuring the speeds of those stars, both bright and faint, whose spectra contain unusually broad and poorly defined lines. For such stars, in many cases, 3-prism spectrograms are unmeasurable: the hazy lines are magnified too highly, and the contrasts between the absorption lines and the continuous spectrum background are not sufficient to define the positions to be measured. Reducing the dispersion to one-third reduces the widths of the broad lines to one-third, and the contrasts are increased sufficiently to let the boundaries of the lines be estimated. The accuracy is not so great as for stars with good lines, but it is sufficient for the solution of a large class of most interesting problems. Here are remarkably accordant observations[24] of three *Pleiades* stars made at the Yerkes Observatory with a 1-prism instrument. Their spectra are of Classes B8p, B5, and B5, respectively.

ATLAS (27 TAURI)		ALCYONE (25 TAURI)		MEROPE (23 TAURI)	
1903 Oct. 30	+14 km.	1903 Oct. 30	+17 km.	1903 Dec. 27	+6 km.
Dec. 25	+12	Dec. 4	+14	1904 Mar. 19	+5
1904 Jan. 29	+15	Dec. 25	+13	Apr. 16	+8
Feb. 26	+10				
Means	+13		+15		+6

[24] *Ap. J.*, **19**, 340-341, 1904.

TABLE V

STELLAR MOTIONS EXCEEDING ± 50 KM. PER SECOND

Object	α (1900)	δ (1900)	Spm.	$D_{K.V.}$	Obs'd V	Corr'd V
					km.	km.
ϕ_2 Orionis	5h 31m.4	+ 9° 15′	K	10°.6	+ 99.4	+ 85.3
θ Canis Maj.	6 49 .5	−11 55	K5	25 .7	+ 96.7	+ 79.8
13 Canis Min.	7 57 .1	+ 2 36	K	27 .8	+ 72.0	+ 58.4
λ Aurigæ	5 12 .1	+40 1	G	30 .9	+ 66.5	+ 60.1
δ Leporis	5 47 .0	−20 53	K	33 .5	+ 99.2	+ 81.6
O_2 Eridani	4 10 .7	− 7 49	G5	36 .1	− 41.6	− 56.4
54 Eridani	4 36 .1	−19 52	Ma	39 .7	− 33.7	− 50.2
β Columbæ	5 47 .4	−35 48	K	48 .2	+ 89.4	+ 71.8
A. G. C. 10120	7 41 .9	−33 59	F8	50 .7	+105.	+ 88.4
**κ Pyxidis	9 3 .7	−25 27	K5p	56 .1	− 42.3	− 56.1
Cord. Zone 5.243	5 7 .7	−45 59	G-K	58 .8	+242.	+225.5
ο Ceti	2 14 .3	− 3 26	Md	61 .0	+ 62.2	+ 53.5
e Eridani	3 15 .9	−43 27	G5	68 .5	+ 87.3	+ 73.3
μ Cassiopeiæ	1 1 .6	+54 26	G5	73 .1	− 97.4	− 92.8
Groom. 1830	11 47 .2	+38 26	G	77 .8	− 97.	− 93.0
ε Andromedæ	0 33 .3	+28 46	G5	79 .6	− 83.5	− 81.1
ν Virginis	11 40 .7	+ 7 5	Ma	80 .9	+ 51.2	+ 50.3
A. G. C. 1345	1 20 .2	−42 1	G	85 .8	+ 72.5	+ 63.4
**α Phœnicis	0 21 .3	−42 51	K	96 .4	+ 76.:	+ 69.7
f Draconis	17 32 .4	+68 12	K	99 .5	− 74.7	− 61.3
η Cephei	20 43 .3	+61 27	K	100 .7	− 87.0	− 73.9
*G. C. 4373	17 58 .6	+66 38	Neb.	101 .3	− 64.7	− 50.9
**A. G. C. 17977	13 8 .0	−58 34	F	107 .5	− 64.4	− 69.3
72 Cygni	21 30 .7	+38 5	K	111 .7	− 65.2	− 52.3
35 Pegasi	22 22 .8	+ 4 12	K	115 .7	+ 54.8	+ 62.4
1 Pegasi	21 17 .5	+19 23	K	124 .5	− 75.8	− 63.0
11 Libræ	14 45 .8	− 1 53	K	128 .0	+ 83.2	+ 93.0
υ_2 Lupi	15 15 .1	−47 57	G	128 .6	− 65.0	− 63.4
ν Pavonis	18 22 .0	−62 20	B8	129 .6	+ 62.	+ 62.0
ζ Herculis	16 37 .5	+31 47	G	130 .7	− 70.	− 53.3
φ Serpentis	15 14 .2	+ 2 9	G	133 .6	+ 53.8	+ 65.4
λ Serpentis	15 41 .6	+ 7 40	G	137 .6	− 65.6	− 51.9
6 Vulpeculæ	19 24 .5	+24 28	Ma	139 .5	− 85.0	− 68.0
37 Libræ	15 28 .7	− 9 43	K	139 .9	+ 48.9	+ 59.5
*N. G. C. 6891	20 10 .4	+12 26	Neb.	141 .9	+ 40.7	+ 55.7
A. G. C. 27600	20 4 .6	−36 21	K5	144 .8	−132.	−125.8
A. G. C. 24321	17 49 .5	−44 19	K	147 .2	+ 46.	+ 51.5
*N. G. C. 6790	19 17 .9	+ 1 19	Neb.	158 .9	+ 48.5	+ 63.8
ν_2 Sagittarii	18 49 .1	−22 47	K	165 .9	−106.	− 94.9
α Scuti	18 29 .8	− 8 19	K	174 .3	+ 36.0	+ 50.4

[Elements of solar motion used: $V_o = -17.77$ km., $\alpha_o = 272°$, $\delta_o = +27°.5$. See CHAPTER V, p. 189.]

*Nebulæ observed by Keeler.

**These three stars were added to the table after the date of this lecture, February 1, 1910.

These useful results, and a long list of similar ones, could not have been obtained so accurately with 3-prism dispersion; and there is the added advantage that exposure-times with 1-prism instruments are not more than one-fifth to one-eighth as long as with 3-prism instruments. The output of results can be correspondingly augmented.

There are occasional stars, perhaps from 1 per cent to 2 per cent of the entire number, whose spectra contain such extremely broad and imperfectly defined lines or bands as to be at present incapable of measurement to any satisfactory degree of accuracy. Once in a great while we photograph a spectrum which seems to be entirely devoid of lines; and with such spectra we can, of course, do nothing; but there is promise that for spectra containing only very wide and very hazy lines we shall be able, by using slow plates for the exposure, and by purely photographic manipulation of the original negatives, to increase the contrasts within the spectrum enough to let fairly satisfactory measures be made. It is not that we dislike to pass by a certain *number* of stars as unmeasurable, but that we prefer not to pass by a certain *class* of stars as indicated by their spectra.

The stellar velocities quoted on preceding pages include a few which equal 100 km. or more per second. Speeds of this order are few in number, but can scarcely be called exceptional. The accompanying table contains a list of thirty-seven stars on my program and three planetary nebulæ whose observed radial velocities are equal to or greater than 50 km. per second, as contained in the last column, after correcting the observed velocities contained in the next to the last column for the direction and speed of the solar motion. The fifth column of the table defines the angular distances of the stars from the Kapteyn vertex of preferential stellar motion, to which reference will be made in a subsequent chapter. The methods of determining and eliminating the solar motion effects are described in Chapter V, and it is sufficient to say that the results in the last column of Table V are the velocities of the stars themselves toward and away from the position which the solar system occupies at any one instant. They present interesting considerations. In the first place, they are not the real velocities of the stars in space but merely the projections of those velocities on the line of sight; for example, the faint star, *Groombridge* 1830, which is known to be one of our near neighbors (parallax, $\pi = 0''.10 \pm$) and to possess high proper motion ($\mu = 7''.05$ per annum), must have a speed in space of approximately 250 km. per second in order to harmonize these observational data. Curtis[25] finds that the star *Cordoba Zones* $5^h.243$, whose observed radial velocity is 242 km. per second (page 114) and whose parallax is 0''.32, must have a velocity in space of approximately 260 km. per second. If the observed parallax of the latter star is approximately correct, it must be an exceedingly small star, for it is of the ninth visual magnitude. It would be interesting to supplement this list of high radial velocities with other stars whose spectrographic velocities are small or medium but whose cross motions, as determined by their parallax and proper motions, must be very great. A case in point is *Arcturus,* with parallax supposed to be approximately 0''.07, and a well-determined proper motion of 2''.26 per year, whose radial velocity is but 6 km. per second approach. To make these elements harmonize, the velocity of *Arcturus* in space must be more than 150 km. per second. It is conceivable that the parallaxes of the three stars referred to in this paragraph may be in error by 25 to 40 per cent; and, if so, the velocities attributed to them would be somewhat in error.

These high-velocity stars are sometimes described as runaways, because they seem to be quite beyond the control of the gravitational power of the universe on any reasonable assumption. Newcomb[26] has calculated that the maximum velocity attainable by a body starting with velocity zero at an infinite distance and passing through a stellar system containing one hundred million stars, each five times as massive as our Sun and distributed throughout a disk-like spheroid whose maximum radii correspond to 15,000 light years, cannot exceed 40 km. per second. *Groombridge* 1830 has a speed nearly nine times this value, and the massive star *Arcturus* a speed probably four times

[25] *Lick Obs. Bull.,* **5**, 133, 1909.

[26] *Popular Astronomy,* Revised 4th Ed., p. 499.

this value. If existing velocities owe their magnitudes to the gravitation of the system, the quantity of attracting matter in the system would have to be at least eighty times that assumed by Newcomb, as, other factors being equal, the velocities are proportional to the square roots of the attracting masses.

The most comprehensive investigation on this subject is that of Kelvin.[27] Assuming the universe to be composed of gravitational matter such as we are acquainted with, in quantity equal to one thousand million times the Sun's mass (twice the mass in Newcomb's assumption), uniformly distributed throughout a sphere whose radius would correspond to 3300 light years (parallax = 0″.001), he finds that the velocity acquired by a body starting originally at rest from the surface of this sphere would, in five million years, be about 20 km. per second, and in twenty-five million years would be about 108 km. per second, provided that the acceleration remained sensibly constant throughout these intervals; or, if conditions were such that the concrete bodies were now about equally spaced throughout the assumed sphere, their mean velocity would be about 50 km. per second. I shall show later that the mean velocity of the stars included on my program of radial velocity determinations is 27 km. per second, as against an average of 50 km. required by Kelvin's assumption of mass and other conditions. Kelvin's assumed mass of the attracting matter in our stellar universe may be regarded as a superior limit, if we agree that velocities are purely gravitational effects, in the ordinary sense; but it is certain that many stellar velocities are greatly in excess of 108 km. per second. If the Sun is a star of average mass, and one hundred million suns are observable in our greatest telescopes, it is clear that by far the major quantity of gravitational matter exists in a form rendering it invisible to us. When we consider the quantity of interplanetary matter within the limits of our own solar system, manifesting itself in the zodiacal light phenomena, and in the form of meteors; and the possibilities of interstellar space as a reservoir for similar minutely divided matter; we should have no difficulty in seeing the reasonableness of Kelvin's results.

[27] *Report B. A. A. S.*, p. 563, 1901.

We pass to another phase of the subject, beginning by way of illustration with the well-known double-star system, *α Centauri*. This double star is our nearest neighbor, so far as known, and is an especially interesting system on that as well as on other accounts. The two stars of the pair, α_1 and α_2, of magnitudes 1.7 and 0.3, and of spectral Classes K5 and G, respectively, are known to revolve about their mutual centre of mass in a period of 81.2 years, and their masses, m_1 and m_2, are supposed to be to each other as about 1.00 to 1.04.[28] Our present problem is to determine whether the system as a whole, in other words, whether the centre of mass of the system, is approaching or receding from us, and at what rate. It is a well-known fact that in every such binary system the two bodies are always on precisely opposite sides of the centre of mass and moving in precisely opposite directions, with speeds inversely proportional to their respective masses, m_1 and m_2. Let these speeds projected upon the line of sight be V_1 and V_2. Knowing the ratio of the masses in this case, as stated above, if we measure the speeds V_{01} and V_{02} of the two components in the line of sight, we shall be able to determine the speed V_0 of the system as a whole in the line of sight. We shall have:

$$V_{01} = V_0 + V_1$$
$$V_{02} = V_0 - V_2$$

If we multiply these equations through by m_2 and m_1, respectively, we shall have:

$$m_2 V_{01} = m_2 V_0 + m_2 V_1$$
$$m_1 V_{02} = m_1 V_0 - m_1 V_2$$

Now the mass of the second times the velocity of the first is always equal to the mass of the first times the velocity of the second. Therefore the first members are equal; and from the equated second members we have:

$$V_0 = \frac{m_2 V_{01} + m_1 V_{02}}{m_1 + m_2} \tag{27}$$

28 There exists some doubt as to their relative masses: see CHAPTER VII, TABLE XXX.

Observations secured by my colleague, Mr. Wright of the D. O. Mills Expedition, on the radial velocities of the two stars are:

α CENTAURI

			V_{01}	V_{02}
1904	Feb.	21		—24.4 km.
	Feb.	25	—19.2 km.	
	Mar.	4	—18.8	—24.4
	June	23	—20.1	—24.8
1905	Jan.	27	—19.4	—25.4
	Mar.	7	—20.3	—24.9
	Apr.	17	—19.0	—25.0
Means			—19.5	—24.8

Substituting in (27) we find that the system is approaching us with a speed

$$V_0 = -22.2 \text{ km. per second.}$$

The two stars of this system are far enough apart to permit their spectra to be observed separately. Suppose they were so close together that they could not be observed individually, but that the light of both stars entered the slit of the spectrograph at the same time; what should we find on the photographic plate? A composite spectrum, with two sets of lines, corresponding to the two speeds observed as above. In this particular case, however, with relative speeds differing only 5 km. per second, the two sets of lines, on the basis of 3-prism dispersion, would not be separated; the composite lines would be only slightly broadened. If the two bodies composing a system are relatively close together, so that they revolve around their common centre of mass very rapidly, say in a few days or a few weeks, the two systems of lines corresponding to the two stars will in general shift rapidly with reference to one another. As the two stars continue to revolve, the two sets of spectral lines will swing past one another very much as two pendulums of equal lengths would, if one pendulum were mounted directly in front of the other and they differed a half cycle in phase. Conversely, if a spectrum is seen to be composite, containing two sets of lines which shift from violet toward red and red toward violet, respectively, in continued succession, this is conclusive evidence that we are dealing with a double star whose two components are not very unequal in brightness. A series of spectrograms of such a composite star, measured, will tell us the form of the orbits of the two bodies about their common centre of mass, and, if the usual comparison spectrum is photographed on the plates, the relative masses of the two stars. We can likewise deduce the speed of the centre of mass of the system; that is, the speed of the system as a whole, toward or from the observer.

The first system of this kind to be discovered, in 1889 by Pickering, was ζ *Ursæ Majoris*,[29] the star at the bend in the handle of the Big Dipper. Its spectrum is of a simple type, the principal lines present being those of hydrogen, magnesium, and calcium. It was observed that these lines were at times widely doubled, again they were single and at other times narrowly doubled. There should be no doubt that this star consists of two component stars nearly equal in magnitude, revolving around their centre of mass, as Vogel later determined,[30] in 20.54 mean solar days. By means of a long series of observations Ludendorff[31] has found that the masses of the two bodies are as 1.01 to 1.00, and that the system as a whole is approaching us with a speed of 12.6 km. per second.

Suppose, now, that one of the components of such a double-star system is considerably fainter than the other component, because it is a smaller body, or because its efficiency as a light radiator is less. The brighter body will revolve in its orbit around the centre of mass of the system, its velocity of approach and recession will vary regularly through a cycle, and the spectrum lines will accordingly shift from red toward violet and from violet toward red of their average positions, in proportion to the changes of radial speed. Conversely, if the measured positions of the lines in the spectrum of any star are appreciably different at different times, indicating a variable velocity, we conclude

29 *Amer. Jour. Sci.*, **39**, 46, 1890.

30 *Ap. J.*, **13**, 328, 1901.

31 *Astr. Nach.*, **180**, 276, 1909.

that this star is attended by an invisible companion, massive enough to swing the observed star around in an elliptic orbit.

I found in 1898 that η *Pegasi* is such a system. The following seven observations, selected from twenty-nine secured altogether, exhibit the alterations of radial velocity with reference to the solar system:

Date Greenwich M. T.			Velocity Km.
1896	August	27.8	+ 7.10
1897	July	8.9	— 6.37
1898	September	4.7	+16.46
1899	January	23.6	— 0.84
	June	21.0	— 8.02
1900	September	25.7	+21.40
1901	May	9.0	— 0.18

The period of revolution of the bright star around the centre of mass of itself and its invisible companion proved to be 818 days. Making use of this period, the twenty-nine observations were charted as in Figure 4, with times as abscissæ and kilometers as ordinates. Crawford's determination of the orbital elements

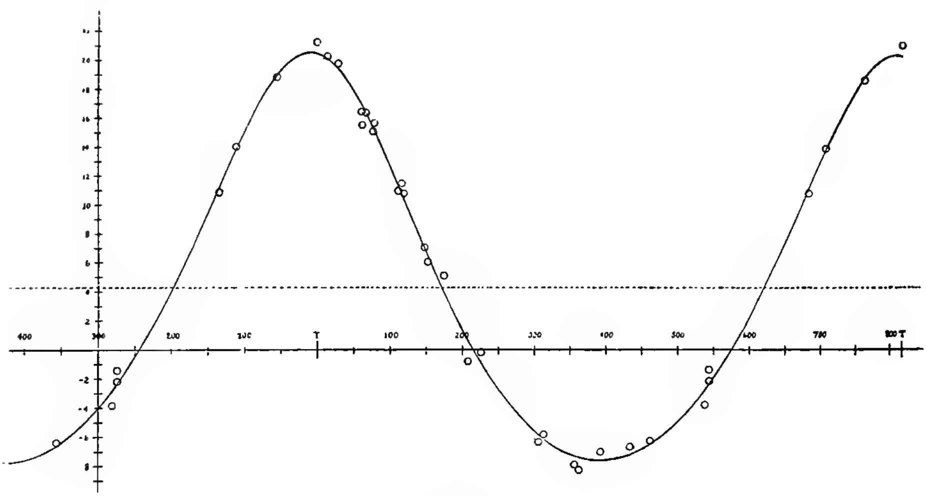


Figure 4

fixed the eccentricity at 0.155 and the velocity of the centre of mass of the system at 4.3 km. per second recession.[32]

In this manner, by means of the synchronous shiftings of two sets of lines, or the shifting of one set of lines, through recurring cycles of change, about 300 such double-star systems, known as spectroscopic binary stars, have been discovered to date. However, at this point in the development of our subject, we are interested in obtaining the velocities of as many stars, or systems of stars, as possible, with a view to using these velocities in solving certain fundamental problems of the stellar system. I have, therefore, introduced the subject of binary systems in this chapter only to illustrate the methods of getting at the velocities of such systems as a whole, *i.e.*, of their centres of mass, and a study of their interesting features will be taken up in another chapter. Of the 300 binary systems known, about sixty-five have had their orbits computed, and, consequently, the velocities of their centres of mass determined. These sixty-five velocities are available for use in the fundamental problems immediately before us, precisely as the unchanging velocities of solitary stars are; but the variable observed velocities of the remaining 240 binary stars have not been fully investigated, though the quite approximate values of the systemic velocities are known in eighty of these cases.

We now have available, from observations made with the Mills spectrographs at Mount Hamilton and at Santiago, the radial velocities of 1340 stars: of these, 160 are binary stars whose velocities, as explained above, are variable, and for which the velocities of the centres of mass remain as yet unknown. About 160 stars have been observed only once, or are on the list of suspected binaries. Deducting these stars, and adding 40 stars observed elsewhere and not at Mount Hamilton or at Santiago, we have 1060 stars whose radial velocities we know.[33] When these velocities are critically examined and compared, certain interesting facts stand out.

32 *Lick Obs. Bull.*, 1, 26-30, 1901.

33 It should be said, however, that many of these stars will undoubtedly prove to be spectroscopic binaries of long period or small range.

If a group of neighboring stars be considered, their velocities will seem, in general, to be unrelated to one another; as, for example, the following for a group of twenty-five stars near the vernal equinox:

STARS NEAR THE VERNAL EQUINOX

— 5.5 km.	—10.8 km.
— 5.4	+12.9
—16.0	+ 4.3
+15.9	—44.9
— 8.5	—22.1
+ 6.0	+18.6
—14.9	+10.5
+ 6.9	+15.0
+ 6.0	+ 1.3
+ 4.8	—19.0
+11.	+32.4
— 4.6	+16.1
— 2.0	

$V_{25} = -0.1$ km.

Here are the velocities of twenty-five neighboring stars near the autumnal equinox:

STARS NEAR THE AUTUMNAL EQUINOX

+ 6.0 km.	+ 4.0 km.
+16.9	+ 4.8
— 8.6	—29.4
— 5.1	+43.5
— 9.9	+ 4.2
— 8.9	+ 3.0
+21.0	—19.2
— 4.4	—19.0
+ 2.0	+17.3
— 4.1	—17.6
+51.2	— 1.2
— 6.0	—13.2
+ 4.9	

$V_{25} = +1.3$ km.

It will be noticed that positive and negative velocities are nearly equal in number in these two lists, and that their algebraic sums differ little from zero.

There is a large region of the sky in which apparent motions of approach plainly predominate. Here is an illustrative group of twenty-five velocities in or near the constellations *Hercules* and *Lyra:*

STARS NEAR HERCULES AND LYRA

—14.5 km.	—58.0 km.
—26.1	—14.0
—16.1	—26.0
—15.6	+22.6
—26.8	—16.4
— 1.5	— 0.8
—21.6	—26.7
—21.8	—25.5
+ 0.4	—51.3
—36.0	—20.3
—14.0	— 9.0
—32.7	—30.3
—22.0	

$V_{25} = -19.9$ km.

The differences of their velocities are as great as before, but the negative sign (indicating approach) plainly predominates, only two being positive; and the mean for the group is large, 19.9 km. per second.

Here is another group of twenty-five velocities in exactly the opposite region of the sky, in or near the constellations *Canis Major* and *Columba:*

STARS NEAR CANIS MAJOR AND COLUMBA

—13.7 km.	+ 2.7 km.
— 4.6	— 1.1
+25.0	— 7.
+35.	+ 8.
— 9.1	+22.5
+99.2	+37.
+89.4	—15.
0.0	+40.
+63.5	+28.
+21.	+21.9
+18.7	+49.
+22.	+28.5
+34.	

$V_{25} = +23.6$ km.

Here again, the velocities seem as unrelated individually, as before; their mutual differences are as large as in other groups, but the positive sign (indicating recession) plainly predominates. The mean velocity is + 23.6 km. per second. Three of the velocities are very large. If we reject them, the average for the remaining twenty-two stars becomes + 15.3 km.

If we form a series of such groupings of velocities, passing in any direction from *Lyra-Hercules*, as the point of departure, over the spherical surface of the sky, to the opposite point, we shall then have the mutual *differences of velocity within the groups* approximately as large as ever, but the *means for the groups* will increase somewhat continuously from approximately — 20 km. up through zero to approximately + 20 km. The significance of these facts is plain: first, the stars have their individual motions, in a large measure apparently at random, but, as we shall see later, not entirely so; second, the stellar system as a whole has an apparent motion or drift away from the *Lyra-Hercules* region. This is as we had expected, for a long list of investigators, from Herschel to the present generation, have determined that the solar system must be carrying terrestrial observers toward the general region of sky that includes *Lyra-Hercules.* If we determine the direction and speed of the solar motion, and correct each observed stellar speed for the effect of the solar motion, the corrected velocities of neighboring stars will differ from each other as widely as ever, but the algebraic sum of the velocities in any small area of the sky, say areas containing twenty-five neighboring stars each, should be approximately zero, *provided* the motions of the stars are at random as to direction and magnitude of speed. We now take up the consideration of the solar-motion problem, approaching it from the proper-motion side, but pursuing this phase of the subject only far enough to supply a natural introduction for the radial velocity method of solving the problem. A comprehensive presentation of the proper-motion determinations of the Sun's course would demand the capacity of a separate volume.

CHAPTER IV

THE SOLAR MOTION AS DETERMINED FROM STELLAR PROPER MOTIONS

We shall gain a better comprehension of the solar motion problem if we assume, first, that the distant stars are really fixed; that is, that they are at rest relatively to one another; and that our star alone is in motion. What should be the observed effects? Using Herschel's original diagram,[1] Figure 5,

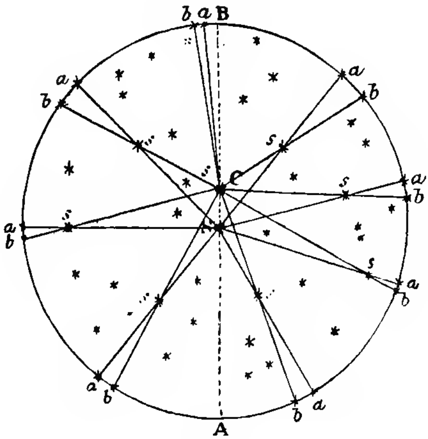


Figure 5

let the Sun have been at S at some past epoch, and let its motion in the interval have carried it along the line SB to its present position C. Let the stars be distributed through surrounding space, in all directions and at all distances from the solar system, as at s, s, s, ... When the Sun was at S, the stars were seen

[1] *Phil. Trans.* (Abridged Ed.), 15, 1783, Fig. 8, Pl. VI.

projected upon the celestial sphere at a, a, a, ... From the Sun now at C, the same stars are seen at b, b, b, ... That is, each star in the sky will appear to have moved away from B, along the great circle drawn from B through the star to A, over the angular distance, ab, ab, ab, ... The point B toward which the solar system is assumed to move is known as the *apex* of the Sun's way, and the point A is the *antapex.* The apparent motion of every star in the sky away from the apex and toward the antapex, due to the observer's motion toward the apex, is variously called, for convenience, the *parallactic component* of the star's motion, or, more briefly, the star's *parallactic motion, parallactic displacement, or secular parallax.* It is clear that this parallactic motion is a function:

1. Of the Sun's speed: a doubling of the Sun's velocity would double the corresponding displacement of every star;

2. Of the star's distance: a doubling of the star's distance would divide the parallactic displacement by two; and

3. Of the star's angular distance from the apex: a star exactly in the apex or antapex would suffer no apparent displacement, and a star 90° from the apex would suffer the maximum displacement. Other conditions being equal, the displacement would vary as the sine of the star's apical distance.

Giving mathematical expression to these relations, in any one of the triangles sSC, let SC, the Sun's motion in the unit of time, be called q; let D be the angular distance sSB of the star from apex B; let υ be the parallactic displacement, CsS of the star s; and let $\rho =$ sC be the star's distance from the solar system. Then we have:

$$q : \rho :: \sin \upsilon : \sin D \tag{28}$$

As υ is always a small angle, we may replace it by $\upsilon'' \sin 1''$. If we express q as the angular speed of the Sun when viewed at right angles from distance unity, we may place $q'' \sin 1''$ for q and (28) becomes

$$\frac{q}{\rho} \sin D = \upsilon \tag{29}$$

This equation expresses the relation which always holds between the Sun's motion, the star's direction and distance, and the resulting parallactic motion of the star.

Let us consider the reverse problem. If observation should show that all the stars are moving toward a common point in the sky, with speeds as the sine function of their distances from that point, we should come to but one reasonable explanation: there must be a motion of our Sun away from that point. In this case, how simple the problem of determining the elements of the solar motion. If for any two stars whose positions (right ascension and declination) are known, we should determine by observation the directions of their apparent motion, the amount of the parallactic motion, v, of one of the stars, and the distance, ρ, of that star, we should be able to solve absolutely and completely the solar-motion problem by a simple calculation: the great circles containing the parallactic motions would intersect in the apex and antapex; the value of sin D in (29) would come from simple computation; and q, the speed of the Sun toward the apex, being the only unknown quantity in (29), would be determined at once.

This simple illustrative hypothesis gives way, in the actuality, to a situation so complex as to exceed the bounds of present comprehension. A century and a quarter of investigation, almost continuous since the days of Herschel, has taught us much concerning the Sun's motion; but the chief result has been to make more acute our sense of the difficulties besetting the problem. From the proper-motion side of approach, at least, the last five years have been especially remarkable for the unexpected complications shown to exist in this problem. To these we shall refer later.

The stars are not at rest, but each has its own motion, not on the surface of the celestial sphere, but in space of three dimensions. These motions, in amount and direction, we shall consider, for the present, as at random, showing no preference for any speed or direction. Let S, S, S, ..., in Figure 6,[2] represent the positions of stars at some past time, as projected upon the celestial sphere. Let the stars have had actual motions in

[2] From Kobold's *Bau des Fixsternsystems,* p. 86.

space from that given time to the present, such that the projections of those motions upon the celestial sphere are represented by the arcs SM, SM, SM, ... These are known as the real or *peculiar* motions (*motus peculiares*) of the stars. Represent the position of the antapex by A and the stars' parallactic motions in the given time interval by SN, SN, SN, ... Then the apparent or observed motions of the stars are the resultants of the peculiar and the parallactic motions, SR, SR, SR, ...; and these are the star's *proper* motions in the time interval.

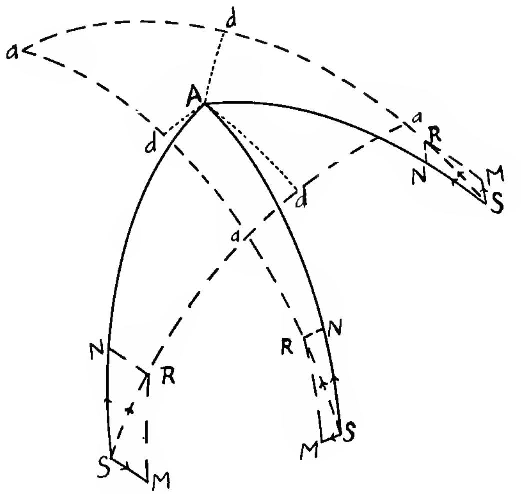


Figure 6

Let the proper-motion arcs be prolonged, two by two, until they intersect in the points a, a, a, ... None of these points coincides with the antapex A, but the points will cluster around A; how closely around A will depend upon the magnitudes of the parallactic components as compared with the peculiar motion components. It was this graphical method of locating A, as the approximate centre of gravity of the intersections a, a, a, ..., that Herschel[3] used with the proper motions of thirteen stars in

[3] *Phil. Trans.* (Abridged Ed.), 15, 402, 1783.

1783—all the proper motions then known—to fix the position of the apex at $\alpha_0 = 262°$, $\delta_0 = +26°$, near the star λ *Herculis*.[4]

A few months after the presentation of Herschel's results to the Royal Society, Prévost[5] presented to the Berlin Academy of Sciences (July, 1783) the results of his discussion of the same proper motions. He concluded that the most probable position of the solar apex was $\alpha_0 = 231°$, $\delta_0 = +25°$ (reduced to equinox of 1900.0).

A critical discussion of Herschel's first results was made by Klügel[6] in 1789. He concluded that Herschel's position of the apex as depending upon the proper motions of the stars employed could not be improved upon.

In the Figure the proper-motion arcs pass at distances Ad, Ad, ..., on one side or the other of A. That position of the antapex is best, in general, which makes these distances Ad, ..., as small as possible. Herschel, in 1805, used, first, the proper motions of six of the brightest stars, *Sirius, Arcturus, Capella, Vega, Aldebaran,* and *Procyon,* to determine a new position of the apex, such that the sum of the six distances, Ad, would be a minimum. The position defined by the six was tested, but not modified, by the proper motions of thirty other and fainter stars—the thirty-six in Maskelyne's catalogue being all that were then regarded as satisfactorily determined. His result was[7] $\alpha_0 = 246°.5$, $\delta_0 = 49°.4$.

This position differs 27° from that adopted in 1783, from thirteen proper motions. The discrepancy represents fairly well the uncertainty remaining in each solution. Herschel was well aware of the weak foundation on which his results rested; for, in reporting those of the year 1783 to the Royal Society, he said, "that we have already some reasons to guess (*sic*) which way the solar system is probably tending its course." [*Phil. Trans.* (Abridged Ed.), **15**, 402, 1783.] That he came so close to the truth was not the result of a guess, but of the philosophical

[4] The coördinates of the apex are reduced to the equinox of 1900.0.

[5] *Mem. de l'Acad. de Prusse,* 1781, p. 445.

[6] *Berliner Jahrbuch,* 1789, p. 214.

[7] *Phil. Trans.,* 1805, p. 256.

ability, amounting to real genius, with which he considered all known facts bearing upon the questions before him.

The 1805 apex has not stood the test of time so well as the 1783 apex.

Herschel employed a purely "cut and try" method for locating the antapex, which would be impracticable if the number of proper motions employed were large. A direct analytical solution for the most probable position of the antapex is obtained as follows: Let ψ be the known position angle of the proper-motion arc SR, χ the unknown position angle of the parallactic motion SN, and D the unknown angular distance of the star from the apex. Then in the right triangle dSA, we shall have, for any star,

$$\sin D \sin (\chi - \psi) = \sin \mathrm{Ad} ;$$

each equation containing as unknowns χ, D, and Ad. Now any value of D and of χ will fix a position for the apex, but the resulting values of Ad will be different for every star. If the number of stars is very large and their motions are entirely at random, the distances Ad will follow the laws of accidental errors, and a solution of all the equations by the method of least squares, making the sum of the squares of all the distances, Ad, a minimum, will give the most probable position of the antapex. That is, the complete solution, in principle, is contained in the equation:

$$\Sigma \left\{ \text{arc} \sin \left[\sin D \sin (\chi - \psi) \right] \right\}^2 = \Sigma \, \mathrm{Ad}^2 = \text{Minimum}. \quad (30)$$

In practice, this equation would have to be transformed considerably before solution. Of course Herschel did not make use of the least-squares principle, as he preceded Gauss, the developer of the method, by a quarter of a century. In effect, he omitted the exponent 2 and made the *sum* of the Ad's a minimum.

Thus far Herschel gave no concern to the different distances of the stars from our system—a factor far from negligible—nor to the speed of the solar motion. His generation was without knowledge of the distance of a single star, and the making of assumptions was unavoidable. He assumed that the six stars, *Sirius, Arcturus, Capella, Vega, Aldebaran,* and *Procyon,* are of equal *absolute* brightness, and, therefore, that their distances are inversely proportional to the square roots of their apparent brightness. In this manner, letting *Sirius* be at unit distance and of unit brightness, he deduced relative distances for the other five stars. Assuming a position for the antapex, A, in Figure 6, he computed for each star the angle NSR $= \chi - \psi$, as before, between the observed direction, ψ, of the proper motion, SR $= \Delta s$ and the great circle SA. (SN is the parallactic component of the star's apparent motion, and SM $= \Delta s \sin (\chi - \psi)$, the star's own or peculiar motion.) Herschel's idea was that the antapex, A, and speed, q, of the solar motion should be so chosen as to make the component remaining as each star's own motion as small as possible. Evidently the quantities $\Delta s \sin (\chi - \psi)$ for the several stars are not homogeneous, as the bodies are at different distances; and in order that all may enter the problem with equal weight, Δs must be multiplied by ρ before using. Thus, the expression to be given as small a value as possible is $\Delta s \rho \sin (\chi - \psi)$.

By trial of various values of the Sun's speed, q, originally with the six first-magnitude stars, and later by permitting the observed proper motions of thirty other fainter stars to check and modify his conclusions, Herschel decided in favor of $q = 0''.89$. The speed, q, is expressed in terms of the average distance of the six first-magnitude stars. That is, there being 206,265 seconds in the unit of length, or distance, the solar motion in one year would carry it $\frac{0.89}{206,265} = 1/230,000$ the distance of a first-magnitude star. Herschel did not know the distances of the first-magnitude stars, and, therefore, he could not translate this result into linear velocity. From the parallaxes now assigned to the six first-magnitude stars used by Herschel, giving average distance thirty-three light years, we find the linear value of $q = 0''.89$ to be 43 km. per second.[8]

[8] To be exact, Herschel expressed the Sun's speed in terms of the distance of *Sirius* taken as unity: $q = 1''.117$. His assumed relative distances of the six stars were, respectively: 1.00, 1.20, 1.25, 1.30, 1.40 and 1.40.—*Phil. Trans.,* 1806, p. 233.

Although the enormous accumulation of proper-motion observations since Herschel's day has rendered his results obsolete, one cannot pass on without registering high admiration for the genius which opened this field of research.

It is necessary that we define more exactly the term "solar motion," for all motion is relative, and we must describe the datum to which it relates. It would be meaningless, for example, to say simply that the solar system is moving toward a point whose R. A. is 270° and Decl. + 30°, with a speed of 19 km. per second. Knowing, as we now do, the motions of many stars, it would be possible to select a list of special stars relatively to which our solar system would be moving in a direction opposite to that quoted above, with a speed of 10 km.; or other groups of stars could be formed, by selection, such that the solar motion with reference to them would be at right angles to the direction quoted, with still different speeds.

The unusual interest taken by astronomers in the solar-motion problem is almost negligibly in that motion on its own account, though this is of wide human interest. We want to know the direction and speed of the solar motion in order that we may eliminate its effects from the apparent motions of the other stars, and thus have left the real motions of the stars. The purpose of our investigations fixes, therefore, the datum to which we would have the solution refer. We would know the elements of the solar motion—the direction and speed—with reference to the centre of mass of the stellar universe and to some definite system of coördinate directions. The universe, at least that part of it which the telescope shows, and for which we have present means of study, is quite generally believed to be finite in extent, and it is here thus assumed. The problem, at first sight, appears to be definite, but it is so only to a limited degree; for we must not overlook the dark and invisible bodies which space near and far is thought to contain. The *complete* solution of the problem involves a knowledge of the mass, and of the direction, distance, direction of motion and speed of motion, at the desired instant of time, of every body in the system, whether visible or invisible. Such a solution is beyond our present powers, and beyond our present conception of future powers; and we must content ourselves with approximate solutions, improving them with the advance of time and the development of better methods. Fortunately, we need not know even the approximate locus of the centre of mass of the system in order to determine and define the present direction and speed of our motion, though this knowledge would be essential in predicting our motion in the distant future. The directions of rectangular coördinate axes in and perpendicular to the ecliptic, or in and perpendicular to the equator, are determined by the geometry of the Earth's motions. A set of axes anywhere in space, parallel to, or definitely related in directions to, these axes will serve as a basis for studies of the stellar system, provided also their intersection, as the origin of coördinates, remains in a fixed, but it may be entirely unknown, relation to the real centre of mass of the system; for the direction and speed of our motion relative to the origin and axes described are identically the same as with reference to axes actually passing through the unlocated centre of mass of the system.

Our available methods of determining the solar motion are necessarily deductive, and dependent upon the observed effects of the solar motion on the apparent motions of the stars. *The results must refer the solar motion exclusively to the system of stars utilized in the solution.* In this and similar problems astronomers should ever hold in mind two salient points:

1. To select stars as a basis for the solution such that their motions, as a small system, will be representative of the entire stellar system, as far as possible.

2. To employ such mathematical processes of solution as shall best eliminate the effects of the individual motions of the stars employed as the basis of the investigation, and leave the parallactic motion as the residuum sought.

Herschel's first solution, described above, based upon the observed proper motions of the thirteen available stars, referred the solar motion to the system composed of those thirteen stars, and no other stars; and it was satisfactory only in so far as those stars were representative of the hundreds of millions of bodies in the stellar system; and similarly for his later solution, based upon the known proper motions of thirty-six stars.

After Herschel, the greatest of observers, came Bessel, the greatest of practical astronomers, with quite a different method of attack. He determined the poles of the great circles which pass, respectively, through the stars in the directions of their known proper motions. If the observed motions of the stars were entirely parallactic, that is, directed exactly toward the antapex, the poles of their proper-motion circles would all lie on one great circle (parallactic circle) whose poles in their turn would be the antapex and apex. In reality, as the observed motions are compounded of the peculiar motions and the parallactic motions, the poles of their great circles would be scattered more or less widely on both sides of the real parallactic circle. Bessel had hoped to draw a great circle amongst the plotted polar positions which would make the sum of the distances of the individual proper-motion poles from this circle a minimum; or, according to more modern methods, the sum of the squares of these distances a minimum. He found on the contrary that the plotted poles did not define the position of such a great circle: the proper-motion poles of the seventy-one stars utilized by him appeared to be distributed at random over the entire sphere. He, therefore, came to the conclusion that there was no justification for assigning a position to the solar apex, as Herschel and others had done. Bessel's position in the profession was so high that his views prevailed for two decades.[9] It appeared nearly a century later that Bessel's negative result was due to his selection of an unfortunate method.

New life was given to the subject in 1837 by Argelander's solution,[10] based upon the well-determined proper motions of 390 stars. He assumed a position of the apex and computed for each

[9] Many authors (*e.g.*, Chauvenet, *Spher. and Prac. Astr.*, **1**, 705, Fifth Ed.) credit Gauss with a solution of the solar motion, leading to the position of the apex $\alpha_0 = 260°$, $\delta_0 = +31°$; but I have not been able to find Gauss's paper, nor is it listed in Houzeau's *Vade-Mecum.*

[10] *Mem. de l'Acad. St. Petersbourg*, **3**, 590, 1837; *Astr. Nach.*, **16**, 45, 1838.

star the value of the expression $\sin(\chi - \psi) \sin D$ (see page 132), and accepted that position of the apex which made the sum of the squares of these values a minimum. His results (for 1900.0) were $\alpha_0 = 261°$, $\delta_0 = +32°$.

A method of solution which proved to be a favorite with many investigators was developed by Airy.[11] He assumed that the real motions of the stars occur at random; that is, that they have no preference for motions in certain directions, and being small quantities they can be treated as ordinary errors of observation, following the usual laws of errors. The equations connecting the three rectangular components of the Sun's motion and the positions of the stars in polar coördinates involve the distances of the stars. If the distances were known, these equations would enable the investigator to separate the peculiar and parallactic components of the motion; and thus afford satisfactory results for the solar motion. Lack of knowledge concerning the stellar distances in this method, as in other methods, imposed limitations upon its efficiency. Airy placed the apex at $\alpha_0 = 262°$, $\delta_0 = +25°$, and the annual speed at $q = 1''.912$.

Passing over the numerous solutions made in the third of a century following Airy, we may say that the chief investigators, since Herschel, have been Kapteyn and Kobold, whose results have appeared from time to time in the past fifteen years. Kapteyn has discussed the characteristics and limitations of the methods developed by Bessel, Argelander, and Airy, and has developed other methods of great merit.[12] His own leading method is in effect an extension of Herschel's, the extension consisting in the combination of the individual directions of all the stars in a small area of the sky into an indication of the direction of the apex from the centre of that small area, combined with a system of weighting whose purpose is to equalize the effects of the near and the distant stars. In outline, the resulting direction of the apex from each area of stars is that which makes the sum of the components of the individual proper

[11] *Mem. R. A. S.*, **28**, 143, 1860.

[12] *Proc. Amsterdam Acad.*, **2**, 353, 1900; *Astr. Nach.*, **156**, 1, 1901; *Astr. Nach.*, **161**, 325, 1903; etc.

motions at right angles to this direction equal to zero for all the stars in the area.

Substantially all of the solutions since the days of Gauss have employed the method of least squares to determine the most probable position of the apex and speed of the solar motion. As usual, there were equations of condition which took different forms in different methods. The logic of all the methods rested on the assumption, to which we have already referred, that the peculiar motions of the stars are at random and, therefore, in accordance with the laws of error underlying the method of least squares. Now this assumption, even if it represents the truth, puts the method of least squares to a heroic test, for these at-random components due to the star's own motions are of the same order of magnitude as the parallactic components due to the Sun's motion, as we shall learn in CHAPTER V. It is only because the numbers of equations involved were very large that the methods were at all satisfactory. All of the methods were embarrassed, though in different degrees, by the presence of a few very large proper motions which have undue influence upon the results; for example, it could readily happen that a star of small mass situated relatively near the solar system would have more influence than a score of very massive stars at great distances. As Kapteyn has pointed out, all of the methods must be very unsatisfactory, owing principally to our ignorance of the distances of the individual stars, if each star is made the basis for one equation of condition in the solutions; but that all the methods must yield very nearly the same results if each equation, so to speak, be made to depend upon a great number of neighboring stars. As a result of the group method, the peculiar components of the proper motions are largely eliminated in the mean for each area, and the mean of the systematic or parallactic components remains. Unknown individual distances, and the mean but unknown distances of the groups are eliminated more or less successfully through the effect of other groups occupying symmetrical positions in the sky.

To illustrate some of these points let us refer to the results obtained by Kapteyn in applying the methods of Argelander, Airy, and himself to the same data—the proper motions of the Bradley stars.[13]

RESULTS FROM THE BRADLEY STARS

Investigator	Method	α_0	δ_0
Kapteyn	Argelander	274°.6	+27°.2
Kapteyn	Airy	275 .0	+28 .5
Kapteyn	Kapteyn	267 .6	+29 .4
	Means	272 .4	+28 .4

It would seem, from these accordant values of α_0 and δ_0, that there remained little to be desired; and this was the all but unanimous opinion of astronomers. It was with something of a shock that the results of Kobold's investigations were received. Kobold employed a development of Bessel's method, and the following positions of the apex are selected from the many deduced by him:

KOBOLD'S RESULTS

α_0	δ_0	
275°.0	+0°.4	213 southern stars
269 .0	—2 .9	1579 stars
270 .2	—2 .3	2262 stars
271 .0	—0 .2	2262 stars

These results, too, are extremely accordant. If they stood alone and we were without Kapteyn's and earlier solutions we should regard them as evidence of a thoroughly satisfactory solution of the problem, as approached from the proper-motion side. However, Kobold's position of the apex is 31° south of Kapteyn's position, and Kapteyn's is about 6° farther south than the weighted mean of the earlier apical positions. The total range in declination for what might be called results by astronomers of great experience is in fact nearly 60°. These immense discordances give rise to the question, Are the mathematical methods at fault, or are the underlying assumptions wrong? That Bessel's method is difficult of application and sensitive to

[13] *Astr. Nach.*, 156, 15, 1901.

the observer's judgment is clear, for when Kobold divided the entire sky into 122 areas, combining the proper motions for all the stars in each area into one resultant motion, and solved for the 122 groups, the declination of the apex came out $+ 16°.5$,—a change of about 20°. To call attention to only one peculiarity of the method: a star having a given proper motion and another star having exactly the opposite proper motion would be moving on great circles having identical poles. However, in Kobold's skilled hands the application of this method raised a question which has become one of the most interesting in the science. Studying the extremely discordant positions assigned to the apex he was led to conclude that "the results could be harmonized on the assumption that the *motus peculiares* of the stars take place in the plane of the Milky Way, some in the direct sense and others in the retrograde sense; the motion of the Sun occurring in a plane which makes an angle of 17° with the plane of the Milky Way."[14] This conclusion was in decided opposition to the assumption that the peculiar motions of the stars have no preference for any special direction or directions, upon which all investigators from Herschel to Kobold had built. The subject was not carried appreciably further for many years, until in 1904 Kapteyn announced the exceedingly important discovery that the stars have decided preferences for motions in two definite directions. To this subject, which is now one of the most interesting in the whole field of stellar astronomy, we shall return a little later.

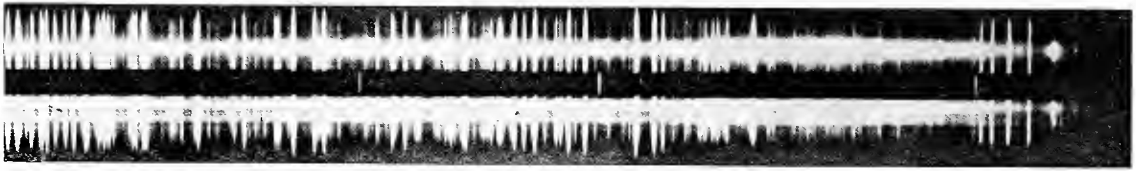
Bravais[15] has developed a method of determining the solar motion, from stellar proper motions, which in theory is entirely independent of the question as to whether the stars move at random or in preferential directions. Weersma, a student of Kapteyn, has made a comprehensive determination of the solar motion upon an adaptation of Bravais's method.[16] In order to obtain a solution which should refer the solar motion to the centre of mass of the stellar system and be independent of the

[14] *Astr. Nach.*, **137**, 393, 1895. See also *Astr. Nach.*, **139**, 65, 1895.

[15] *Jour. de Math.*, 1843, p. 435.

[16] *Publ. Astr. Lab. Groningen*, 21, 1908.

Class P. Spectrum of Great Nebula in *Orion*



Class P. Spectrum of the Planetary Nebula, B. D. −12°1172

Class P. Spectrum of Planetary Nebula, N. G. C. 7027 character of stellar motions, we should require to know, in addition to the observed proper motions, the masses, the distances, and the radial velocities of all the stars. This information being almost totally lacking, it was evidently unavoidable that Weersma should make assumptions, as reasonable as possible, concerning the masses, distances, and radial velocities. Under these conditions the results cannot of course be independent of the doctrines of random or preferential stellar motions, yet we cannot doubt that Weersma's solution possesses unusual merit. Up to date it is the final word on the subject.[17]

To test Bravais's modified method, he determined the position of the apex upon the basis of the Bradley proper motions, which Kapteyn had used for the three results quoted on page 139. Bravais's method gave,

$$\alpha_0 = 270^\circ.2, \ \delta_0 = +30^\circ.8.$$

That the results from the four methods are in remarkable accord is due, as Kapteyn and Weersma have remarked, to the fact that the stars were not permitted to form *individual* equations of condition but that their proper motions were combined into a *few groups,* involving the use of correspondingly few conditional equations. If the proper motions of a large number of neighboring stars are combined into one resulting proper motion, the method of solution employed is really of minor importance. The resultant proper motion will be nearly free from the individualities in the motions of the separate stars, and the effect of unknown distances will be largely eliminated through the use of symmetrically situated groups.

Basing another solution by Bravais's method upon all available proper motions, Weersma assigns the apex to

$$\alpha_0 = 268^\circ.0 \pm 0^\circ.8, \delta_0 = +31^\circ.4 \pm 1^\circ.1. \ (1900.0)$$

In TABLE VI we quote determinations of the solar motion made up to the present time, but it should be said that no effort has been made to have the list complete. In fact, a complete list

[17] Note added August, 1910.—Professor Boss has just published [*Astr. Jour.*, 26, 112, 1910] a solution based upon the proper motions of 5413 stars, with results: $\alpha_0 = 270^\circ.5$, $\delta_0 = +34^\circ.3$, $q = 24$ km. per sec.

TABLE VI.—PARTIAL LIST OF SOLAR

	α_o (1900)	δ_o (1900)	q	Materials
Herschel	262°.	+26°.4		13 stars
Prévost	231	+25 .		
Herschel	247 .	+49 .		6 "
Herschel			0″.89	36 "
Gauss	260 .	+31 .		
Argelander	260 .9	+32 .4		390 "
O. Struve	262 .3	+37 .5	0 .34	400 "
Mädler	262 .4	+39 .8		2163 Bradley stars
Airy	262 .1	+24 .7	1 .91	113 stars
Dunkin	264 .3	+25 .0	0 .41	1167 "
L. Struve	274 .2	+27 .3	0 .34	2509 "
Stumpe	287 .7	+44 .9		551 stars $\mu = 0''.$
Stumpe	282 .5	+43 .5		340 " 0 .
Stumpe	280 .6	+33 .5		105 " 0 .
Stumpe	285 .6	+30 .4		58 " >1 .
Porter	282 .0	+53 .7		576 " <0
Porter	280 .8	+40 .1		533 " 0 .
Porter	285 .3	+34 .0		142 " 0 .
Porter	277 .1	+34 .9		70 " >1
Newcomb	276 .9	+31 .4		644 " large μ
Newcomb	272 .5	+31 .3		2527 " small μ
Kapteyn	273 .8	+29 .5		Extensive
Boss	283 .5	+44 .1		273 stars, Dec. =
Boss	288 .9	+51 .5		247 " "
Kapteyn	275 .0	+29 .2		1809 "
Kapteyn	274 .6	+27 .2		All Bradley stars
Kapteyn	267 .6	+29 .4		" " "
Kapteyn	275 .0	+28 .5		" " "
Kobold	275 .0	+ 0 .4		213 stars, souther
Kobold	269 .0	− 2 .9		1579 stars
Kobold	270 .3	0 .0		1579 " in 122 g
Kobold	270 .2	− 2 .3		2262 "
Kobold	271 .0	− 0 .2		2262 "
Kobold	270 .4	+16 .5		2262 " in 122 g
Kobold	265 .1	− 8 .1		144 " large μ
Kobold			0 .23	Stars near Vernal
Kobold			0 .79	" " Antape
Kobold			0 79	" " Autum
Kobold			0 .25	" " Apex
Kobold			0 .22	" in Hemisph.
Kobold			0 .82	" " "
Weersma	270 .2	+30 .8		Bradley stars
Weersma	268 .0	+31 .4	14.9 km.	All available (190

sed	References
	Phil. Trans., 1783 (Abridged Ed.), p. 402
	Mem. de l'Acad. de Prusse, 1781, p. 445
	Phil. Trans., 1805, p. 256
	Phil. Trans., 1806, p. 233
	Astr. Nach., **16**, 48, 1838
	Astr. Nach., **21**, 73, 1843
	Dorpat Beob., **14**, 227, 1856
	Mem. R. A. S., **28**, 160, 1860
	Mem. R. A. S., **32**, 27 and 31, 1864
	Mem. de l'Acad. St. Petersbourg (VII), 35, p. 24
; to 0″.32	*Astr. Nach.*, **125**, 425, 1890
? to 0 .64	" " " " "
t to 1 .28	" " " " "
3	" " " " "
)	*Astr. Jour.*, **12**, 93, 1892
) to 0 60	" " " " "
) to 1 .20	" " " " "
)	" " " " "
	" " **20**, 4, 1899
	" " " " "
	Astr. Nach., **156**, 17, 1901
°+ to +5°	*Astr. Jour.*, **9**, 165, 1890
" to "	" " " " "
	Astr. Nach., **161**, 347, 1903
	" " **156**, 15, 1901
	" " " " "
	" " " " "
	" " **144**, 39, 1897
	" " **144**, 40, 1897
ups	" " " " "
	" " **150**, 269, 1899
	" " **150**, 279, 1899
ups	" " **150**, 272, 1899
	" " **166**, 3, 1904
quinox	*Bau des Fixsternsystems*, p. 131
	" " " " " "
l Equinox	" " " " " "
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f Vernal Eq.	" " " " " "
Autum. Eq.	" " " " " "
	Publ. Astr. Lab. Gron., No. 21, p. 21, 1908
μ's	*Publ. Astr. Lab. Gron.*, No. 21, p. 57, 1908

would contain fully twice as many names and more than twice as many entries. Numerous results by Kapteyn and Kobold for special lists of proper-motion stars are not quoted. The references to their more comprehensive results will lead the reader to the omitted ones.

We stop for a few comments on these results. The right ascensions vary from Herschel's 247° to Boss's 289°, or 42°; the declinations range from Porter's + 54° to Kobold's — 8°, total 62°.

The determinations based upon only a few stars, upon stars distributed over a small part of the sphere, or upon stars whose proper motions are of selected dimensions, should have small weight. Some of the discordances in the table must be due in part to systematic errors in the catalogues of star places as well as to small errors in the precession factors. Kapteyn has shown, for example, that the discordant apical declinations assigned by several determinations of greatest weight are made more accordant by allowing for systematic errors in the declination components of the proper motions, and by the use of a corrected value for the precession constant. The values of q, the annual solar motion, are in terms of the average distance of a 1.0 magnitude star. Results for the speed of solar motion obtained by several computers, but not quoted in the table, are very discordant, some of the results being ten times as large as others. The uncertainties in the assigned values are perhaps indicated in fair measure by Kobold's results: the stars in one hemisphere of the sky assign a solar velocity three and a half times as great as that assigned by the stars in another hemisphere of the sky.

The weighted mean of the determinations would seem to place the apex in Right Ascension 271°, Declination + 31°. It is a convenience, however, to assume it in position $\alpha_0 = 270°$, $\delta_0 = + 30°$; and this is certainly well within the limits of probable error. The definitive proper motions of more than six thousand stars contained in Professor Boss's *Preliminary General Catalogue,* now almost through the press, should form the basis for a solution of the solar-motion problem which will undoubtedly render existing solutions in a sense obsolete.

Kapteyn[18] announced, in 1904-1905, that his researches on proper motions during preceding years not only confirmed Kobold's conclusions as to the non-random character of stellar motions, but justified him in announcing the directions to which stellar motions give preference. He had used again the proper motions of more than 2400 Bradley-Auwers stars, extending from the North Pole to Declination $-31°$. Dividing this three-quarters of the sky into twenty-eight compact areas, averaging nearly 100 stars to each area, he determined the prevailing tendencies in each area, by a method which we shall not here take time to present. The outcome was that these prevailing direction-tendencies divided themselves quite clearly into two groups; one group pointing to a certain not very large area of the sky and the other group to another well-defined area. These two areas, widely separated, represent the convergent points of the preferential proper motions. Kapteyn assigned the position of one area at $\alpha = 85°$, $\delta = -11°$, and of the other area $\alpha = 260°$, $\delta = -48°$. While the apparent motions of the stars favored one or the other of these centres, we must not overlook the effect of the solar-motion component. When this component is eliminated the mathematical principles involved leave us no recourse but to assume that the stars, with reference to the stellar system as a whole, have a preference for motion in diametrically opposite directions, either toward a point at $\alpha = 91°$, $\delta = +13°$ in the northern edge of *Orion,* or toward the antipodal point at $\alpha = 271°$, $\delta = -13°$. These points Kapteyn has denominated the "vertices of preferential motion." It must not be understood that the individual stars according to this theory are actually moving parallel to the straight line joining these vertices, but simply that their components of motion parallel to this line are considerably greater, on the average, than in any other direction. We may visualize these ideas in the following manner:

Assume the existence of a great cluster of stars, spherical or otherwise, distributed through space uniformly or otherwise,

18 *Congress of Arts and Sciences, St. Louis,* 4, 413, 1904; *Report B. A. A. S.,* 1905, p. 257.

whose individual motions were at random in both magnitude and direction. Suppose, further, that an entirely similar group of stars, whose members were moving at random, occupied another volume of space. Let these two groups of stars approach each other and more or less completely interpenetrate. Necessarily the resultant preferential motion in the *combined system* is in the direction of the line of approach of the two systems. There are stars moving in all directions, with speeds of all dimensions within certain limits, and yet there exists a preference in the combined system for motion along and parallel to the line which originally joined the centres of the two groups. This is the line joining the vertices. Assume now that our Sun is moving through the combined group in a direction making a considerable angle with the line of preferential motion. The motions of the individual stars as observed from the solar system would have preferences for two directions very different from the line joining the vertices. We should then have in all essential matters the conditions which Kapteyn, working backwards so to speak, has described. Neither Kapteyn nor any other investigator, so far as I know, has expressed or holds the belief that the stellar system is the result of the combination of two primordial star groups which, originally separate, are now combined, and the description is intended to be merely a hypothetical help to the imagination.

To this tendency of the stars to move in opposite directions, Kapteyn has applied the term "star streaming," and it is customary to speak of two "streams" of stars which have preferential motions in opposite directions. Eddington has used the term, two "star drifts," in this connection.

Eddington, of Greenwich, gave substantial confirmation to Kapteyn's conclusions, based upon a study of the proper motions of about 4500 stars lying within 52° of the North Pole, which had been observed by Groombridge early in the nineteenth century and re-observed at Greenwich late in that century. He found strong evidence for the existence of two star drifts, substantially alike as to details of composition, the directions of the drifts agreeing closely with Kapteyn's directions.

Dyson, astronomer royal for Scotland, based a study of the subject upon about 1900 stars whose proper motions are greater than 20″ of arc per century. The characteristics of the two streams were brought out with great clearness. In fact, for the stars whose proper motions are greater than 100″ per century, he thought he was able to assign a majority definitely to one or to the other of the two streams. The theory was well illustrated by means of his tabulation of the apparent directions of the individual proper motions with reference to lines drawn from the separate stars to the assumed vertices of the two streams. The proper motions whose directions make small angles with the lines through the vertices are most numerous, and as the deviations from these directions increase more and more the number of corresponding stellar motions decreases almost continuously. Calling the speed of the solar system unity, Dyson's investigation led him to the conclusion that the relative speed of the two streams is 2.6; that is, for a solar speed of 19 km. per second, the separation-speed of the streams would be 48 km. per second.

Schwarzschild, of Göttingen, introduced a promising hypothesis in connection with this problem. There is a fair chance that it bas greater probability of conforming to the reality. He leaves the stars in one system instead of dividing them into two streams, or drifts, and assumes that the components of the real stellar motions are *on the average* greater in one direction than in any other; and that the actual stellar motions, as functions of their directions, can be represented in amount and in direction by all the radii of an ellipsoid whose longest axis coincides with the direction of relative motion in Kapteyn's two-stream theory. Assuming this hypothesis to be correct, and that our solar system is travelling through the stellar system not at right angles to the long axis of the ellipsoid, we should have another representation of the preferences of the observed proper motions for the two directions which Kapteyn had determined.

The difference between Kapteyn's and Schwarzschild's hypotheses may be more apparent than real. So far as we can now see, two streams of stars, thoroughly intermingled, with preferential motions in opposite directions, are essentially equivalent to opposite prevailing motions in one system, at least for periods of time which are in effect instantaneous.

Below are solutions for the apex of the Sun's motion and for the vertices of the two star streams, by Kapteyn, Eddington, and Dyson; and for the positions of the solar apex and of the vertices of Schwarzschild's ellipsoid of preferential motion, by Schwarzschild, Beljawsky, and Rudolph.

	Apex		Vertex	
	R. A.	Dec.	R. A.	Dec.
Kapteyn—Bradley stars*			91°	+13°
Eddington—Groombridge stars**	266°	+31°	95	+ 3
Schwarzschild—Groombridge stars†	266	+33	93	+ 6
Dyson—Stars of large proper motion††	281	+42	88	+24
Beljawsky—Porter's stars‡	281	+36	86	+24
Eddington—Zodiacal stars**			109	+ 6
Rudolph—Bradley stars§	268	+26	96	+ 7
Rudolph—Bradley stars§			87	+11
Means	272	+34	93	+12

The vertex, as Kapteyn noted, is in the Milky Way; that is, the mutual axis of the two streams lies in the plane of the Milky Way. This is not out of harmony with the view that prevailing stellar motions, if gravitational effects, should have their governing centres in the plane of the Milky Way.

It was thought by some, for a time, that the two apparent star streams might not have an objective existence, but that they could be explained on the basis of systematic errors in star catalogues of position, yielding erroneous stellar proper motions. However, the question has been investigated from so many points of view, upon the basis of observational data taken from independent sources, all leading to positions for the vertices of the streams in remarkable accord, that we cannot hesitate to accept

* *Report B. A. A. S.*, 1905, p. 257.
** *Mon. Not. R. A. S.*, **68**, 602, 1908.
† *Göttingener Nach.*, 1907, pp. 628, 631.
†† *Proc. Roy. Soc. Edinburgh,* **28**, 231, 1908, and **29**, 376, 1909.
‡ *Astr. Nach.*, **179**, 298, 1908.
§ *Astr. Nach.*, **183**, 5-6, 1909.

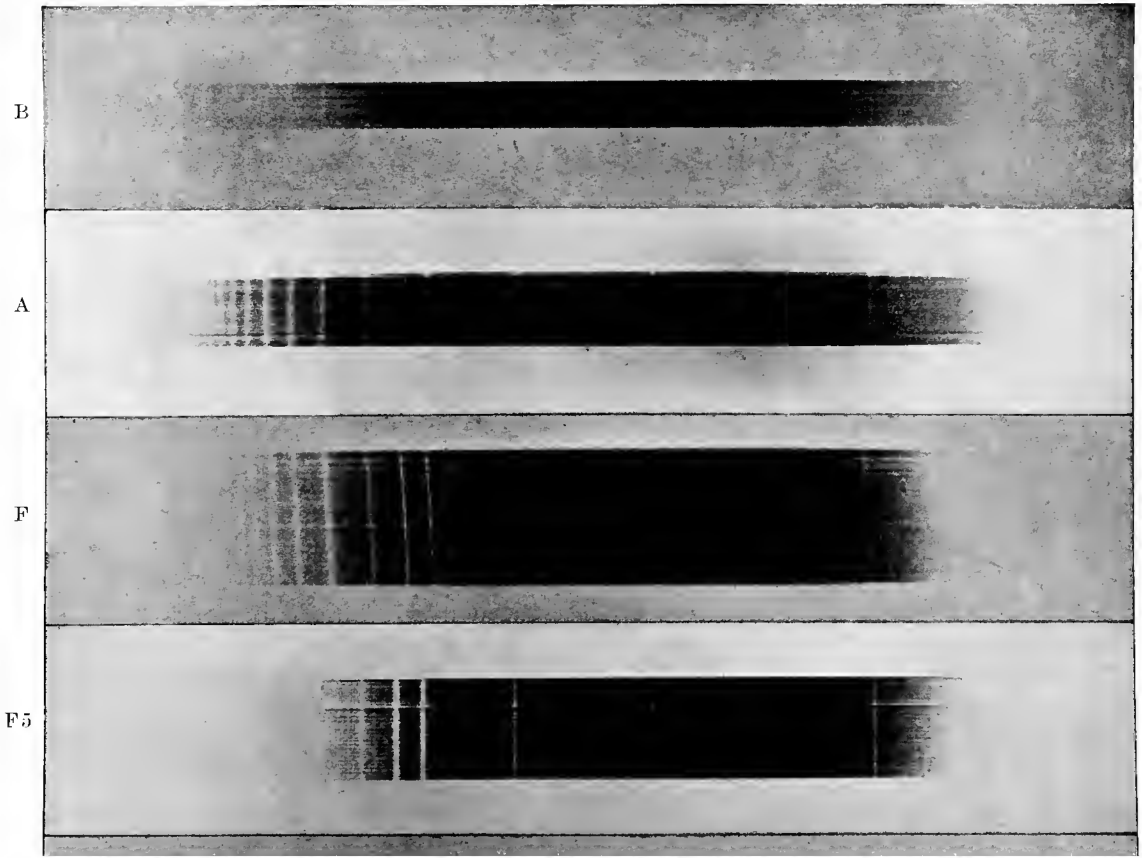
the theory as representing a cosmical truth. Later, we shall consider the subject in the light of radial velocity methods and results.

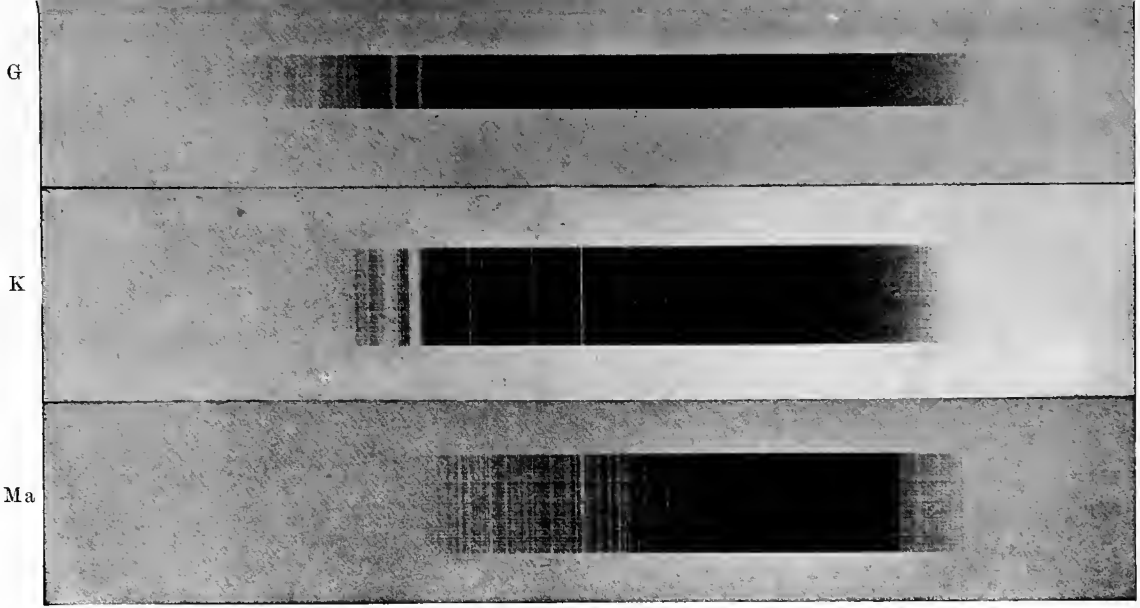
We should inquire as to whether and to what extent the stars whose proper motions have been utilized in determining the solar motion are representative of the general stellar system. It is well known that our sidereal universe appears to have vastly greater extension in the plane of the Milky Way than in any other direction. The prevailing idea of the form of the universe is, that it is roughly an ellipsoid whose minor axis at right angles to the plane of the Milky Way is extremely short in comparison with the major axes in the plane of the Milky Way. Bearing vitally upon this question are the "star gauges" of the Herschels, father and son; and we may recall in passing that it was Sir William Herschel who first brought the question of the form of the universe under quantitative consideration. Using a reflecting telescope of 18 inches aperture, 20 feet focus, and magnifying power of 180 diameters, Herschel counted the total number of stars visible in the field of view, 15′ in diameter, in about 5000 regions distributed over the northern sky. It was his habit to average the results for one to ten or more closely neighboring areas into one result, and in this manner the counts formed 1088 star gauges.[19] Sir John Herschel employed identically the same instrument at the Cape of Good Hope in counting the number of stars in 2299 areas uniformly distributed over the southern sky, suitable precautions being taken to avoid or to eliminate the effects of fields which happened to be extraordinarily rich or poor in stars. The limit of visibility for the telescope used was probably in the vicinity of 13½ visual magnitude.

These invaluable data have been made the basis for several investigations of stellar distribution, with reference to the Milky Way as a plane of symmetry. Wilhelm Struve[20] made a statistical study of Sir William Herschel's star gauges. He expressed

[19] *Publ. Washburn Obs.*, 2, 113-173, 1883; 405 "gauges" were here published for the first time.

[20] *Etudes D'Astron. Stellaire*, 1847, pp. 71-72; using the 683 gauges published by Herschel in *Phil. Trans.*, 1785.


B
A
F
F5



Typical Stellar Spectra.—Harvard College Observatory

Class B, ε *Orionis*
" A, α *Canis Majoris*
" F, α *Carinæ*
" F5, α *Canis Minoris*
" G, α *Aurigæ*
" K, α *Bootis*
" Ma, α *Orionis*

the law of their distribution, in terms of galactic latitudes, as follows:

North Galactic Latitude	Average number of stars per field 15′ in diameter
90°	4.15
75	4.68
60	6.52
45	10.36
30	17.68
15	30.30
0	122.00

Sir John Herschel published[21] his results for the Southern Hemisphere not for definite latitude values, but as averages for latitude zones 15° wide. By means of Struve's formula he expressed Sir William Herschel's results in the same manner. In the following table the first six entries contain Sir William's data for the zones north of the central plane of the Milky Way, and the last six Sir John's for the six zones south of the Milky Way.

North Galactic Latitude Zones	Average number of stars per field 15′ in diameter
+90° - +75°	4.32
+75 - +60	5.42
+60 - +45	8.21
+45 - +30	13.61
+30 - +15	24.09
+15 - 0	53.43
0 - —15	59.06
—15 - —30	26.29
—30 - —45	13.49
—45 - —60	9.08
—60 - —75	6.62
—75 - —90	6.05

A glance at the table shows that similar laws of density prevail in the two hemispheres with reference to the plane of the Milky

21 *Outlines of Astronomy*, 1849, p. 535.

Way, though the number of stars in the Southern Hemisphere appears to be slightly greater. Suitable allowance having been made for the greater transparency of the air at the southern observing station, the generally accepted explanation of the discrepancy is that the solar system is not in the central plane of the Milky Way, but that it lies a little to the north of it. Struve's statistical studies led him to the conclusion that the effective central line of the Milky Way is not a great circle of the sphere but is a small circle lying at a distance of 92° from the North Pole of the galaxy and 88° from the South Pole.[22]

The extremely rapid increase of star density with decreasing galactic latitudes is undoubtedly due to the greater extent of the stellar universe in the direction of the Milky Way, though Struve believed that the stars are somewhat more closely crowded together in space as we approach the Milky Way.

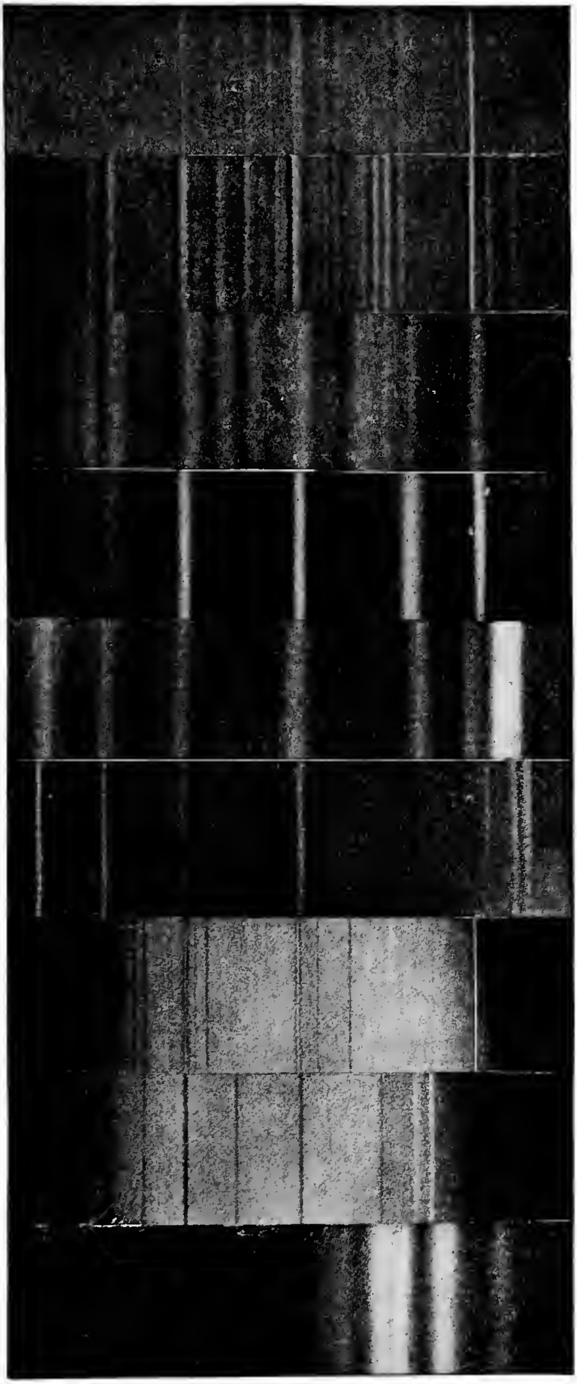
The distribution of naked-eye stars with reference to the Milky Way as the plane of symmetry was well brought out by Houzeau, as in the following table,[23] in which the first six magnitudes are quoted for nine zones of galactic latitude each 20° wide.

Galactic Latitude Zones	Numbers of Stars of Visual Magnitudes							
	1	2	3	4	5	6	Total	Density
+90°– +70°	0	1	4	8	42	86	141	0.113
+70 – +50	3	6	18	42	74	295	438	0.122
+50 – +30	1	4	21	70	148	439	683	0.124
+30 – +10	4	7	34	114	190	625	974	0.145
+10 – —10	7	11	46	108	243	730	1145	0.160
—10 – —30	3	11	34	111	257	619	1035	0.154
—30 – —50	0	7	28	65	141	465	706	0.129
—50 – —70	2	2	13	65	81	281	444	0.124
—70 – —90	0	2	2	12	37	100	153	0.125
Total	20	51	200	595	1213	3640	5719	0.139

The totals for the nine zones should, of course, differ, even for uniform distribution, because the areas of the zones are unequal,

[22] *Etudes D'Astron. Stellaire,* 1847, p. 61.

[23] *Annales de l'Observatoire Bruxelles,* 1, 51, 1878.



η Carinæ
Nova?

Nova
Aurigæ

Nova
Normæ

Nova
Sagittarii

N. G. C. 7662
Gaseous
Nebula

N. G. C. 7009
Gaseous
Nebula

μ Centauri
Hydrogen lines
bright

ζ Puppis
Secondary Hydrogen lines

π_1 *Gruis*
Spectrum
Peculiar

PECULIAR STELLAR SPECTRA.—Harvard College Observatory but the increase of star density with decreasing latitudes is demonstrated in the last column. A further condensation of the table,[24] as below, is illuminating.

Galactic Latitude Zones	Magnitudes 1+2+3		Magnitudes 4+5+6	
	Number of Stars	Density	Number of Stars	Density
+90°-+30°	58	0.00563	1204	0.117
+30 - —30	157	0.00761	2997	0.145
—30 - —90	56	0.00543	1247	0.121

From the relative densities, we see that the stars in both groups show a moderate preference for low galactic latitudes. The central zone, + 30° to — 30°, embracing one-half of the sky, contains 3154 stars, whereas the other two zones together, embracing the other half of the sky, contain only 2565 stars.

Investigations analogous to those described have been made by several astronomers, for both the brighter and the fainter stars, but we have space only for a few of Seeliger's results, as published in numerous papers presented, for the most part, to the Munich Academy of Sciences. Among other sources of information, he used the Herschel star gauges, and the *Bonner Durchmusterung,* which Argelander and Schönfeld intended should include all stars, to the ninth magnitude, from the North Pole of the heavens to Declination — 24°. Seeliger divided the sky into nine galactic zones, each 20° wide, as in the first column of the following table. The average numbers of Durchmusterung stars down to magnitude 9.0 per square degree in the several zones are in the second column, headed "DM. Density." The third column contains the Durchmusterung densities after due allowance has been made for the fact that the brightnesses were overestimated, in effect, in the regions containing few stars, and underestimated in the denser regions lying in the Milky Way. The last column contains the Herschelian densities.

[24] *Annales de l'Observatoire Bruxelles,* **1**, 52, 1878.

Galactic Latitude Zones	DM. Density per sq. degree	Corrected DM. Density	Herschelian Gauges
+90°- +70°	3.06	2.78	107
+70 - +50	3.24	3.03	154
+50 - +30	3.80	3.54	281
+30 - +10	5.34	5.32	560
+10 - —10	7.36	8.17	2019
—10 - —30	5.94	6.07	672
—30 - —50	3.99	3.71	261
—50 - —70	3.56	3.21	154
—70 - —90	3.51	3.14	111

Extremely important facts concerning the stellar universe are apparent from Houzeau's and Seeliger's tables. Stars to the sixth magnitude have a small but certain preference for the galaxy. Stars down to the ninth magnitude have threefold greater density of distribution in the zone containing the plane of the Milky Way than in the zones containing the galactic poles. The stars down to the 13½ magnitude are shown by Herschel's gauges to have nearly twenty times greater density of distribution in the Milky Way than at the galactic poles. These facts make the principal basis for existing ideas of the form of the stellar universe.

Professor Pickering's studies on the distribution of the spectral classes, especially with reference to the Milky Way, have been very extensive and fruitful. Not to enter upon the details, he wrote in 1891: "It appears that the number of stars of the (Secchi) second and third type is nearly the same in the Milky Way as in other parts of the sky. Considering, therefore, only the stars whose spectra resemble that of our Sun, we should find them nearly equally distributed in the sky. The stars of Class A on the other hand are twice as numerous in Region M (the Milky Way areas) as in Region N (outside the Milky Way), and in the case of Class B this ratio exceeds four. The Milky Way is, therefore, due to an aggregation of stars of the first type, a class to which our Sun seems to bear no resemblance as regards its spectrum. Spectra of Class B seem to conform still more closely to the region of the Milky Way, although probably they are not sufficiently numerous to materially affect its light. The Milky Way must, therefore, be described as a distinct cluster of stars to which, from its composition or age, the Sun does not seem to belong.

"The proportion of stars of Class A in the Milky Way appears to be greater for the faint than for the bright stars."[25]

Pickering has tabulated the numbers of stars of the different spectral classes, brighter than visual magnitude 6.25, in terms of their galactic latitudes, as in TABLE VII. By way of explanation, the latitudes quoted in the first column are the average latitudes of each of eight approximately equal zones, beginning with that which surrounds the north galactic pole and ending with that which surrounds the south galactic pole. If the stars of the different spectral classes were uniformly distributed over the sky, the numbers for the eight zones, in each column, should be equal. It is seen that they are very unequal, most of all for Class B stars; being more nearly equal as we pass through the advancing spectral classes to Class M.

Pickering draws these conclusions from the table: "It seems probable that its (the Milky Way's) effect extends over the entire sky. Even in Class A, to which the Milky Way is mainly due, the falling off in numbers, as the latitude increases numerically, shows itself in all except the first line. This would indicate that the Milky Way consisted of a layer of stars rather than a ring. In all the classes, the number of stars is greater when the latitude is small, or all are affected by the Milky Way."[26]

TABLE VII

GALACTIC LATITUDES OF STARS BRIGHTER THAN 6.25

Latitude	B	A	F	G	K	M	All
+62°.3	8	189	79	61	176	56	569
+41 .3	28	184	58	69	174	49	562
+21 .0	69	263	83	70	212	57	754
+ 9 .2	206	323	96	99	266	77	1067
— 7 .0	161	382	116	84	239	45	1027
—22 .2	158	276	117	100	247	69	967
—38 .2	57	161	94	59	203	59	633
—62 .3	29	107	77	67	202	45	527
Sums	716	1885	720	609	1719	457	6106
Mean	90	236	90	76	215	57	763

25 *Annals H. C. O.*, 26 (1), 152, 1891.

26 *Annals H. C. O.*, 64, 143-144, 1909.

I have been interested in tabulating the numbers of stars of Classes B, A and K contained in the *Revised Harvard Photometry,* with the following results:

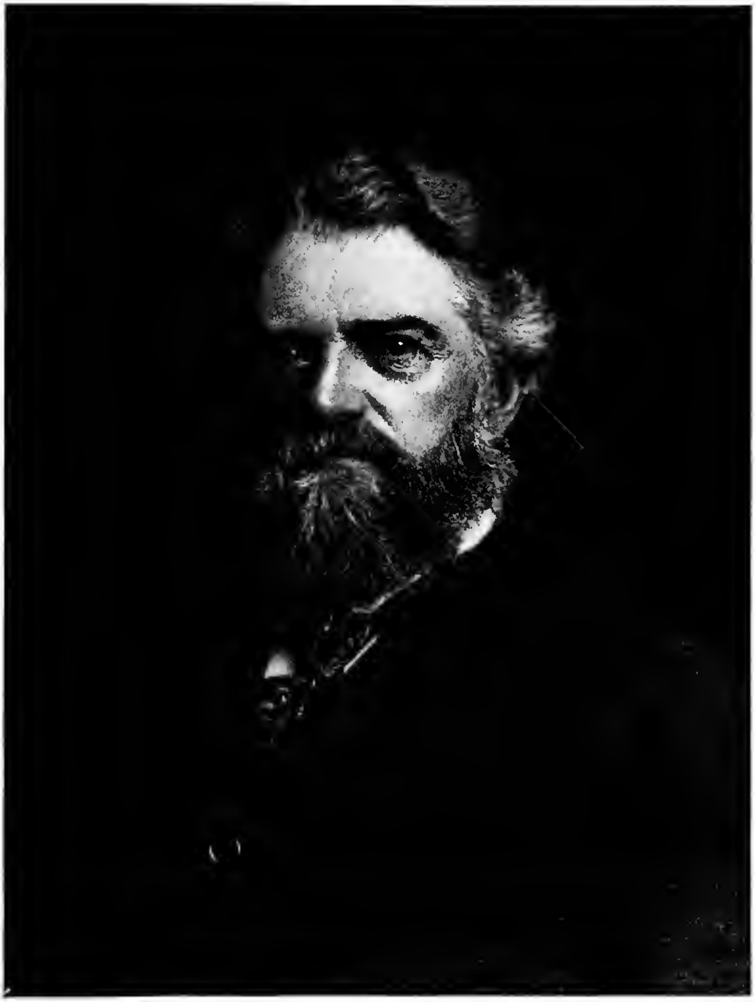
Vis. Mag.	B	A	K
> 5.01	350	318	403
5.01–5.50	217	346	318
5.51–6.00	163	790	560
6.01–6.50	164	1423	939

The rapid decrease in the number of Class B stars with decreasing brightness, and the rapid increase of Class A stars with decreasing brightness are striking. Pickering has said of the Class B stars, that "of the bright stars one out of four belongs to this class [B], while of the stars of the sixth magnitude there is only one out of twenty; and that few [of Class B], if any, will be found fainter than the seventh or eighth magnitude."[27] The figures are, of course, in complete harmony with Pickering's statement quoted above, that the Milky Way is composed largely of Class A stars.

The solar-motion solutions described above have necessarily been limited to those stars which have moved perceptibly in the interval covered by accurate observations of their positions. Do the available proper motions refer only to those stars which are relatively close to us, or may we consider them as representative also of the distant stars in the Milky Way? The best available test for nearness, aside from direct measures of parallax for the nearest hundred stars, more or less, lies in the proper motions. Large proper motions are in general indicative of proximity, though there are known exceptions to the rule. Kapteyn is of the opinion that the stars whose proper motions exceed 5″ per century are distributed nearly uniformly over the sky; that is, with little tendency for clustering in or near the Milky Way; but for proper motions smaller and smaller, there is a continuous tendency for increasing numbers as we approach the Milky Way. Newcomb, writing ten years ago,[28] questions the latter

[27] *Annals H. C. O.*, 56 (II), 37, 1905.

[28] See Newcomb's *The Stars,* 1901, pp. 252-256.



SIMON NEWCOMB, 1835-1909

statement, and is convinced that "if we should blot out from the sky all the stars having no proper motion large enough to be detected, we should find remaining stars of all magnitudes; but they would be scattered almost uniformly over the sky and show little or no tendency to crowd toward the galaxy unless, perhaps, in the region at nineteen hours of right ascension. From this again it follows that the stars belonging to the galaxy lie further away than those whose proper motions can be detected."

The term "Milky Way stars," or "stars belonging to the galaxy," is understood to have the following significance: Let the form of the sidereal system be assumed to be an ellipsoid, with one relatively very short axis, $2a$, passing through the present position of the solar system at right angles to the plane of the Milky Way, and with relatively very long axes in the plane of the Milky Way. All stars at a distance from the solar system greater than a, *i. e.*, all stars lying outside of the sphere of radius a, whose centre is at the solar system, may be described in position as "Milky Way stars."

Eddington's evidence as to the mean distances of the Groombridge stars[29] affords interesting testimony on the question. "The mean parallax steadily increases with the distances from the galaxy, a result which is in accordance with the generally accepted ideas of the distribution of stars, viz., that the increased number of stars in the low galactic latitudes is due to additional more distant stars being visible, and not to any crowding among the nearer stars."

These differences should be resolved to a considerable extent when the proper motions of Boss's catalogue become available. It must be regarded as uncertain to what extent the proper-motion stars, extending down to the 9.5 magnitude in some cases, can be considered as representative of the great majority of stars.

Certainly, to take extreme but actual cases, a very distant star of unusually great mass, though possessing average real motion, cannot enter the solar-motion solution, for its proper motion is imperceptible, whereas a small star close to us, yielding a large proper motion, may enter the solution powerfully. It is difficult

29 *Mon. Not. R. A. S.*, **68**, 104, 1907.

to realize the inequality of stellar brightness. In the days of Herschel it was supposed that *Rigel* and *Canopus,* for example, were our near neighbors, but the fact is they are immeasurably distant. Sir David Gill has been unable to obtain an appreciable or certain parallax for them. The parallaxes can scarcely exceed .01 or .02 of a second as a maximum. We can scarcely doubt that *Canopus* is radiating certainly 1000 and perhaps 10,000 times as much light as the Sun. If the effective radiating power of its surface equals that of the Sun, the surface must be fully 1000 times as great as the Sun's. Its corresponding volume would be 31,000 solar volumes. Its mass must greatly exceed the Sun's mass, probably between 1000- and 30,000-fold.

At the other extreme is the large proper-motion star *C. Z.* $5^h.243$, with parallax $0''.32$. It is one of our nearest neighbors, yet its visual magnitude is but 9.2. Its output of light—visual radiations—cannot well exceed 1/100 of our Sun's. Its spectrum is of the solar type, and its surface area is, therefore, probably related to our Sun's surface area, roughly, as their luminosities. On this supposition, its radius is but 1/10 and its volume but 1/1000 of the Sun's. Its mass is probably less than 1/1000 of the solar mass, for its internal forces of gravitation may be very small. There is scarcely any doubt that this star is decidedly less massive than the planet *Jupiter*. In *Canopus* we, therefore, have a star whose mass may be 1000 times 1000, or 1,000,000 times as great as that possessed by the little neighbor referred to.

It is of course unsafe to make a positive statement on this or other subjects involving a relationship between mass and luminosity. For example, the companion of *Sirius* is perhaps the star of least relative luminosity thus far recognized. Its output of light is certainly not 1/100 that of our Sun, yet according to Auwers its mass is 4 per cent greater than our Sun's mass. Unfortunately we know nothing as to its spectrum.

We cannot overlook the fact that the question of stellar masses must some day be taken into account in statistical researches on the solar motion. Up to the present time, investigators have depended upon the doctrine of averages to smooth away the influences of excessive stellar mass and excessive proper motion, and this condition will no doubt continue into the future. As our knowledge of proper motions increases, and as information concerning stellar distances, densities, and masses becomes available—it is now pouring in from many directions—the solutions can be made more and more representative of the stellar system, and therefore possess increasing weight.

Kapteyn has investigated[30] the question of stellar luminosities, in terms of the absolute brightness of the Sun as unity. He assumed that the faint stars are more distant than the bright ones, and that the stars with small proper motions are more distant than those with large motions. His classification as to relative distances, according to these two criteria, was calibrated, so to speak, by means of the magnitudes, proper motions, and distances of those stars whose parallaxes are fairly well determined. Assuming further that the mixture of stars of different luminosities is everywhere the same throughout a sphere whose radius is 555 light years (comprising all stars whose parallaxes are equal to a greater than 0″.006), he obtained the following table of relative luminosities for the brighter stars in this sphere:

No. of Stars	Times more luminous than Sun
1	100,000 to 10,000
46	10,000 to 1,000
1300	1,000 to 100
22000	100 to 10
140000	10 to 1
430000	1 to 0.1
650000	0.1 to 0.01

Kapteyn has not intended that these numbers shall be taken literally, but merely as a rough approximation to the truth. The smaller stars are much the more numerous. Data concerning the numbers, proper motions, and parallaxes of still fainter stars are wanting, but it is not probable that another line, in continuation of the table, should be added. It should be noted that a parallax of 0″.006 confines the classification to the nearer stars—to those well within the minor dimension of the oblate

30 *Congress of Arts and Sciences, St. Louis*, 4, 407, 1904.

spheroid of stars, and has little connection with the stars composing the extension of the universe in the direction of the Milky Way.

Newcomb estimates[31] that a sphere of radius 3300 light years ($\pi = 0''.001$) must include essentially all stars in the direction of the poles of the Milky Way. On the surface of such a sphere a star moving with average speed would have a mean proper motion of about 0″.6 per century. The proper motions of such distant stars cannot be known satisfactorily for one or two centuries, at least through present methods.

We may remark, in passing, that the calibration of magnitudes and proper motions by means of known parallaxes is a weak point in Kapteyn's and Newcomb's structures, for most of our known parallaxes are of stars selected for observation on account of large proper motions and presumed nearness. They are thus not representative of the stellar system as a whole, for their large proper motions are in many cases due to linear speeds greater than the average for the stars in general.

It is interesting to compare Newcomb's and Kapteyn's estimates of density of distribution of the stars; Newcomb's based chiefly upon determined parallaxes and general considerations; and Kapteyn's upon visual magnitudes and proper motions, as calibrated by known parallaxes.

Newcomb estimates[32] that there is one visible star for each volume of space equal to a sphere of radius corresponding to a parallax 0″.5, in our region of the universe. This sphere is of 6½ light-year radius. The volume of Kapteyn's sphere is $\left(\frac{555}{6.5}\right)^3 = 625{,}000$ times Newcomb's unit sphere; and this would be Newcomb's number of stars in Kapteyn's sphere. Kapteyn's luminosity table sets down 1,250,000 ± for this volume, not including possible stars of less than .01 the Sun's luminosity. These numbers are not unreasonably discordant, considering that neither investigator defines clearly the limiting magnitudes included in the calculation. They can be usefully revised in

[31] *The Stars*, p. 316.

[32] *The Stars*, p. 310.

the light of information as to stellar distances supplied by line-of-sight results soon to be available.

Another idea, that there may be a slow rotation of our stellar system around an axis passing through the centre of mass of the system at right angles to the plane of the Milky Way, has long been held, and several experienced investigators have given it careful consideration. This idea is *a priori* not an unreasonable one. The spiral nebulæ, such as the *Andromeda* nebula or those in *Ursa Major* and *Canes Venatici*, are with little doubt in slow rotation about axes through their centres at right angles to the planes passing through their major dimensions; they are in general form somewhat the same as the supposed form of our universe; and, in harmony with spectroscopic observations, they either are now or in the distant future will be great systems of stars. May not our system, as viewed from tremendously distant space, preferably at right angles to the plane of the Milky Way, present the appearance of a great spiral nebula? To Sir John Herschel we owe a general statement of the problem.[33] Schönfeld was, perhaps, the first to give it mathematical expression.[34] It readily appears that a study of such minute rotational effects as are in question involves the true value of the precession constant as well as the speed and direction of the solar motion. The observed proper motions of the stars have been utilized by half a dozen astronomers in efforts to uncover the first indications of a rotation effect. Kobold[35] collected eleven results for the deduced angular speed of rotation; but as six of these indicate rotation toward the east, and five rotation toward the west, all being of minute value, we can say that the evidence in support of a rotatory motion of the stellar system is negligible. However, the problem is not an unpromising one for the distant future.

In this connection we merely mention Mädler's study of stellar motions in search of a great central sun, about which the individual stars may revolve. He named *Alcyone* in the *Pleiades* as

33 *Outlines of Astronomy,* 1849, p. 588.

34 *V. J. S. Astron. Gesell.,* **17,** 255, 1882.

35 *Bau des Fixsternsystems,* p. 118.

the central body. Mädler's idea appealed strongly to man's imagination, and it has become widely circulated in the more popular literature of science. More complete studies of stellar motions showed that the idea is without foundation. In fact, the position of *Alcyone,* far outside the plane of the Milky Way, as a centre about which the myriads of stars in the Milky Way are in revolution, is sufficient to stamp the idea as absurd.

CHAPTER V

THE SPECTROGRAPHIC DETERMINATION OF THE SOLAR MOTION

We now take up the solar motion and related problems from the radial velocity side. This method has several weighty advantages over the proper-motion methods, with apparently but one serious disadvantage; and this one, we hope, is of a temporary and passing nature.

1. Proper motions are measured and expressed in angle, and for linear motions of given dimensions these angles vary inversely as the stellar distances. Now the distances are unknown except in relatively few cases, and this lack of knowledge is a constant source of embarrassment, and a limitation, in the proper-motion methods of determining the Sun's way and speed. The spectrographic method, on the contrary, is independent of the distances of the stars; their velocities in the line of sight are determinable as readily and as accurately for an extremely distant star as for a near one, provided they are both bright enough, and provided further their spectra contain measurable lines. Distance is eliminated, save as distance governs the quantity of light delivered to the spectrograph.

2. The determination of proper motions requires that a long interval of time elapse between two accurate position observations. Auwers's revision of Bradley's great star catalogue, based upon meridian observations of 3222 stars extending from the North Pole down to Declination — 31°, made at Greenwich about 1755, with accuracy wonderful for that time, serves as the invaluable starting point for the proper motions of the brighter stars. However, the interval of 155 years to date is all too short as a base line for determining the proper motions of a considerable proportion of the stars in Bradley's catalogue. Radial velocities, on the contrary, can be measured accurately at once,

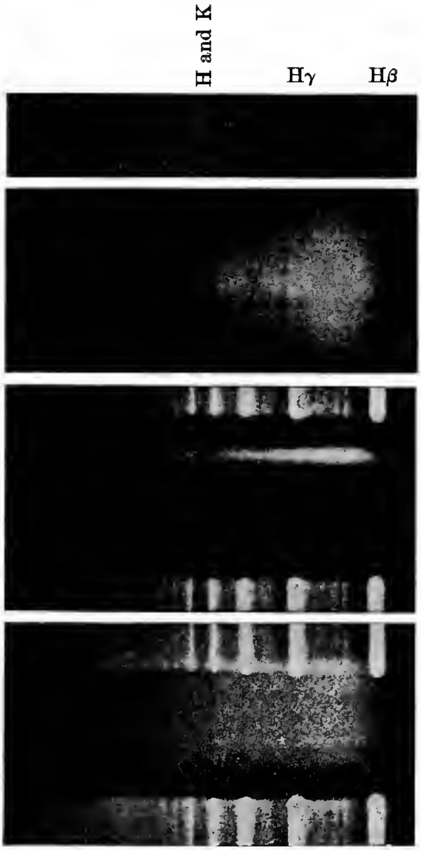
and expressed in definite and absolute units, say in kilometers per second.

3. Proper-motion determinations in the Southern Hemisphere are more than a half century behind those in the northern sky based upon Bradley's catalogue. This is a weakness we cannot overcome except as we give the stars in the Southern Hemisphere time to move. The contributions to this subject by Professor Boss under the auspices of the Carnegie Institution of Washington, in both the Southern and the Northern Hemispheres, should prove exceedingly valuable not only in the distant future but within a few years, though nothing except the lapse of time, so far as we now are able to say, can make up for the deficit of knowledge concerning the proper motions of southern stars.

4. Proper motions have been determined for the nearer stars only, excepting a few distant ones whose lateral motions are abnormally large, and they are, therefore, not representative of distant stars. The great mass of the stars are so far away that generations must come and go before they will have changed their directions appreciably. Solutions of the solar motion based upon proper motions refer the Sun's motion to the stars immediately around us.

If the cloud-like forms, which we are accustomed to call the Milky Way, were swept aside, we could still mark out its position by the larger number of naked-eye stars located near one great circle of the sphere. This great circle would make an angle of not over 5° with the real galactic circle. The excess of bright stars as we approach the Milky Way is made up of distant stars, of great luminosity, and presumably of great mass, whose proper motions are minute or uncertainly determined, and, therefore, of little or no influence in proper-motion solutions. Radial velocity determinations, on the contrary, include these stars as readily as fainter close stars, and are, therefore, more representative of the general stellar system.

5. If there is an absorption or obstruction of light in its transmission through interstellar space, as a function of the distances of the stars, any results involving assumptions as to the relations between stellar distances, stellar magnitudes, and



SPECTRA OF SUN, NEBULÆ AND CLUSTER.—Fath proper motions will be vitiated, unless the effects of such an absorption or obstruction can be allowed for. This subject, though an old one, has come prominently into the foreground within the past three years, for it is clearly recognized that proper-motion researches have reached the point where interstellar absorption should be taken into account, or shown to be non-existent. One should expect *a priori* that the transmission of energy through space would require that toll be paid, to some extent, but the insensitive methods thus far applied have not certainly detected such an effect. Radial velocity methods appear to have the advantage, unless, perhaps, a dispersing effect in interstellar space really exists, in accordance with the announcements of Nordmann and Tikhoff, already considered (page 87). If space absorption is in reality space obstruction by concrete particles of considerable size, it does not seem that radial velocities are affected with error on this account.

6. Only the brighter stars are thus far amenable to radial velocity measurement, it is true, for the light is weakened by spreading it over the large area of the spectrum, whereas the point image of a very faint star is available for proper-motion determination. The number of well-determined radial velocities is now in the neighborhood of 1100, and is rapidly increasing. Inside of ten years the accurately determined radial motions may well exceed the number of known proper motions of corresponding accuracy. The processes are straightforward, more powerful telescopes can be constructed, and the number is merely a function of the energy devoted to the subject.

However, the greatest value of radial velocities lies not in their advantages over proper motions, but in aiding proper motions to come into their own strategic worth. The radial and cross motions are mutually supplementary, and when they are skillfully combined, as perhaps 1500 should be within the next five years, after Boss's proper motions in both Northern and Southern Hemispheres shall have been published, and current programs of radial velocities shall have been carried through and published, the results for the solar motion should be of a weight far surpassing any hitherto deduced.

Several determinations of the solar motion have been made in the last quarter century from radial velocity determinations, with results as below:

SPECTROSCOPIC DETERMINATIONS

	α_0	δ_0	V_0	
Kövesligethy*	(261°)	(+35°)	64$^{km.}$	About 70 stars, visual
Homann**	320	+41	39.3 ± 4.2	Greenwich observations, visual
	310	+70	48. ±23.1	Huggins's observations, visual
	279	+14	24.5 ±15.8	Seabroke's observations, visual
Kempf†	206	+46	18.6 ± 3.0	51 stars, photographic
	160	+50	13.0 ± 3.3	41 stars, photographic
	(267)	(+31)	12.3 ± 3.0	41 stars, photographic
Risteen††	218	+45	17.5	42 stars, photographic
Campbell‡	277.5	+20	19.9 ± 1.5	280 stars, photographic

Here V_0 is equivalent to q in TABLE VI (CHAP. IV) of proper-motion solutions. Kövesligethy's solution is founded upon the visual measurements of radial velocities made at Greenwich prior to 1881. The right ascension and declination of the apex (261°), (+ 35°), do not depend upon the radial velocity results, but were assumed from previous proper-motion solutions. The failure of the visual method is apparent from the deduced value of the Sun's velocity, 64 km. per second, which is certainly three times too great.

The details of Homann's investigation, published in his inaugural dissertation, Berlin, 1885, are not accessible to me; but the solution depending upon Seabroke's observations is certainly remarkable for its agreement with recent solutions based upon a great mass of spectrographic observations. Whether the agreement is accidental, or indicates freedom from systematic errors, is uncertain, especially in view of the large probable error assigned to the velocity, V_0.

* *Astr. Nach.*, **114**, 327, 1886.
** *Astr. Nach.*, **114**, 25-26, 1886.
† *Astr. Nach.*, **132**, 81-82, 1893.
†† *Astr. Jour.*, **13**, 75, 1893.
‡ *Ap. J.*, **13**, 83, 1901.

Kempf's solutions, based upon the Potsdam spectrographic velocities obtained in 1888-1891, place the apex of the solar motion approximately midway between the positions of the apex and antapex assigned by proper-motion solutions. Under this condition, the fair agreement of the solar velocities 18.6 and 13.0 km. with recent determinations of greater weight has little significance. Kempf's first solution depends upon all of the fifty-one Potsdam velocities. In the second solution the radial velocities of five *Orion* stars were combined into one resulting velocity; and similarly for five *Ursa Major* stars and for three *Leo* stars; inasmuch as the individual stars in each of the three groups seemed to have related and essentially equal velocities.

Assuming the proper-motion position of the apex as in the parentheses, Kempf's third determination of the solar speed is 12.3 km. per second.

Risteen's solution is based in effect upon forty-two Potsdam spectrographic velocities. The velocities of five *Ursa Major* stars were combined and used as the velocity of the centre of gravity of the group. The velocities of *Sirius* and *Procyon* were rejected on account of known orbital irregularities not yet investigated. Three uncertain velocities were also rejected.

Up to the end of the year 1900, the radial velocities of somewhat over three hundred stars had been measured at Mount Hamilton, by means of the Mills spectrograph attached to the 36-inch refractor. Rejecting about twenty-five stars whose velocities were variable under the attractions of unseen massive companions, there remained 280 stars whose velocities were apparently constant.[1]

The 280 stars, in order to form the basis of a solution of the solar motion, were divided into eighty groups, each group representing the mean right ascension, declination, and observed velocity of from two to seven neighboring stars; on the average, three and a half stars in each group. These means are exhibited in TABLE VIII. It will be noticed that the observed velocities are prevailingly positive, indicating recession, in the hemisphere from 0 to 12 hours of right ascension; and prevailingly negative

[1] A great many of these have since been found to be variable.

in the other hemisphere. In the former only ten groups out of forty have negative velocities, and in the latter only five out of forty are positive.

TABLE VIII

No. of Stars	Mean R. A.	Mean Dec.	Mean Observed Velocity	No. of Stars	Mean R. A.	Mean Dec.	Mean Observed Velocity
	h m		km.		h m		km.
2	0 26.5	−14°.0	+17.0	3	12 20.4	+43°.0	− 4.7
6	0 40.0	+58 .8	−10.3	3	12 35.7	−22 .5	− 1.7
5	0 45.0	+28 .2	−24.2	3	12 48.1	+ 4 .9	−17.0
3	1 23.0	−10 .1	+12.7	2	13 6.2	+23 .2	− 8.2
4	1 35.5	+43 .9	+ 0.2	2	14 9.2	− 7 .6	+ 3.0
3	1 43.7	+10 .6	+ 7.3	5	14 14.9	+18 .1	− 3.9
2	1 47.4	−19 .0	+ 2.5	2	14 29.5	−25 .6	+11.8
2	2 2.6	+24 .2	−19.0	7	14 58.0	+29 .6	−13.1
9	2 38.5	−14 .1	− 3.2	4	15 2.4	+46 .0	−28.5
3	2 54.4	+ 0 .1	+21.7	4	15 23.2	+ 4 .8	− 7.0
5	2 55.4	+50 .2	+13.6	4	15 27.4	+65 .8	−14.2
4	3 3.8	+41 .2	+ 4.5	4	15 41.4	−12 .9	− 1.0
3	3 16.9	+13 .6	+ 6.7	4	16 17.1	−20 .8	−11.1
3	3 45.8	− 9 .2	−10.7	4	16 32.8	− 8 .5	− 7.0
3	4 3.2	−17 .3	+34.8	3	16 38.0	+13 .3	−26.0
5	4 16.6	+18 .0	+35.5	3	16 49.5	+35 .9	−29.0
2	4 29.7	+79 .8	− 3.0	3	17 44.7	+53 .6	−22.3
4	4 46.8	+44 .7	+ 8.2	4	17 53.6	+29 .5	− 9.9
3	5 15.7	+37 .4	+30.7	3	17 53.8	+ 5 .3	− 7.5
2	5 17.0	+ 7 .1	+22.0	6	18 18.6	− 7 .2	− 5.4
4	5 23.5	−20 .9	+ 1.8	6	18 36.4	−23 .5	−23.5
3	5 32.6	− 7 .5	− 0.7	3	18 38.6	+19 .0	−28.5
3	5 51.5	+57 .7	+ 5.0	2	18 43.0	+41 .2	−20.5
4	6 18.1	+25 .0	+24.8	6	18 48.9	+69 .6	+ 6.4
2	6 41.0	−15 .6	+50.5	4	19 42.1	+ 6 .2	−19.9
3	6 43.3	+16 .7	+ 6.3	5	19 42.6	+23 .6	−11.6
3	7 32.4	+27 .0	+11.3	4	20 3.6	+55 .4	−44.2
2	7 33.8	−25 .1	+42.0	3	20 19.3	− 9 .1	− 8.7
2	7 47.0	+ 9 .3	+34.0	3	20 48.9	+43 .3	+ 1.0
2	7 50.6	− 6 .0	+21.0	5	20 58.5	+12 .8	−28.2
4	8 56.6	+10 .5	+26.9	5	21 1.3	+31 .9	− 2.6
2	8 57.8	+32 .0	+25.2	4	21 24.6	−15 .8	− 2.9
4	8 58.0	+65 .7	− 4.0	3	22 0.2	+12 .7	− 7.3
2	9 10.2	+47 .2	+19.0	2	22 13.6	+54 .7	−14.2
3	9 27.2	− 3 .7	+12.3	4	22 32.6	− 6 .7	−11.3
2	9 57.2	+22 .3	−15.2	5	22 52.9	+25 .4	− 0.2
5	10 40.2	−15 .2	+19.6	2	23 10.6	+71 .4	−26.5
5	10 44.7	+38 .1	− 2.8	4	23 10.7	−18 .7	+ 5.6
2	11 11.6	+66 .1	± 0.0	2	23 16.0	+43 .8	− 2.0
6	11 32.8	+ 6 .2	+ 3.7	4	23 31.0	+ 5 .0	− 0.9

The determination of the elements of the solar motion from radial velocity data, such as those contained in TABLE VIII, has the great merits of directness and extreme simplicity. We shall now develop the simple equations required for the solution.

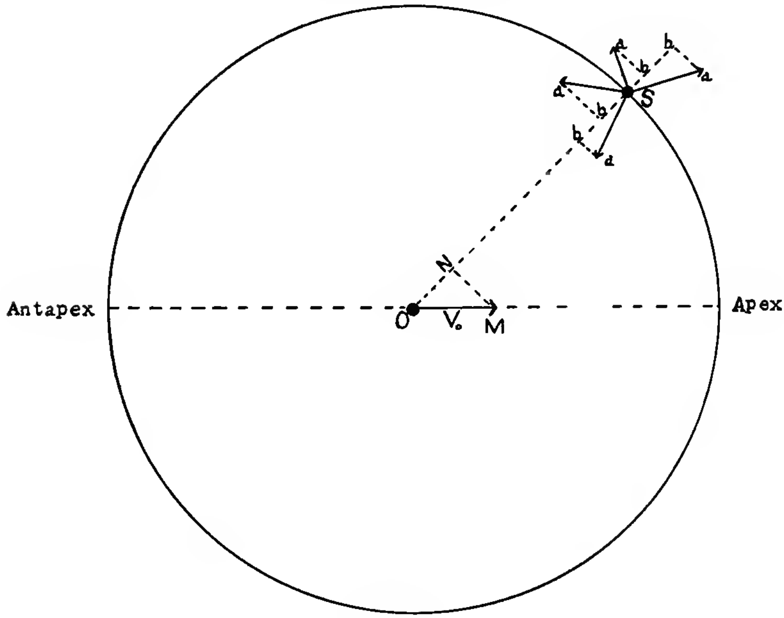


Figure 7

Let V_0 be the Sun's unknown speed with reference to the entire system of observed stars; V the observed speed of a star with reference to the solar system; and D the unknown angular distance of the star from the apex of the solar motion. Now, if the star is assumed to be at rest with reference to the whole system of stars, its apparent velocity with reference to the observer in the solar system will be

$$V_0 \cos D.$$

But every star has a motion of its own, v, of unknown amount. Let us suppose in Figure 7 that the small region of sky at S contains n stars, whose individual observed radial velocities are V_1, V_2, . . . V_n, and whose radial velocities with reference to the stellar system are v_1, v_2, . . . v_n. Then we may write the n equations

$$
\begin{aligned}
V_0 \cos D_1 &= V_1 - v_1 \\
V_0 \cos D_2 &= V_2 - v_2 \\
\cdots \quad & \quad \cdots\cdots \\
V_0 \cos D_n &= V_n - v_n
\end{aligned}
$$

Now the chances are assumed to be equally in favor of motions of approach and recession with reference to the stellar system, and small motions are assumed to be more numerous than large ones. As these are the conditions governing the occurrence of accidental errors, we cannot at present do better than to express the total radial motion of the n stars by the equation

$$V_0 \Sigma \cos D = \Sigma V,$$

obtained by summing the n individual equations.

If it is desired to write one equation for each observed velocity, we may assume that the equation

$$V_0 \cos D = V \tag{31}$$

is true except for the accidental error which represents the radial velocity of the star with reference to the system; and as there will be similar equations for neighboring stars, each affected by a corresponding assumed error of observation, the combination of the individual equations into a normal equation will eliminate, more or less successfully, the errors involved in the equations for the separate stars.

Let α_0, δ_0 be the coördinates of the unknown apex, and α and δ the known coördinates of a star. Then the value of $\cos D$ for the star will be defined by the well-known equation expressing the distance between two points,

$$\cos D = \cos \delta_0 \cos \delta \cos (\alpha_0 - \alpha) + \sin \delta_0 \sin \delta.$$

If we place

$$
\begin{aligned}
x &= V_0 \cos \alpha_0 \cos \delta_0, \\
y &= V_0 \sin \alpha_0 \cos \delta_0, \\
z &= V_0 \sin \delta_0,
\end{aligned}
$$

equation (31) takes the form:

$$\cos\alpha\cos\delta\ x + \sin\alpha\cos\delta\ y + \sin\delta\ z - V = 0, \tag{32}$$

from which the values of x, y and z may be determined. The values of V_0, α_0, and δ_0 may then be found from the relations

$$\left.\begin{aligned} V_0^2 &= x^2 + y^2 + z^2, \\ \tan\alpha_0 &= \frac{y}{x}, \\ \sin\delta_0 &= \frac{z}{V_0}. \end{aligned}\right\} \tag{33}$$

Forming the equation of condition (32) for each observed star or group of stars, and combining these into the three usual normal equations, we may solve the values of x, y and z, and thence by (33) for the direction of the solar motion as defined by α_0 and δ_0 and for the speed of the solar motion, V_0.

The right ascensions, declinations, and observed radial velocities of the 280 stars would have enabled us to construct the same number of equations of condition on the basis of equation (32). However, as explained in connection with proper-motion solutions of the solar motion, it is better to combine the results for a group of neighboring stars into a mean result, in order to eliminate in good measure the individualities of stellar motions and leave the mean result more truly representative of the system. Eighty equations formed from the mean results in the table were solved for the most probable values of α_0, δ_0 and V_0, with the results recorded in the last line in the table on page 167.

The right ascension of the apex, 277°.5, is in fair accord with the proper-motion results, but the declination of the apex, + 20°, places it further south than the proper-motion solutions, excepting Kobold's. The right ascension value has fair weight, for the observations were well distributed completely around the northern sky, east and west, so as to give symmetry to the solution in this coördinate. The declination result, while it may be near the truth, cannot be assigned so grêat a weight, for the southern third of the sky was completely unrepresented in the solution, no radial velocity observations having up to that time been attempted on the far south stars. It was to remedy this foreseen defect in the solution and to secure data for a later solution, as uniformly distributed over the sky as possible, that the D. O. Mills Expedition to the Southern Hemisphere was organized.

It was stated in CHAPTER III that the radial velocities of 1020 stars were known up to January 1, 1910, as measured with the Mills spectrograph at Mount Hamilton in the Northern Hemisphere, and by the D. O. Mills Observatory in the Southern Hemisphere. This list does not include about 200 stars which have been discovered in the progress of the observations to be spectroscopic binaries whose systemic velocities have not yet been determined; nor does it include about 200 other stars whose spectra contain lines of quality so poor that the spectrograms with moderate and high dispersion are not accurately measurable. Velocities of other stars not observed as yet by us, but measured at other observatories and published to date, number about forty. Adding thirteen of the results[2] for nebulæ obtained by Keeler, using visual methods, we have a total of 1073 radial velocities available for a first approximation in the solution of certain fundamental problems of the stellar system.

Practical questions exist as to the proper weights to assign to results of different degrees of accuracy when it is desired to combine them statistically. The apparent speeds of the brighter solar type stars, or those containing numerous well-defined lines, have been determined well within a kilometer per second; whereas the speeds of stars containing only broad and hazy lines may be in error up to a maximum of 6 or 8 km. per second; it being understood that stars containing only extremely broad and indefinite absorption bands, whose observed velocities might be in error 10 or 15 km., have been rejected from this investigation. Again, the speeds of the fainter stars, determined with 2-prism and 1-prism instruments, considered individually, have smaller weights than 3-prism results. When we combine the observations for all the stars, shall the weights assigned to individual

[2] Omitting the single rough measure of G. C. 5851 (R. A. $= 17^h\,8^m$).—*Publ. Lick Obs.*, 3, 205, 1894.

results be proportional to the inverse squares of their probable errors, as is usually the case? I think not, for we are dealing with special conditions. We desire that the solar motion shall refer to an observed program of stars which shall be as representative as possible of the entire stellar system. If we were to divide the stellar system into three sub-systems, one containing the bright solar type stars, a second the bright stars with hazy lines in their spectra, and a third containing the great mass of faint stars, and give greater weights to the observed velocities of the stars in one of these groups, the solar motion deduced would have discriminated against the other groups. The solution must refer to stars with hazy lines, or to faint distant stars, as truly as to bright solar type stars. One poorly determined result for stellar velocity used alone should have small weight, but a large number of such determinations should be given considerable weight, proper care being taken to avoid systematic error. Inasmuch as the results, even for the poorer stars, appear, at this stage of our knowledge, to be sensibly free from systematic error which can affect the solar-motion problem, and the number of such stars is much smaller than the number whose spectra admit of accurate measurement, I have assigned equal weights to all the velocities admitted into the solution. A study of the results of the present investigation will no doubt serve as a basis for a possibly more logical assignment of weights, when the time arrives for making another solution based upon a greater number of observed stars.

Before proceeding to apply this method to the data of observation at hand, it is necessary to decide whether any of the velocities for individual stars should be rejected, for one reason or another. For example, if there are groups or families of stars travelling along parallel lines, toward the same goal, and the computer should let each star, or several stars of such a group, enter his solution for the Sun's motion, his results would be vitiated in a small and corresponding degree: the velocities of such stars would not be independent of each other, as demanded by the basis of the solution. To illustrate by an extreme case: If one-tenth the stars used individually in the solution were members of a closely related system, moving through space along parallel or approximately parallel lines, with a speed equal to the average speed of the stars, whereas the remaining nine-tenths were moving in a truly haphazard manner, the one-tenth might influence the solution more than the other nine-tenths combined. Now several systems of related stars, aside from ordinary and extremely wide double stars, are known to exist, and the investigator of stellar problems should consider them.

Proctor[3] called attention to such a system, comprising five out of the seven bright stars in the *Big Dipper*. He found that all but the first and last of the seven stars possess nearly equal and parallel proper motions, and he concluded that they are, with little doubt, travelling along lines very nearly parallel, with linear speeds essentially equal.

Klinkerfues, before the days of accurate radial velocity determinations, had investigated this system. Höffler repeated the investigation when the Potsdam photographic radial velocities of four of the stars, β, γ, ϵ and ζ *Ursæ Majoris*, became available.[4] Vogel and Scheiner had assigned to β, γ, ϵ and ζ radial velocities in close accord, averaging — 30 km. per second, but they were unable to determine the velocity of δ because of the poor quality of the lines in its spectrum. Höffler found that the observed radial velocities and the observed proper motions conform, within the limits of unavoidable observational errors, to the hypothesis that these stars are travelling with equal linear velocities along parallel lines whose apparent "radiant"—the point at infinite distance from which the parallel lines seem to radiate—is at $\alpha = 123°.7$, $\delta = +34°.9$, and that they have an average parallax, $\pi = 0''.0165$.

Following the improvements in spectrographs and spectrographic methods, Ludendorff has recently secured long series of radial velocity measurements of β and ϵ *Ursæ Majoris*, both of which he found to be spectroscopic binary systems, and of ζ *Ursæ Majoris*, which is a well-known spectroscopic binary. As

3 *Proc. Royal Soc.*, **18**, 169, 1869.

4 *Astr. Nach.*, 144, 369, 1897.

TABLE IX.—URSA MAJOR GROUP.—LUDENDORFF

Star	α (1900)	δ (1900)	μ			ψ			Radial Velocity				H. R.	Remarks
			Obs.	Com.	O—C	Obs.	Com.	O—C	Com.	Lud.	V. & S.	L. O.	Spm.	
									km.	km.	km.	km.		
β *Urs. Maj.*	163°.95	+56°.92	0″.0871	0″.0853	+0″.0018	72°.3	70°.7	+1°.6	−16.8	−16.8	−29.3	−11.5	A	Sp. Bi., Ludendorff
γ " "	177 .14	+54 .25	0 .0944	0 .0996	−0 .0052	88 .5	86 .8	+1 .7	−15.4		−26.6		A	Spectrum v. v. poor
ζ "	182 .62	+57 .59	0 .1095	0 .1060	+0 .0035	88 .6	87 .9	+0 .7	−14.6			−11. ::	A2	Spectrum v. v. poor
ε " '	192 .41	+56 .50	0 .1140	0 .1159	−0 .0019	95 .6	96 .9	−1 .3	−13.2	−13. :	−30.3	− 9. :	Ap	Sp. Bi., Ludendorff
δ " "	199 .98	+55 .45	0 .1252	0 .1233	+0 .0019	101 .8	103 .5	−1 .7	−12.0	−12.6	−31.2	− 7.3	Ap	Sp. Bi., Harv. Col. Obs.
α " "	164 .39	+62 .29	0 .1414	0 .1428	−0 .0014	239 .5	238 .7	+0 .8	− 8.8	− 8.8	−11.9	− 9.7:	K	Vis. and Sp. Bi., Lick Obs.
η " "	205 .90	+49 .81	0 .1174	0 .1157	+0 .0017	260 .2	260 .8	−0 .6	−14.		−26.2		B3	Spectrum poor

the radial velocities of the three stars from the later observations are less than one-half those used in Höffler's study of the *Ursa Major* system, Ludendorff has made a complete re-investigation of the subject.[5] Assuming that the five stars are travelling along parallel lines with equal velocities, he finds that all observed data are satisfied within the limits of unavoidable error by an apparent radiant point at

$$\alpha = 123°.2 \pm 3.0, \delta = +36°.6 \pm 3.5 \text{ (mean error)};$$

an average parallax of the system

$$\pi = 0''.0352 \pm 0''.022;$$

and an actual velocity of the system (relative to the Sun), 20.7 km. per second.

The position of the radiant is in remarkably good accord with that assigned by Höffler, but the parallax is more than twice Höffler's value, chiefly because the radial velocities used by Ludendorff were less than one-half those used by Höffler.

TABLE IX exhibits the extent to which the hypothesis satisfied the observational data. The significance of the various columns will be clear from their headings. The differences between observation and computation in both proper motion and radial velocity are a little larger than we should expect in the hypothesis of motions absolutely parallel and equal. A share of the discordances may readily be due to the unavoidable errors of observation. Another share may arise from an error in the assumed position of the radiant, or in the assigned parallaxes; and of course the chief source of discrepancy may lie in the fact that the motions are not strictly equal and parallel. Under the heading, Radial Velocities, are given, respectively, the theoretical velocities as computed by Ludendorff, and the velocities as observed by Ludendorff, by Vogel and Scheiner (1888-1891), and by the Lick Observatory (up to 1912). There can exist no doubt that the five *Ursa Major* stars form a closely related system.

The last two lines of the table contain corresponding data for α and η *Ursæ Majoris*. Ludendorff's studies of the motions of

[5] *Astr. Nach.*, 180, 265, 1909.

these two stars led him to conclude that they are probably travelling with equal velocities along another set of parallel lines whose radiant point is situated at

$$\alpha = 270^\circ.2, \quad \delta = +36^\circ.5;$$

in which case the average parallax of the two stars is

$$\pi = 0''.0360,$$

agreeing almost perfectly with the parallax of the other system of five stars; and the actual velocity in space of the two stars (with reference to the Sun) is 20.8 km. However, some doubt must exist as to the close relationship of α and η, for the radial velocity of η is essentially unknown; and, as Ludendorff pointed out, the radiant practically coincides with the solar apex, the velocity of the two stars is nearly equal to the solar velocity, and the observed motions of α and η *Ursæ Majoris* are but a reflex of the Sun's motion.

From a consideration of the visual magnitudes and the parallaxes of the *Ursa Major* stars Ludendorff concluded that all are exceedingly brilliant stars. Their luminosities are to that of our Sun as assigned in the following table:

α	*Ursæ Majoris*	126 ☉
β	" "	72
γ	" "	66
δ	" "	32
ϵ	" "	105
ζ	" "	87
η	" "	95

Inasmuch as the five stars composing one system are moving as one star, it would not be permissible to let all enter as individual stars into the solar-motion problem. However, they represent five stars in mass—much larger stars than the average, we have strong reason to believe—and it would be unjust to let but one-fifth of this mass enter. Again, there are undoubtedly many cases of closely related stars, as yet undetected, in other parts of the sky for which the individual velocities will be used in the solution. A fair compromise, I have decided, is to let two of the five stars enter the solution; δ, with undetermined velocity, and two others, γ and ϵ, are rejected.

Since some uncertainty existed as to whether α and η have motions in common, both were utilized in the solution.

Shortly after the publication of Ludendorff's paper, Hertzsprung called attention[6] to the fact that eight other prominent stars, including *Sirius*, appear to be moving along lines parallel to those followed by the five *Ursa Major* stars, and with equal linear speeds. By trial he determined that position of a radiant point on the great circle defined by the proper motion of *Sirius*, near the intersection of that circle and the great circle representing the average proper motions of the five *Ursa Major* stars, which would make the sum of the angular deviations of the remaining proper-motion circles (omitting that of β *Eridani*) a minimum. This placed the radiant at

$$\alpha = 127^\circ.8, \qquad \delta = +40^\circ.2 \ (1900).$$

Table X shows the agreement between the observed and computed values of the proper-motion position angles, ψ, and between the observed and computed radial velocities, the latter by Ludendorff, miscellaneous observers, and the Lick Observatory, respectively. The parallaxes and the computed radial velocities were deduced, by intention, to agree as exactly as possible with Ludendorff's observed velocities.

In the cases of the first two stars on the list, β *Eridani* and β *Aurigæ*, the discordances in position angle are large enough to raise a question as to whether these stars belong to the *Ursa Major* system. On the basis of probabilities, we should have to expect that a certain number of stars whose proper motions and radial velocities are known would have proper-motion position angles differing not more than 7° from any assigned position angle; but the probabilities would be small that two stars as

[6] *Ap. J.*, **30**, 135, 1909.

TABLE X.—URSA MAJOR GROUP.—HERTZSPRUNG

Star	α (1900)	δ (1900)	μ	ψ Obs.	ψ Com.	ψ O−C	Radial Velocity Com.	Radial Velocity Lud.	Radial Velocity Misc.	Radial Velocity L. O.	Computed Parallax π	H. R. Spm.	Remarks
							km.	km.	km.	km.			
β Eridani	75°.73	− 5°.22	0″.1188	228°.1	221°.3	+6°.8	− 7.5			− 8.::	0″.034	A2	Spectrum v. v. poor
β Aurigæ	88 .05	+44 .94	0 .0455	260 .1	265 .2	−5 .1	−16.0		−18.1[1]	−18.4	0 .024	Ap	Sp. Bi., Harv. Col. Obs.
α Can. Maj.	100 .19	−16 .58	1 .3235	203 .6	203 .6	0 .0	− 8.5			− 7.4	0 .387	A	Vis. and Sp. Bi.
37 Urs. Maj.	157 .18	+57 .60	0 .0756	61 .8	60 .0	+1 .7	−16.6			−14.3	0 .045	F	
β Urs. Maj.	163 .95	+56 .92	0 .0871	72 .3	70 .0	+2 .3	−16.1	−16.8		−11.5	0 .047	A	Sp. Bi., Ludendorff
δ Leonis	167 .20	+21 .07	0 .2017	132 .5	128 .8	+3 .6	−14.4		−14. [2]	−12.::	0 .084	A2	Spectrum v. poor
γ Urs. Maj.	177 .14	+54 .25	0 .0945	88 .5	87 .5	+1 .1	−15.0				0 .042	A	Spectrum v. v. poor
δ Urs. Maj.	182 .62	+57 .59	0 .1095	88 .6	87 .6	+1 .0	−14.4			−11.::	0 .045	A2	Spectrum v. v. poor
Groom 1930	191 .08	+60 .87	0 .075	88 .5	91 .4	−2 .9	−13.4			− 5.	0 .028	F	
ε Urs. Maj.	192 .41	+56 .50	0 .1140	95 .6	96 .9	−1 .3	−13.2	−13.:	− 8.2[3]	− 9.:	0 .042	Ap	Sp. Bi., Ludendorff
78 Urs. Maj.	194 .11	+56 .91	0 .115	99 .5	98 .0	+1 .5	−13.0			− 7.0	0 .042	F	
ζ Urs. Maj.	199 .98	+55 .45	0 .1250	101 .8	103 .4	−1 .7	−12.2	−12.6		− 7.3	0 .043	Ap	Sp. Bi., Harv. Col. Obs.
α Coronæ	232 .61	+27 .05	0 .1580	128 .5	132 .0	−3 .4	− 2.2		+ 0.4[4]		0 .041	A	Sp. Bi., Hartmann

1. $\frac{1}{3}$ (−19.8 Potsdam −18.2 Pulkowa −16.3 Allegheny) = −18.1.
2. Vogel and Scheiner.
3. $\frac{1}{2}$ (−7.1 Allegheny −9.3 Yerkes) = −8.2.
4. $\frac{1}{2}$ (+0.4 Allegheny +0.5 Ottawa) = +0.4.

bright as these, whose observed and computed radial velocities are in close accord, should deviate only five and seven degrees respectively from the positions of great circles passing through the *Ursa Major* radiant.

It is indeed remarkable that stars so widely separated in space should be travelling along parallel lines with equal speeds. The two stars β and ζ *Ursæ Majoris* are separated by an angle of approximately 20°, which means that the minimum distance between β and ζ must exceed one-third the average distance of the system from us; that is, the distance between these two stars is such that light requires thirty years as a minimum to travel over it. The angular separation of *Sirius* and ζ *Ursæ Majoris* is approximately 90°, which means that the distance between the two stars is greater than the distance of either from the solar system.

The large cluster of stars known as the *Pleiades* concerns our problem in the same manner. The relative positions not only of the naked-eye stars but of many between the sixth and ninth magnitudes, in this group, were accurately determined by Bessel[7] in the years 1829-1841. Subsequent observations by Gould, Jacoby, and especially by Elkin[8] in 1884-1885, failed to show any certain changes in their relative positions. Fully three score members of the cluster have a common proper motion; they appear to move on through space with a uniform speed of 5″.3 per century. Only the half dozen most brilliant members of the group are brighter than 5.01 visual magnitude, and the spectra of all these contain only a few poorly defined lines. It was expected that their velocities would be essentially equal. This expectation was not fully realized when the speeds of six of the brighter stars in the group were measured recently by Adams[9] at the Yerkes Observatory. The measures were made with 1-prism dispersion, on account of the hazy character of their lines, and the results are therefore subject to larger errors than usual.

7 *Astr. Nach.*, **18**, 353, 1841.
8 *Trans. Yale Col. Obs.*, **1**, 1, 1887.
9 *Ap. J.*, **19**, 338, 1904.

BRIGHTER PLEIADES STARS

		α (1900)	δ (1900)	Spm.	V	μ_α	μ_δ
Electra	(17 *Tauri*)	$3^h38^m.9$	+23°.8	B5	+15 km.	$+0^s.0014$	—0″.050
Taygeta	(19 *Tauri*)	3 39 .3	+24 .2	B5	+ 3	8	48
Maia	(20 *Tauri*)	3 39 .9	+24 .1	B5	+21 to —7	21	45
Merope	(23 *Tauri*)	3 40 .4	+23 .6	B5	+ 6	17	54
Alcyone	(25 *Tauri*)	3 41 .5	+23 .8	B5	+15	14	48
Atlas	(27 *Tauri*)	3 43 .2	+23 .7	B8p.	+13	+0 .0014	—0 .050
				Average	+10	+0 .0015	—0 .049

The parallax of the *Pleiades* group is unknown, beyond the fact that it is small. It can scarcely exceed 0″.02. The annual proper motions of the six bright stars, in right ascension and declination, are quoted in the last two columns of the table (changed in March, 1910, by the substitution of the proper motions from Boss's *Preliminary General Catalogue*). The average value of the six resultant proper motions is 0″.053, and the proper-motion vectors point nearly to the antapex of the Sun's way. The mean of Adams's radial velocities for the six stars, + 10 km. per second, is almost exactly the Sun's velocity with reference to the point occupied by the *Pleiades*, for they are approximately 60° from the antapex. Several writers have noted that the *Pleiades* group must be nearly fixed in position, with reference to the stellar system. The values of the proper motions and radial velocities quoted in the table are in harmony with this view. If we assume the group to be at rest in the sidereal system, equation (29) enables us to compute the distance of the group from the point occupied by the solar system as 215 light years, or parallax 0″.015. The *Pleiades* stars are certainly of great luminosity. Corresponding to the distance which we have computed, *Alcyone* must be nearly 200 times as luminous as our Sun, and the average luminosity of the six bright stars must be in the neighborhood of 100 times that of the Sun.

The velocity of *Maia* is variable. Differences between the velocities assigned to other stars in the group seem to be real in good part. However, if these differences, of the order of 10 km. per second, are a fair measure of differential motions existing within the *Pleiades* system, then the *Pleiades* must have a parallax much smaller than 0″.015, in order to account for the essential absence of relative motions in the sixty years between Bessel's and Elkin's surveys. If we increase the estimate of distance, we must increase the estimated luminosities even more rapidly.

Here again we must not use the radial velocities of all the individual stars, for several of them are approximately equal, and are no doubt the related velocities of a system; nor must we confine our solution to one star-mass. We decide to let *Electra, Alcyone,* and *Atlas* enter as one star; to let *Taygeta* and *Merope* enter as one star having their mean velocity; and to omit *Maia,* whose speed is variable and whose systemic velocity is as yet unknown. Thus the massive *Pleiades* cluster enters as only two stars.

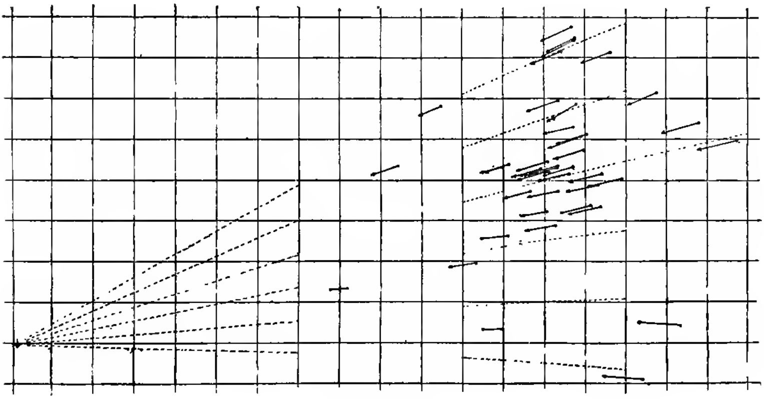


Figure 8

More remarkable than either of these groups, in some ways, is an approximately globular cluster, about 15° in diameter, containing thirty-nine or more stars whose proper motions are such as to carry them toward a common converging point, as discovered[10] by Boss of Albany. Figure 8 shows these stars in their present relative positions as round dots, and the arrows

[10] *Astr. Jour.,* **26,** 31, 1908.

represent by their directions and lengths the apparent motions of these stars in 50,000 years. The stars, at their present distances from us, are ranked as of the third to the seventh magnitudes. In the distant future, after 65,000,000 years elapse, Professor Boss computes, this cluster, now 15° in diameter, will have condensed into a cluster not over one-third of a degree in diameter, with the stars reduced to the ninth to twelfth magnitudes, and occupying the relative positions indicated in Figure 9. It is not expected that they will be nearer each other than they are now; but, moving along parallel to each other, as they are supposed to do, they will converge as parallel lines drawn to infinity. The radial velocities of these stars are of interest. Three were observed at Bonn and seven at Mount Hamilton, in the prosecution of their regular programs; and at the Yerkes Observatory the task of observing all of the stars known to be in this group has recently been assumed. The following are available to date (1910):

	α (1900)	δ (1900)	Bonn	Lick	Spm.	μ_α	μ_δ
γ *Tauri*	$4^h 14^m.1$	+15°.4	+38.6 km.	+38.0 km.	G	$+0^s.0081$	$-0''.027$
δ *Tauri*	4 17 .2	+17 .3	+39.8	+38.6	K	77	33
68 *Tauri*	4 19 .7	+17 .7		+35.	A	75	24
ϵ *Tauri*	4 22 .8	+19 .0	+38.4	+39.2	K	80	38
θ_1 *Tauri*	4 22 .9	+15 .7		+37.5	K	72	28
θ_2 *Tauri*	4 23 .0	+15 .6		Variable	A5	72	25
c *Tauri*	4 32 .6	+12 .3		Variable	A5	+0 .0071	−0 .010

The Yerkes Observatory has found that a large proportion of the stars in this group have variable velocities; that is, are spectroscopic binaries: but for most of them—perhaps for all—the speeds of the centres of mass appear to be approximately 35 to 40 km. per second, recession. These radial velocities therefore support the proper-motion indications that the stars in question have equal and parallel motions. Professor Boss was able on this hypothesis to determine all the principal elements of the motion of this group, as well as the mean distance of the group. The observed proper motions and radial velocities were harmonized within the limits of unavoidable error by assuming:

1. A common convergent point at $\alpha = 92°$, $\delta = +7°$;
2. The angle (at present) between the line of sight and the direction of the group motion, 28°.9;
3. The mean parallax of the group, 0″.025;
4. The linear speed of the group (with reference to the solar system), 45.6 km. per second.

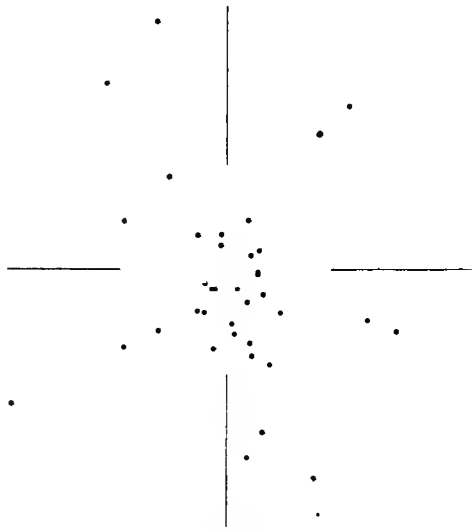


Figure 9

This cluster includes and surrounds the *Hyades* cluster. Kapteyn has recently determined the parallax of the *Hyades* group by ordinary methods, and finds it to be 0″.023.[11] The agreement with Boss's value, 0″.025, is remarkably close.

Our question is again, To what extent shall we let these stars enter into the determination of the solar motion? Of the thirty-nine stars at present thought to be members of the cluster, fourteen are as bright as the 5.0 magnitude, which is the limit for our present purpose. Compromising between the equal and parallel motions of these stars on the one hand, and their large

[11] *Pub. Astr. Lab. Groningen*, No. 23, 1909.

masses on the other, we have decided to enter three individual radial velocities as representative of the fourteen bright members of the system.

There are several cases, throughout the sky, of two stars, fairly close together in appearance, whose proper motions are seemingly equal and parallel, and whose radial velocities we have found to be equal; such as ζ_1 and ζ_2 *Reticuli*,[12] in the Southern Hemisphere.

	α (1900)	δ (1900)	μ_α	μ_δ	V	Spm.
ζ_1 *Reticuli*	$3^h15^m.6$	−62° 57′	$+0^s.1947$	+0″.677	+13. km.	G
ζ_2 *Reticuli*	3 16 .0	−62 53	+0 .1924	+0 .692	+12.	F 8

The two stars, 5′ apart, are closely related, and their radial velocities were combined and used as if for one star in our present problem.

It will happen occasionally, without violating the probabilities, that two or more neighboring stars will have essentially equal and parallel proper motions, as viewed from the position of the solar system, whereas if viewed from a quite different direction in space they would clearly be unrelated. The spectrograph has supplied the equivalent of the different viewpoints for several other supposedly related groups, only to prove them unrelated.

In the case of the few double stars for which we measured the radial velocities of both components, the clear course to pursue is to use each pair of stars as one star whose velocity is the velocity of the centre of mass of the double system. Such was the method followed for α *Centauri*, γ *Virginis*, *Castor*, and one or two others.

There is another kind of discrimination which we must exercise in selecting and rejecting stars in connection with the solar-motion problem. Certain stars are travelling at very high speeds, both in and across the line of sight. Some of them appear to be sporadic cases, not representative of general prevailing conditions. For example, the star *Cordoba Zones* $5^h.243$, moving away from the Sun at the rate of 242 km. per second, is one of our

12 The proper-motion equality of the two stars was pointed out by Mr. E. J. Stone.—*Mon. Not. R. A. S.*, 40, 26, 1879.

nearest neighbors; the fifth star from us, to the best of our knowledge. Its visual brightness is of only the 9.2 magnitude, and it must in reality be a very small body. Now if we were to let this negligible mass with abnormal velocity enter the problem it might influence the result more powerfully than twenty stars of normal masses and velocities. It should clearly not be used. Another runaway star, of relatively small mass, is *Groombridge* 1830, moving toward us 97 km. per second, and across the line of sight perhaps 250 km. per second. This, too, should be excluded as abnormal. However, it will not do to reject arbitrarily, and a definite rule of rejection has been formulated and followed. My preliminary solution of the Sun's motion, in the year 1900, based on the velocities of 280 stars, gave a speed $V_0 = 19.9$ km. per second, in direction $\alpha_0 = 277°.5$, $\delta_0 = +20°$ (see page 167). Combining this determination of direction with the many determinations based on proper motions, we assumed, for the present purpose, that the solar motion is toward $\alpha_0 = 275°$, $\delta_0 = +30°$. If D is the angular distance of a star (α, δ) from this apex, $V_{\alpha\delta}$ the star's radial motion with reference to the stellar system, and V the star's observed radial motion with reference to the solar system, then we have the observed velocity of the star

$$V = V_{\alpha\delta} - 19.9 \cos D;$$

or the star's motion with reference to the stellar system is

$$V_{\alpha\delta} = V + 19.9 \cos D.$$

Before beginning the present solution for the solar motion, based upon the materials now available, we applied the term $+ 19.9 \cos D$ to all the *large* observed velocities and obtained a closely approximate value of these individual stellar velocities with reference to the *stellar* system. Further, from the early solution, the velocities of the stars to and from the solar system, freed from the solar-motion component, were as follows:

151	positive velocities, average	+17.01 km.
129	negative velocities, average	—17.10 km.
280	numerical average	17.06 km.

It has seemed to me that all stars whose radial speeds with reference to the stellar system exceed four times the average observed speed 17, or 68 km., would unduly influence the computed value of the solar motion and that they should be excluded from the solution. This is substantially in accord with Peirce's criterion and other criteria for the rejection of discordant observations. Further, there are indications that the stars having extremely rapid motions are in general of small mass, though there are exceptions, and stars of known small mass should not enter with their full value. Accordingly, I decided to reject all radial velocitics (freed from the correction for solar motion) which equal or exceed three and a half times the average velocity; that is, all greater than 60 km. per second.

My original aim was to have all stars brighter than the 5.01 visual magnitude, *Revised Harvard Photometry,* included in the solution. A small number of such stars, chiefly those winter stars whose spectra contain poorly defined lines, have not yet been sufficiently observed. On the other hand, especially in the Southern Hemisphere, a number of stars fainter than the fifth magnitude were observed because their proper motions are large. Now large proper motion means, in part, proximity to the solar system, and, in part, great linear speed. An examination of these stars shows that if we exclude all whose speeds are more than 60 km., according to the above ruling, the remainder of the proper-motion stars fainter than the fifth magnitude have radial speeds only slightly larger than ordinary. In many of these cases the greater than average proper motions must be due to proximity, in which case their masses are relatively small. Because of their presumably small masses and because they were added to the observing program for special reasons, I have let only about one-half of these stars enter into the solution.

Deducting twenty-six velocities, rejected for reasons described in the preceding paragraphs, from the original 1073 observed velocities, we have left as a basis for the solution 1047 individual results. These are distributed not uniformly but nevertheless quite satisfactorily over the entire sphere. They present stars of essentially all spectral classes, including thirteen nebulæ observed by Keeler; but stars of spectral Classes B and A have perhaps the greatest lack of homogeneity. The 1047 stars were combined into 172 groups of neighboring stars, each group representing on the average slightly more than six stars. It was then a question of forming the equation of condition (32), from the average data for each of the groups; 172 equations in all, each involving three unknown quantities.

Combining the 172 equations into three normal equations and solving, we obtained the following elements of the solar motion:

$$\alpha_0 = 272° \; 0' \qquad \pm 2°.50$$
$$\delta_0 = +27°26' \qquad \pm 3°.00$$
$$V_0 = -17.77 \text{ km.} \pm 0 .62 \text{ km.}$$

I was surprised at the smallness of the resulting V_0. I had expected the value of the velocity to exceed rather than to be smaller than the value —19.9, deduced in the year 1900. Yet the observational data cannot be made to yield an appreciably different result, as we shall see in the next paragraphs.

Another solution was made with an equation of condition for each of the observed velocities. That is, there were 1047 separate equations. Solving them by the method of least squares we obtained the following elements:

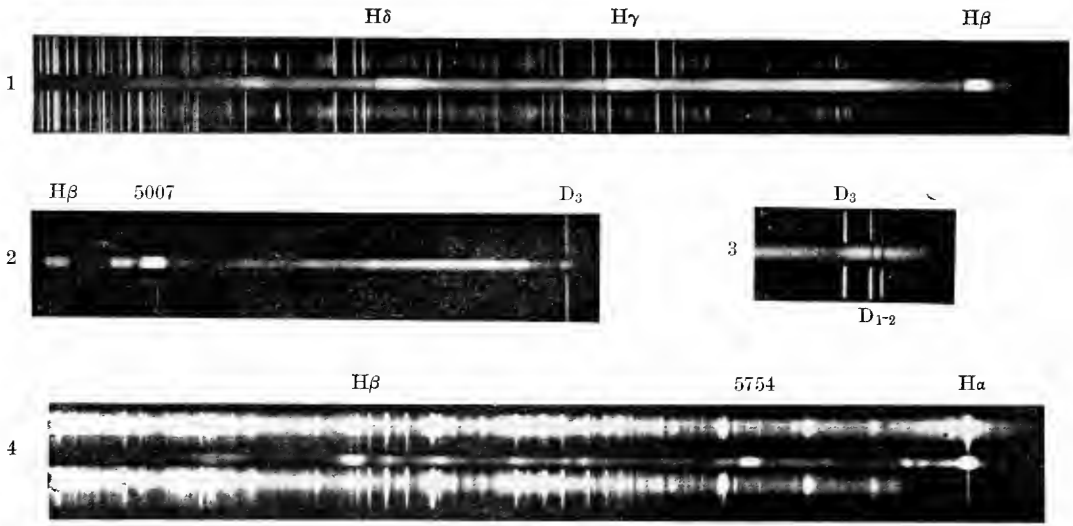
$$\alpha_0 = 273°.5$$
$$\delta_0 = 28°.0$$
$$V_0 = -17.73 \text{ km. per second.}$$

The two solutions, one based on individual velocities and the other upon group velocities, are in remarkable accord.

Still another form of solution for the velocity of the solar motion was employed, as follows:

Neglecting the thirteen nebulæ, there remained the observed velocities of 1034 stars. Three hundred and thirty of these are of spectral Classes O, B, A and F-F4, inclusive, which we may say constitute Secchi's TYPE I. The remaining 704 are of spectral Classes F5 to M, inclusive. These conform to Secchi's TYPE II, except[13] that Class K5 and Class M stars fall in Secchi's TYPE III. The angular distances D of these stars from the

[13] Only about 1/7 of the 704 stars are of TYPE III.



1. Spectrum of *Nova Persei*, 1901, March 18
2. " " " " 1901, August 11
3. " " " " 1901, March 3
4. " " *Nova Lacertæ*, 1911, January 1

deduced position of the apex $\alpha = 272^\circ.0$, $\delta = +27^\circ\ 26'$, were computed. The observed velocity of each star was freed from the solar-motion component by applying correction $-17.77 \cos D$ km. per second. The stars were then tabulated as in the accompanying table, in terms of their apical distances and corrected radial velocities. For instance, in the zone whose limiting apical distances are 60° and 65°, there are eleven stars of Secchi's TYPE I, whose average apical distance is 62°.3, and whose average radial velocity with reference to the solar system (freed from the solar-motion component) is +7.9 km. per second; and in the same zone there are twenty stars of Secchi's TYPE II whose average apical distance is 62°.6, and whose average radial velocity with reference to the solar system is + 6.1 km. per second. If the elements of the solar motion used as a basis for correcting the velocities here tabulated can be regarded as satisfactory, then the residual velocities in the hemisphere whose apical distances are between 0° and 90° should "balance" the residual velocities in the hemisphere whose apical distances are between 90° and 180°. It is noticed immediately that the signs of the average velocities quoted in the table are prevailingly positive; a small positive average for the TYPE II stars, but a large positive average for the TYPE I stars. However, the mean velocity of the 506 stars in the hemisphere surrounding the apex of the Sun's way agrees well with the mean velocity of the 528 stars in the hemisphere surrounding the antapex. Each average residual velocity is entitled to have weight in determining a correction to the deduced velocity of the solar system, —17.77 km. per second, in proportion to the number of velocities which have combined to form the mean, and to the cosine of its distance from the apex. The expression for the correction to the velocity forms the left member of the following equation. The numerical values of the corrections supplied by the data for TYPE I stars and for TYPE II stars are as set down respectively in the right members of the equation.

$$\frac{\Sigma \frac{n\,V}{\cos D} \cos D}{\Sigma\, n \cos D} = \begin{cases} +\,0.08 \text{ km. for Classes B to F4G stars (Secchi's TYPE I)} \\ -\,0.29 \text{ km. for Classes F5G to M stars (Secchi's TYPE II)} \end{cases}$$

TABLE XI

CORRECTED VELOCITIES IN TERMS OF APICAL DISTANCES

($\alpha_0 = 272° 0'$, $\delta_0 = + 27° 26'$, and $V_0 = - 17.77$ km.)

Apical Distances	Type I Stars n	Type I Stars D	Type I Stars V	Type II Stars n	Type II Stars D	Type II Stars V
			km.			km.
0°— 5°	1	4°.4	— 3.9	2	4°.0	+17.2
5 — 10	2	8 .4	+10.2	6	7 .4	—10.8
10 — 15	4	12 .4	— 2.9	4	11 .3	+ 4.3
15 — 20	5	17 .3	+ 0.1	9	17 .9	— 7.7
20 — 25	3	23 .9	— 3.4	12	23 .2	+ 2.7
25 — 30	4	27 .8	+ 3.8	19	28 .0	— 3.6
30 — 35	9	33 .0	+ 4.6	13	32 .1	— 4.8
35 — 40	8	36 .4	— 0.6	23	37 .2	+ 0.8
40 — 45	8	41 .6	+ 4.1	17	42 .7	+ 4.5
45 — 50	10	48 .0	+ 6.9	17	47 .8	+ 1.5
50 — 55	13	52 .7	— 1.1	28	52 .6	+ 2.4
55 — 60	8	58 .2	+ 1.9	27	57 .1	+ 2.5
60 — 65	11	62 .3	+ 7.9	20	62 .6	+ 6.1
65 — 70	12	67 .3	+ 4.0	31	67 .6	+ 0.8
70 — 75	11	72 .8	+ 4.6	30	72 .2	+ 0.8
75 — 80	15	77 .5	— 2.8	21	77 .4	— 3.5
80 — 85	17	83 .0	+ 2.9	33	82 .3	+ 2.6
85 — 90	20	87 .4	+ 1.0	33	87 .8	+ 1.6
90 — 95	13	92 .6	+ 0.5	27	92 .4	+ 5.6
95 —100	10	97 .6	— 1.8	30	97 .6	— 6.7
100 —105	8	101 .4	+ 4.2	28	102 .3	+ 0.8
105 —110	11	107 .7	+ 0.7	25	107 .3	+ 1.2
110 —115	12	112 .6	+ 0.8	24	112 .3	+ 3.4
115 —120	23	117 .2	+ 2.7	26	117 .4	+ 2.3
120 —125	10	122 .9	+ 0.3	24	122 .3	— 1.8
125 —130	11	127 .1	+ 1.8	34	127 .5	+ 0.4
130 —135	12	132 .6	+ 2.4	22	132 .4	+ 6.5
135 —140	9	137 .0	+ 2.5	23	137 .2	— 3.0
140 —145	7	142 .2	— 5.6	26	142 .1	+ 9.1
145 —150	7	147 .5	+ 2.5	15	146 .6	— 1.7
150 —155	10	153 .3	+ 4.2	13	152 .7	+ 3.4
155 —160	11	157 .1	+ 3.0	21	157 .1	— 1.1
160 —165	6	162 .8	+ 8.2	5	162 .8	+ 3.2
165 —170	6	167 .0	+10.0	14	167 .9	+ 0.1
170 —175	3	170 .4	+ 3.5	2	172 .1	—11.2
175 —180	0			0		
Totals	330			704		

The 330 stars of Secchi's TYPE I yield a corrected value of the solar velocity $V_0 = -17.77 + 0.08 = -17.69$ km. The 704 stars of Secchi's TYPE II yield a corrected value of $V_0 = -17.77 - 0.29 = -18.06$ km. Weighting these results in proportion to the number of component velocities, we obtain $V_0 = -17.94$ km. per second.

The most probable value of the solar velocity obtainable from the observational data is approximately $V_0 = -17.85$ km. per second. However, the discussions which follow in the next chapter have been based upon the velocity -17.77 km. as determined by the group method of solution.

We may say that the position of the apex as determined from the radial velocity data agrees satisfactorily in right ascension with the average of the best proper-motion apices; but that the radial velocity apex is three or four degrees south of the proper-motion apex of greatest weight. The radial velocity data are, perhaps, a little nearer homogeneity in right ascension than in declination, for the Mount Hamilton observations have been made throughout the twenty-four hours of right ascension with the same spectrographs, and, in general, with the same personal equations in the plate measurements; and the same conditions hold for the observations in the Southern Hemisphere, obtained by the D. O. Mills Expedition. It is not impossible that small systematic differences in the personal equations of the observers in the two hemispheres may be responsible for a part of the discrepancy between the radial velocity and proper-motion declinations of the apex, but there is an even stronger probability that systematic differences in the proper motions assigned to stars in the Northern Hemisphere and the Southern Hemisphere may be responsible for an appreciable share of the discrepancy. Again, it seems certain that the radial velocity data extend out further amongst the Milky Way stars than do the proper-motion data. The proper-motion apex and the radial velocity apex may both be correct, for they refer only to the systems of stars actually used in the solutions; but the radial velocity solution may be the more representative of the stellar universe. We must wait for the future to decide.

The deduced speed, — 17.77 km. per second, carries the solar system a distance of 560,000,000 km. per year, or 3.75 times the Earth's mean distance from the Sun.

These are frequent and legitimate questions: Is the solar system moving in a simple orbit, such as a conic section? Will it eventually complete a circuit in this orbit and return to the part of its orbit where it is now? The idea of affirmative answers to these questions appears to be prevalent in the human mind. It is natural to think that we must be moving on a great curve—perhaps closed like an ellipse, or open like a parabola—the centre of mass of the universe being in the curve's principal focus. The attraction which any individual star is exerting upon us is certainly slight, owing to its enormous distance, and the resultant attraction of all the stars may not be very much greater; for since we are believed to be somewhere near the centre of our stellar system, the attractions of the stars in the various directions should nearly neutralize one another, in accordance with the principle that a body situated within a concentrically homogeneous sphere is effectively acted upon only by the gravitational matter nearer the centre of the sphere than itself. Even though we may be following a definite curve at the present time, there is, in my opinion, little doubt that we shall be prevented from continuing upon it indefinitely. In the course of our travels we should be carried, sooner or later, relatively close to some individual star whose attraction would be vastly more powerful than that of all the other stars combined. This would draw us more or less from our present curve and cause us to follow a different curve. At a later date our travels might carry us into the sphere of attraction of some other great sun which would send us away in a still different direction. Thus, the chances are, in my opinion, that our path would, in time, be made up of a succession of unrelated curves.

The results deduced above define the direction and speed of the solar motion along a straight line; and, as a single line does not fix the position of a plane, we are without knowledge as to the plane in which the solar system is moving. It is of great interest that the present *line* of motion lies nearly in the plane of the Milky Way, making in fact an angle of about 17° with the central line of the Milky Way. We need not concern ourselves at present with the question of the plane of our orbit, for the curvature of our path is undoubtedly so slight that we may consider it as a straight line for many generations of astronomers to come.

When my solution for the solar motion, as based upon 1047 radial velocities, was under way and nearly concluded, there appeared a paper[14] on the same subject by Hough and Halm of the Cape of Good Hope Observatory. It is based upon their velocities of 166 of the brightest southern stars, for 50 per cent of which they had secured more than one spectrogram; plus the radial velocities of forty-five stars, mostly northern, published by various observatories in the past seven years; plus the radial velocities of 280 stars, four or more spectrograms each, north of Declination —30°, which I published as 80 mean velocities nine years ago (TABLE VIII); a total of about 460 velocities, deducting duplicates. There has not been an opportunity since receiving and reading the paper to make a critical analysis of the results, for this would require rather extensive computations; and this is impossible, as the velocities of the individual stars have not been published. Their results are quoted here in order that the subject may be brought up to date (January, 1910).

Hough and Halm's deduced speed of the solar motion, —20.85, is 3 km. per second greater than mine. The explanation of a part of the discrepancy seems to me to be clear. The observational data are very far from homogeneous. The forty-five miscellaneous velocities include twenty stars of the *Orion* type, as observed at the Yerkes Observatory, and a large proportion of the remaining twenty-five are Class B spectroscopic binary systems. We shall show in the next chapter that the radial velocities of Class B stars are, for some unknown reason, observed too great to the extent of about 5 km. per second, positive. Half of the forty-five stars concerned are situated relatively near the antapex, where, in common with stars near the apex, they have the maximum weight in determining the

[14] *Mon. Not. R. A. S.*, 70, 85, 1909.

solar velocity. As the observed velocities of these stars are on the average abnormally great, a large value of the solar speed naturally follows.

The right ascension and declination of the apex, $\alpha_0 = 271°.2 \pm 3°.3$, $\delta_0 = +25°.6 \pm 3°.7$, are in remarkably close accord with my results depending upon 1047 velocities.

CHAPTER VI

STUDIES OF THE STELLAR SYSTEM

We recall that the equations used in solving for the elements of the solar motion were developed on the assumption that the motions of the stars, with reference to the sidereal system, follow the laws of accidental errors. This procedure is, of course, not permissible unless the radial velocities are distributed in accordance with such laws. However, this assumption cannot lead to results seriously in error provided that in each small area of sky we consider the motions of approach equal to motions of recession, on the average, even though the average approach and average recession in different areas of the sky may be quite different, as must certainly be the case if Kapteyn's conception of two star streams is correct. Further, it has been shown by Weersma[1] that the three normal equations resulting from the individual equations of condition for observed stars, as developed and used in CHAPTER V, are in reality independent of the supposition that the radial velocities are distributed according to the laws of accidental errors. It was not my purpose to regard the assumption referred to as final; but it was intended to investigate the law according to which the stellar radial velocities, with reference to the sidereal system, are really distributed. This we shall now undertake.

Each observed stellar velocity was freed from the solar-motion component by applying the correction —17.77 cos D km. per second. The results are the velocities of the individual stars with reference to the system of stars employed. These residual velocities have been arranged as in TABLE XII, with reference to their numerical magnitudes and their spectral types. Stars of spectral Classes O to F4 inclusive have been tabulated as

[1] *Publ. Astr. Lab. Groningen,* **21**, 59, 1908.

TABLE XII

DISTRIBUTION OF STELLAR VELOCITIES WITH RESPECT TO SPECTRAL TYPES

Residual Rad. Vel.	I n	II n	I+II n
Above +80 km.	0	5	5
+70 to +80	0	3	3
+60 to +70	1*	3	4
+50 to +60	0	7	7
+40 to +50	0	18	18
+35 to +40	1	7	8
+30 to +35	4	14	18
+25 to +30	1	19	20
+20 to +25	17	32	49
+15 to +20	21	53	74
+10 to +15	48	71	119
+ 5 to +10	49	61	110
+ 0 to + 5	73	78	151
— 0 to — 5	50	84	134
— 5 to —10	19	70	89
—10 to —15	17	53	70
—15 to —20	13	49	62
—20 to —25	9	20	29
—25 to —30	5	20	25
—30 to —35	5	19	24
—35 to —40	1	10	11
—40 to —50	2	13	15
—50 to —60	1	4	5
—60 to —70	0	4	4
—70 to —80	0	1	1
Below —80	0	5	5
Totals	337	723	1060

Column I includes Classes B to F4 (Secchi's TYPE I).
Column II includes Classes F5 to M (Secchi's TYPE II).
The nebular velocities are not included in this table.

* Only two spectrograms of this star secured; a good chance that the velocity is variable.

Secchi's Type I, and stars of Classes F5 to M inclusive as Secchi's Type II. Strictly, the Classes K5 and M stars belong to Secchi's Type III, but as their number is relatively small we have in these preliminary studies entered them in Type II. The significance of the table can, perhaps, be most definitely stated by describing the contents of one horizontal line; for example, of stars whose residual radial velocities lie between + 20 and + 25 km. per second there are seventeen of Type I and thirty-two of Type II; and of stars whose residual velocities lie between — 20 and — 25 km. per second, there are nine of Type I and twenty of Type II.

Two facts appear prominently on the face of the table.

1. The number of positive velocities is considerably greater than the number of negative velocities for both types, but especially in the case of Type I stars. Of Type II stars, 371 have positive velocities and 352 have negative velocities. Of Type I stars, 215 have positive velocities and 122 have negative velocities. We shall consider this discrepancy a little later, in greater detail.

2. There are no velocities amongst the Type I stars exceeding + 70 or — 70 km. per second, whereas there are fourteen Type II stars with residual velocities greater than these limits. Exceeding the limits ± 50 km., there are two stars of Type I and thirty-two stars of Type II. Exceeding the limits ± 25 km., there are twenty-one stars of Type I and one hundred and fifty-two stars of Type II. The proportion of small velocities of Type I stars is much greater than in the case of Type II stars.

The data in Table XII have been plotted as in Figure 10, first multiplying the number of Type I stars in the different compartments by the ratio of the number of stars of the two types, 723/337, in order to make the data for the two types comparable. The ordinate of each black circle represents the number of Type I velocities whose arithmetical mean velocity is the abscissa of that circle, as determined from the individual velocities which lie in the corresponding compartment of the table. Each open circle represents the corresponding number and average arithmetical velocity of Type II stars whose individual velocities fall in the corresponding compartment of the table.

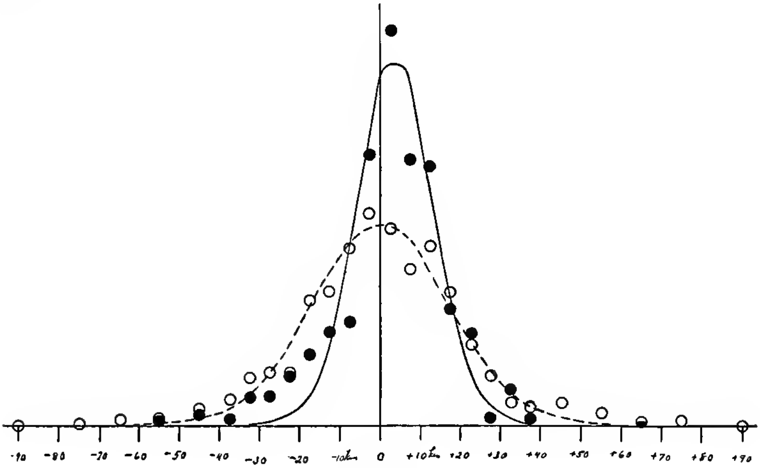


Figure 10

If these stellar velocities are distributed according to the laws of accidental errors we should be able to represent them reasonably well by means of probability curves with suitable constants. The dotted curve in Figure 10 corresponds to the equation

$$y = 2.5\,e^{-0.75\,x^2} \tag{34}$$

It represents extremely well the radial velocities of TYPE II stars with reference to the stellar system. Apparently the use of the method of least squares, so far as the TYPE II stars are concerned, was entirely justifiable.

The full curve in Figure 10 corresponds to the equation

$$y = 4.5e^{-2\,(x-0.17)^2} \tag{35}$$

It does not represent the velocities of TYPE I stars very satisfactorily; yet it would be difficult and perhaps impossible to find a symmetrical and reasonably simple curve which would represent them better. It may be that the number of TYPE I stars observed, 337, is too small to serve as a statistical basis, or, more probably, that the data included under TYPE I are not homogeneous and comparable. The sudden decrease in the number of stars of TYPE I whose velocities are between — 5 and — 10 km., as compared with the number in the 0 to — 5 compartment, is very striking.

The value of the unit in the two equations and their curves is 20 km. The numerical term, — 0.17, in the exponent of the TYPE I equation corresponds to a positive displacement of the curve amounting to 3.33 km. per second.

It is apparent from Figure 10, as well as from TABLE XII, that the numerical average velocity of the TYPE II stars is much in excess of the average velocity of the TYPE I stars. The number of stellar velocities upon which the results rest, 1060, is so large that the discrepancy in the average velocity of the two types can scarcely be otherwise than a fact of nature.

In order to explain, if possible, the prevailing positive tendencies of the mean velocities in TABLE XII, especially for the TYPE I stars, the mean residual velocities were arranged in greater detail as in TABLE XIII. The stars were divided into four spectral classes, B to B9, A, A2 to F8, and G to M. The numbers of stars of the four classes in each zone of apical distance, 5° wide, and the mean residual velocities of these stars, are as quoted in the table. The numbers of stars in the four divisions and the corresponding averages of the residual velocities are given at the foot of the table. We see that the averages are prevailingly positive: very small for Class A velocities; + 0.60 km. for Classes A2 to F8; + 0.91 km. for Classes G to M; and + 4.93 for the 138 stars of Classes B to B9 inclusive. These positive tendencies prevail alike in the hemispheres of the apex and of the antapex. The mean residuals for the Class A and for the Classes A2 to F8 are perhaps no greater than could be ascribed to casual velocity distribution and personal equation of measurement; but the residual, + 0.91 km., for Classes G to M can scarcely be explained in this manner; and the residual, + 4.93 km., for the Classes B to B9 must certainly seek some other explanation.

Let us review some possible explanations of the Classes B to B9 discrepancy.

TABLE XIII

STELLAR VELOCITIES WITH REFERENCE TO SPECTRAL CLASSES AND APICAL DISTANCES

$\alpha_0 = 272° \; 0', \delta_0 = +27° \; 26', V_0 = -17.77$ km.

Apical Distance	Classes B–B9		Class A		Classes A2–F8		Classes G–M	
	n	V	n	V	n	V	n	V
		km.		km.		km.		km.
0°– 5°	0		0		2	+ 7.2	2	+17.7
5 – 10	2	+10.2	0		0		6	−10.8
10 – 15	1	− 3.9	2	+ 0.4	2	+15.8	3	− 7.7
15 – 20	3	+ 6.6	1	−12.1	1	− 7.1	9	− 7.7
20 – 25	1	+12.2	1	−16.6	2	−11.3	11	+ 4.5
25 – 30	2	+ 1.0	1	+ 8.7	6	− 1.9	14	− 3.8
30 – 35	1	+21.2	4	+ 6.0	6	+12.3	10	−10.0
35 – 40	3	− 2.5	1	− 8.5	7	− 2.0	20	+ 2.2
40 – 45	1	+24.6	4	+ 2.4	5	+ 0.5	15	+ 4.9
45 – 50	3	− 2.4	4	+ 9.4	8	+ 4.2	12	+ 1.2
50 – 55	5	+ 0.4	2	− 5.2	9	− 1.6	25	+ 3.1
55 – 60	2	+ 1.9	2	+ 8.1	7	+ 2.3	24	+ 1.9
60 – 65	4	+ 8.0	3	− 4.0	7	+15.5	17	+ 4.8
65 – 70	6	+ 9.7	2	+ 2.4	10	− 5.0	25	+ 2.4
70 – 75	6	+ 2.4	3	+ 8.2	7	+10.3	25	− 1.4
75 – 80	4	− 2.0	5	−11.3	8	+ 5.9	19	− 5.1
80 – 85	7	+ 4.4	2	− 1.2	16	+ 1.5	25	+ 3.4
85 – 90	11	− 2.6	3	+14.4	9	+ 1.4	30	+ 1.5
90 – 95	3	+ 6.6	1	− 5.5	12	− 2.8	24	+ 7.4
95 –100	3	+ 1.0	3	− 3.5	5	+ 0.7	29	− 7.3
100 –105	1	+ 8.3	3	+ 5.8	8	− 1.9	24	+ 1.8
105 –110	5	+ 1.0	3	− 9.6	3	+10.5	25	+ 1.2
110 –115	9	+ 3.2	0		4	− 4.0	22	− 0.9
115 –120	6	+ 5.5	6	+ 1.9	15	+ 0.5	22	+ 3.2
120 –125	5	+ 5.2	3	− 3.2	4	−10.2	22	− 0.7
125 –130	4	+ 9.7	1	+ 9.3	12	− 8.2	28	+ 2.3
130 –135	7	+ 7.5	3	−13.7	5	+ 1.0	19	+ 8.2
135 –140	5	+11.3	1	−23.7	5	− 0.3	21	− 3.7
140 –145	3	+10.6	2	+ 3.7	6	−12.2	22	+10.2
145 –150	5	+ 0.7	1	+ 9.0	6	+ 6.1	10	− 5.7
150 –155	5	+ 8.8	1	+ 8.0	6	+ 4.1	11	+ 0.8
155 –160	4	+ 6.8	3	+ 7.9	7	− 1.6	18	− 1.1
160 –165	5	+ 8.2	1	+ 8.0	1	+44.1	4	− 7.0
165 –170	3	+21.6	2	− 6.3	4	− 6.0	11	+ 3.0
170 –175	3	+ 3.5	0		1	−26.7	1	+ 4.4
175 –180	0		0		0		0	
Means	138	+ 4.93	74	+ 0.18	216	+ 0.60	605	+ 0.91

The nebular velocities are not included in above table. The twenty-six velocities rejected for reasons explained in Chapter V are not included. One additional velocity was omitted by mistake.

1. If we give a literal interpretation to this result, it signifies that the universe of Classes B to B9 stars is expanding, with reference to the instantaneous position of the solar system as a centre, at the rate of 4.93 km. per second. Pickering has shown that the Class B stars are strongly clustered in the Milky Way and vicinity, and quite irregularly in galactic longitudes, though all parts of the galaxy and vicinity are fairly well represented. It is exceedingly improbable that the Class B stars in all the considerable areas of the sky where they are found are travelling outwardly from the point in space which we happen to occupy, as shown by the average residual velocities in TABLE XIII.

2. A personal equation in the measurement of the spectrograms, systematically positive, amounting to 5 km. per second, cannot be regarded as possible. Observations of a few Class B star spectra at other observatories have been published, and the results are either in good agreement with those obtained at Mount Hamilton and in Chile, or these published velocities are in general larger than ours.

3. A more probable explanation, it seems to me, is that the wave lengths of the lines in the Class B spectra, adopted by the radial velocity observers, err in being too small. An average increase of 0.07 A in the wave lengths of all the lines utilized would fully explain the phenomenon. Unfortunately, there is no apparent means of testing this question directly; but the question of causes is an interesting one. It is recognized that high pressures in radiating or absorbing media not only broaden spectral lines but shift their apparent centres in general in the direction of greater wave lengths. The absorption lines—or absorption bands preferably—in Class B spectra are usually of considerable breadth. It appears that axial rotations of the stars can be but minor factors in the broadening of lines. It seems not impossible that the conditions existing in Class B stars are such that the absorptions are effective at great depths in their atmospheres under high pressures, as well as in the surface strata under low pressures. If we grant the efficiency of this factor, the systematic positive tendency given to observed radial velocities of Class B stars is in a fair way to be explained.

4. Another hypothesis, perhaps simpler, should be mentioned, but it is not considered of great weight. Many of the helium lines, which are the most prominent lines in Class B stars, occur in pairs in the laboratory spectrum of helium;[2] as examples, the lines at 4026 A, 4120 A, 4471 A, and 4713 A, in the region utilized by radial velocity observers. In every case the more refrangible component is stronger than the less refrangible. The laboratory line at 4471.646 A has intensity 6, and its companion at 4471.858 A has intensity less than 1; and somewhat similarly in other cases. The interval between the components of the pairs corresponds approximately to a radial velocity difference of 15 km. In most of the radial velocity determination, at the Yerkes Observatory, at the Lick Observatory, at Santiago, Chile, and perhaps elsewhere, the wave length of the helium line in this region of spectrum of the Class B stars has been assumed to be 4471.676 A, obtained by giving weight 6 to the wave length of that component whose intensity is 6, and weight 1 to that of the component whose intensity is about 1. If conditions in Class B stars are such that the relative intensities of the red components of the helium pairs are considerably augmented, so that the effective wave length of a pair is greater than we have assumed it to be, from laboratory measurements, it is possible we should not need to look further for the explanation of the positive discrepancy. It is known to experienced radial velocity observers that the systems of wave lengths adopted for the lines of different elements may be satisfactory for one Class B spectrum, and apparently quite unsatisfactory for another Class B spectrum. Here may exist a fruitful field for investigation. Adopted wave lengths for the helium lines must of course harmonize with wave lengths adopted for the hydrogen, oxygen, silicon, and other lines existing in the same spectrum, so that all the lines in a given spectrum will yield equal radial velocities.

5. There can be little doubt that the Class B stars of the *Orion* region are or have been intimately associated with the great nebulous structures which we know to exist there. The observed velocities for the densest part of the *Orion* nebula, as

[2] Runge and Paschen, *Ap. J.*, 3, 11, 1896.

obtained by Keeler, Vogel and Eberhard, Wright, Frost and Adams, are in excellent agreement, with mean value + 17.4 km. per second. The observed radial velocities of the Class B stars in the *Orion* region, though differing in essentially the usual amounts from one another, average about + 22½ km. Here again we have an indication, more or less weighty, that the observed radial velocities of Class B stars are for some unknown reason about 5 km. too great.

Of all the explanations suggested, that of a pressure effect in the extensive atmospheres of Class B stars appears to be by far the most probable one.

It is not improbable that the excess of positive velocity, 0.91 km. for the stars of Classes G to M, is due to the same cause.

The peculiarities of the data for TYPE I and TYPE II stars in TABLE XII, as further illustrated in Figure 10, are clearly in confirmation of previous indications that the stars of early spectral classes are travelling more slowly than those of later classes. To test this question, and at the same time that of stellar velocities as a function of visual magnitudes, the residual radial velocities were tabulated as in TABLE XIV. The visual magnitudes in the first column are from the *Revised Harvard Photometry*. The two columns under Secchi's TYPE I include, for stars of Classes O, B, A and F to F4 inclusive, both the number of stars in each magnitude division and their average residual radial velocity. The two columns under Secchi's TYPE II contain corresponding data for Classes F5 to F9, G, K and M. The last two columns combine the data for TYPE I and TYPE II stars.

It appears, in brief, that 330 stars of TYPE I have an average residual radial velocity of 10.25 km. per second; and that 704 TYPE II stars have an average residual radial velocity of 15.08 km. per second; that is, the TYPE II stars in the present list have radial velocities nearly 50 per cent greater than those of the TYPE I stars.

Recalling that the 280 radial velocities published by me in the year 1901, which were chiefly of the G, K and M types, averaged 17.06 km. per second; that the radial velocities of twenty Class

B stars, published by Frost and Adams of the Yerkes Observatory in the year 1904, averaged only 7 km. per second; and considering the additional fact that a hasty tabulation of about sixty Class M velocities (Secchi's Type III) in my present list shows an average velocity of about 17 km. per second: I am led to the remarkable conclusion that the velocities of the stars must be functions of their spectral types; that is, of their effective ages. To recapitulate: we have the average radial velocities of twenty Class B stars published by Frost and Adams, 7 km. per second; of 330 Secchi's Type I stars, 10.25 km. per second; of

TABLE XIV

AVERAGE VELOCITIES IN TERMS OF VISUAL MAGNITUDES AND SPECTRAL TYPES

Vis. Mag.	Type I n	Type I V	Type II n	Type II V	Types I and II n	Types I and II V	Vis. Mag.	n	Avg. V
		km.		km.		km.			km.
Above 1.50	9	10.5	8	16.8	17	12.2	Br. than 3.50	179	12.3
1.51 to 2.50	26	9.2	22	12.3	48	10.7			
2.51 to 3.50	40	9.4	74	15.0	114	13.0			
3.51 to 4.50	116	8.9	239	15.1	355	13.0	3.50 to 5.50	817	13.6
4.51 to 5.50	126	11.3	336	15.1	462	14.1			
5.51 to 6.50	9	19.1	18	18.5	27	18.7	Ftr. than 5.50	39	16.7
Below 6.50	4	11.3	8	12.5	12	12.1			
Means	330	10.25	704	15.08	1034	13.51			

704 Secchi's Type II stars, 15.08 km. per second; of the Class M stars in the present list, numbering about sixty, 17 ± km. per second; and of the 280 stars considered in 1900, consisting mostly of Classes G, K and M, and from which no rejections were made on account of abnormally high velocities, 17.06 km. per second. *The progression of average velocity with advancing spectral type is clear and unmistakable.*[3]

[3] Footnotes added after the date of the lecture:

(A) As the question of priority in making this discovery is of interest to some writers, I make the following statement:

Aside from the presentation of all the above results, including Tables

The table of radial velocities which exceed ± 50 km. per second, in CHAPTER III, page 115, bears strongly upon this subject. With the exception of one star each of Classes B8, F and F8, all the stars in that table are of Classes G, K and M; and the chances are reasonably strong that the high velocity of the Class B8 star, *υ Pavonis*, R. A. = $18^h22^m.0$, will prove to be orbital, rather than systemic, as only two spectrograms of this star have been secured. It would scarcely be possible to secure

XII, XIII and XIV, and Figure 10, in the Silliman Lecture of January 31, 1910, they were discussed with those of my colleagues who had assisted in the computations and in forming the tables and the figure, all before January 17, 1910; on January 18 and 19, in San Francisco, I informed certain high officials of the University of California of the discovery that the motions of the stars increase in speed with increasing age; and Figure 10 and TABLES XII, XIII and XIV were shown and their significance explained to leading astronomers in the eastern part of the United States between February 6 and February 10, 1910.

(B) Returning to Mount Hamilton on February 24, 1910, I had the velocities with reference to the stellar system tabulated on the basis of the Harvard classification of spectra, as follows:

Spectral Classes	Number of Stars	Average Radial Velocities
O and B	141	8.99 km.
A	133	9.94
F	159	13.90
G and K	529	15.15
M	72	16.55
Nebulæ (Keeler)	13	23.4

The increase of stellar velocity with advancing type was seen to hold on the Harvard classification, as well as for the Secchi classification upon which the discovery had been made.

It was a surprise to find the average velocity of the thirteen nebulæ, as observed by Keeler, in excess of the averages for the stellar spectral types. This may or may not indicate that the number of nebulæ available is too small to serve as a basis for averages. If from the list of thirteen nebulæ we remove the *Orion* nebula, which has radial velocity nearly zero, the average residual radial velocity for the remaining twelve nebulæ is in excess of 25 km. Here we may have evidence of great strength and importance, in support of a hypothesis that the planetary nebulæ have been a stronger bit of evidence that high stellar velocities appertain to the later spectral classes and abhor the early spectral classes.

The velocities of the nebulæ, as observed by Keeler, have not been included in the results thus far described in this chapter. Here is a list of these nebulæ, with the observed velocities in the next to the last column, and the velocities with reference to the stellar system in the last column. Keeler noted that the observed velocity of the *Great Nebula in Orion* must be due

formed from stars through processes arising from collisions with or close approaches to other massive bodies. The zero velocity of the *Great Nebula in Orion* is not out of harmony with the hypothesis.

(C) In February and March, 1911, the elements of the solar motion were re-determined according to a rather extensive program, as outlined in TABLE XV. The number of stars for which spectrograms had been secured with the Mills spectrographs on Mount Hamilton and on Cerro San Cristóbal, plus additional stars whose radial velocities have been observed and published elsewhere, was in excess of 1700. Excluding those stars whose spectral lines are too indefinite for high dispersion measurement, and those spectroscopic binaries whose systemic velocities either had not been determined or could not be estimated from the available data, but including thirteen nebular velocities, there were available for this investigation the radial velocities of 1193 objects.

Twelve solutions for the velocity of the solar motion as a function of spectral classes, as described in the first twelve lines of the following table, were made. Two solutions for the direction and speed of the solar motion, based upon the radial velocities of all spectral classes, are described in the last two lines. [The last two solutions, based upon the 1193 velocities, include the stars used in the first twelve solutions, and in addition, 13 nebulæ, 3 Wolf-Rayet stars, 1 Class G star, 1 Class K star, and 5 Class O stars.] In the first thirteen solutions a constant term K was introduced to represent any systematic tendency of the velocities, such as that which is apparent for the Class B stars in Figure 10 and TABLE XIII. The position of the apex quoted in the table as $\alpha_0 = 272°.5$, $\delta_0 = +34°.3$ is assumed from Professor Boss's proper-motion solution (*Astr. Jour.*, **26**, 112, 1910). The apical position $\alpha_0 = 270°$, $\delta_0 = +30°$ was assumed as a satisfactory mean of the positions determined from proper-motion and spectrographic data.

The average radial velocities of the stars of different spectral classes, with reference to the sidereal system, are as quoted in the last column of the table.

The resulting velocity of the solar motion, V_0, appears to be a function of the spectral class of the stars upon which it rests, at least as far as the brighter stars are concerned.

TABLE XV.—SOLUTION FOR SOLAR MOTION BY SPECTRAL CLASSES

	Class	No. Stars	No. Groups	Method	Apex		V_o	K	Average Radial Velocity
					α_o	δ_o	km.	km.	km.
1	B-B9	222	35	Group	270°.5	+34°.3	−19.9	+3.94	.....
2	B-B5	177	32	Group	270 .5	+34 .3	−20.7	+4.70	6.21 B-B5 (177 stars)
3	B-B9	225	35	Group	270 .0	+30 .0	−20.2	+4.07	6.47 B-B5 (180 stars)
									6.68 B8-B9 (45 stars)
4	A & B8-B9	222	32	Group	270 .5	+34 .3	−16.8	+1.62	6.66 B8-B9 (45 stars)
									10.48 A-A9 (177 stars)
5	A	177	..	Individual	270 .0	+30 .0	−16.8	+0.95	10.95 A-A9 (177 stars)
6	F	185	..	Individual	270 .0	+30 .0	−15.8	+0.06	14.37
7	G	128	..	Individual	270 .0	+30 .0	−16.0	−0.20	14.97
8	G	123	..	Individual	270 .0	+30 .0	−13.9	+0.22	12.9
9	K	382	..	Individual	270 .0	+30 .0	−21.2	+2.82	16.8
10	K	369	..	Individual	270 .0	+30 .0	−18.9	+1.91	15.1
11	M	73	..	Individual	270 .0	+30 .0	−22.6	+3.93	17.14
12	M	70	..	Individual	270 .0	+30 .0	−20.2	+4.59	15.4
13	All	1193	80	Group	268 .5	+25 .1	−19.5	+1.91	
14	All	1193	80	Group	268 .5	+25 .3	−19.5	Omitted	

almost entirely to the motion of the solar system,[4] a fact which the table confirms. If we were to assume the fixity of the *Orion* nebula and were further to assume that the solar motion is toward an apex at $\alpha = 272°$, $\delta = +27°.5$, the radial velocity of the *Orion* nebula as observed would yield a velocity of the solar system equal to 19.0 km., agreeing well with our present estimate of the solar speed. Keeler further noted, to quote his own words, that "the nebulæ are moving in space with velocities of the same order as those of the stars."[5]

TABLE XVI.—OBSERVATIONS OF NEBULÆ

Object	α (1900)	δ (1900)	D	Observed V	Corrected V
				km.	km.
G. C. 826	4h 9m.6	—13° 0′	148°.6	—10.1	—25.3
Orion Nebula	5 30 .4	— 5 27	156 .3	+17.4*	+ 1.1
G. C. 2102	10 20 .0	—18 8	121 .9	— 6.0	— 3.4
G. C. 4234	16 40 .3	+23 59	20 .1	—34.3	—17.6
[G. C. 5851	17 8 .4	—12 48	42 .8	—51.5	—38.4**]
G. C. 4373	58 .6	+66 38	39 .1	—64.7	—50.9
G. C. 4390	18 7 .1	+ 6 50	20 .6	— 9.7	+ 7.0
N. G. C. 6790	19 17 .9	+ 1 19	30 .9	+48.5	+63.8
G. C. 4510	38 .3	—14 24	47 .2	—16.7	— 4.6
G. C. 4514	42 .1	+50 17	28 .9	— 5.3	+10.3
N. G. C. 6891	20 10 .4	+12 26	32 .2	40.7	+55.8
G. C. 4628	58 .7	—11 46	56 .8	—49.7	—40.0
N. G. C. 7027	21 3 .3	+41 50	38 .2	+10.1	+24.1
G. C. 4964	23 21 .1	+41 59	63 .6	—11.4	— 3.5

[Elements of Solar Motion used: $V_0 = -17.77$ km., $\alpha_0 = 272°.0$, $\delta_0 = 27°.5$; D is the angular distance from the Apex.]

Unfortunately, the number of nebular velocities available is small; probably too small to have serious weight in statistical investigations. Four of the velocities are very small, and four

* Mean of results by all observers.

** Keeler described this as a "rough" measure, not repeated; and it has not been used in the present discussions of radial velocities.

[4] *Publ. Lick Obs.*, **3**, 198, 1894.

[5] *Publ. Lick Obs.*, **3**, 228, 1894.

Class O. Wolf-Rayet Spectrum, B. D. +30°3639, 1908, July 4 (showing extension of the Hydrogen lines)

Class Oa. Wolf-Rayet Spectrum, B. D. +35°4013, 1893, June 17 (low dispersion)

Class Oa. Wolf-Rayet Spectrum, B. D. +35°4013 may be called very large. In the light of the usually accepted hypothesis that the stars whose spectra are simple in type are not far removed from a nebular origin, we should perhaps expect, from the results described in preceding paragraphs, that the nebulæ in general would have low velocities. Certainly the large proportion of high nebular velocities on the list seems to put difficulties in the way of the conclusion that new stars, if formed from nebulæ, are travelling with smaller speeds than stars of great effective ages. However, the discrepancy in this case, if of sufficient statistical weight to be significant, as in other similar cases, may have future value in determining the relationships existing between the nebulæ and the stars. There certainly is no more pressing need at present than for a greatly increased number of nebular radial velocities.

The leading students of stellar proper motions have, of course, noted that the proper motions of Secchi's TYPE II stars greatly exceed the proper motions of Secchi's TYPE I stars. As Miss Clerke has said: "Indeed, the solar type appears to be more often associated with high velocities than any other. Stars with banded and gaseous spectra, and variables of all classes, mostly exhibit but slight signs of displacement; but this may be an effect of remoteness rather than of genuine inertness."[6] The inequality of the proper motions for stars of TYPE I and TYPE II was the chief factor in Kapteyn's conclusion that the mean parallaxes of TYPE II stars are two and a quarter times as great as the mean parallaxes of TYPE I stars; or that the TYPE I stars are on the average two and a quarter times as far away from us as the TYPE II stars of corresponding visual magnitudes.[7]

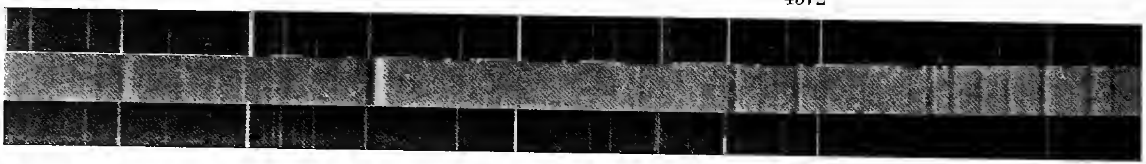
The results in TABLE XIV do not strongly support the indications of the 1900 data that the fainter stars are travelling more rapidly than the brighter ones. Such differences as exist in the averages for the seven magnitude classifications may be accidental, due to the small number of objects in some of the compartments, or to the fact that the data are not strictly

[6] *The System of the Stars,* 1905, p. 327.

[7] *Publ. Astron. Lab. Groningen,* No. 8, 24, 1901; also see Newcomb's *The Stars,* 1901, p. 313.

Class Ob. Wolf-Rayet Spectrum, A. G. C. 8631

Class Ob. Wolf-Rayet Spectrum, B. D. +37°3821



Class B4, peculiar. P *Cygni*, 1911, July 3 homogeneous. When we reduce from seven compartments of magnitude to three, as in the last three columns of the table, the average velocity does appear to increase slightly with decreasing magnitude; but the chief factor in the discrepancy probably lies in the comparative absence of observed stars of Secchi's TYPE I fainter than 5.50.

We now know that the strong evidence of increasing velocity with decreasing brilliancy afforded by the earlier investigation must have been due largely to the fact that the *brighter* stars of the slowly moving Classes B and A were observed on the early programs, whereas the *fainter* stars of these classifications were not yet observed. As a result, the average velocities observed for the brighter stars were reduced in magnitude over those for the fainter stars. This is one of many pieces of evidence indicating the importance of having the observed data of the utmost homogeneity. We may now say that the stellar velocities are not certainly functions of the visual magnitudes.

If an infinite number of stars are moving at random as to direction and speed, their directions may be represented by all the radii which can be drawn from the centre of a sphere to its surface, the radius of the sphere being equal to the average of all the radial velocities.

Let us determine the relations existing between the average speed of all the stars in space, the average radial velocity of all the stars, and the average of all the components of velocity at right angles to the line of sight. Let

$r =$ the average space velocity of the stars, moving at random; that is, the radius of the sphere described above;

$i =$ the angle which a space velocity vector makes with the line of sight; and

$\phi =$ the angle which a plane passing through the observer and a velocity vector makes with a reference plane through the line of sight.

The number of velocity vectors which we can draw to an element of the spherical surface is

$$r^2 \sin i \, di \, d\phi.$$

The average value of the velocity components parallel to the line of sight is defined by the expression

$$V_m = \frac{\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} r^3 \sin i \cos i \, di \, d\phi}{\frac{1}{2}\pi r^2} = \tfrac{1}{2} r.$$

That is, for an infinite number of stars moving at random in direction and speed the average radial velocity, V_m, is equal to one-half the average space velocity, V_s; or

$$V_s = 2 V_m. \tag{36}$$

The average component of velocity at right angles to the line of sight, *i.e.*, tangential to the celestial sphere, is defined by the expression

$$V_t = \frac{\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} r^3 \sin^2 i \, di \, d\phi}{\frac{1}{2}\pi r^2} = \frac{\pi}{4} r.$$

That is, the average linear component of velocity, as we see these components projected on the celestial sphere, is equal to $\frac{\pi}{4}$ times the average space velocity; or

$$V_t = \frac{\pi}{4} V_s = \frac{\pi}{2} V_m. \tag{37}$$

The average angle, i_0, which the stellar motion vectors make with the line of sight, is defined by the expression

$$i_0 = \frac{\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} r^2 \, i \sin i \, di \, d\phi}{\frac{1}{2}\pi r^2} = 1 \text{ radian} = 57^\circ.3. \tag{38}$$

That is, if an infinite number of stars are moving at random, the average of the angles which their motion vectors make with the line of sight is 57°.3.

Applying equation (36) to the average radial velocities quoted above, we obtain the corresponding average stellar velocities in space with reference to the sidereal system:

20	Class B stars (Frost & Adams),	14. km.
330	Type I stars (Campbell),	20.5 km.
704	Type II stars (Campbell),	30.2 km.
70±	Type III stars (Campbell),	33. km.

Our Sun having a speed of approximately 18 km. per second with reference to the sidereal system is thus one of the slow-moving stars; its speed is only 60 per cent of the average speed of the brighter solar type stars.

It is not easy to explain why the velocities of stars should increase with their effective ages, for we are accustomed to think of all matter as equally old gravitationally. Why should not the materials composing a nebula or a Class B star have been acted upon by gravitational forces as long and as effectively as the materials in the Class M stars? Are stellar materials in the ante-stellar state subject to Newton's law of gravitation? Does gravitation become effective only after the processes of combination are well under way? Is it possible that the gaseous matter composing a nebula is acted upon as effectively by radiation pressure as by gravitational attraction? The observed fact of the dependence of stellar velocity upon the spectral class is so new that these comments and questions make no pretensions to the status of a solution; but I am unable to suggest any other directions in which we should seek for the explanation.

It is of interest to examine the residual radial velocities to see how they would bear upon the question of two stellar drifts, which Professor Kapteyn had announced in 1905 to exist. The proper-motion investigations of Kapteyn, Eddington, and others, place the vertices of preferential proper motions at Right Ascension 93°, Declinatiou + 12°, and at the antipodal point of the celestial sphere. If the stars seen in projection upon the various large areas of the sphere have in reality equal average velocities *in space*, but with proper-motion preferences for the two vertices, the residual radial velocities of stars near the vertices should on the average be greater than for the stars in the zone midway between these vertices. The 1034 residual radial velocities were tabulated in terms of the angular distances of the stars from the vertex at Right Ascension 93°, Declination + 12°, as in Table XVII. The numbers of stars in the zones 10° wide, and their average velocities, plus and minus, are tabulated in the second, third, fourth, and fifth columns, and the average velocities, irrespective of sign, for all the stars in the several zones are given in the sixth column. The significance of the results is brought out more clearly by combining the eighteen zones into six zones, each 30° wide. The numbers of stars in each of the six zones and their average velocities are assigned in the seventh and eighth columns. Plotting the six velocities and constructing a curve representing them, the ordinates to the curve corresponding to the two vertices and to the great circle

TABLE XVII

Corrected Velocities in Terms of Angular Distance from Assumed Vertex at R. A. = 93°, Dec. = + 12°

Distance	No.	V_m km.	No.	V_m km.	V_m km.	No. of Stars	V_m	km.
0°– 9°	2	+19.7	2	—13.2	16.4			(16.7)
10 – 19	14	+13.8	10	—11.3	12.6	74	16.1	
20 – 29	31	+16.9	15	—19.1	18.0			
30 – 39	23	+14.3	19	—18.7	16.5			
40 – 49	34	+15.6	17	— 7.6	11.6	164	14.0	
50 – 59	40	+13.2	31	—14.0	13.6			
60 – 69	44	+11.5	35	—12.1	11.8			
70 – 79	42	+10.8	33	—14.2	12.5	249	12.6	
80 – 89	51	+14.6	44	—12.2	13.4			
90 – 99	46	+11.7	48	—12.6	12.2			(12.4)
100 –109	46	+11.9	34	—11.9	11.9	259	12.9	
110 –119	49	+13.9	36	—14.8	14.4			
120 –129	37	+11.3	43	—10.3	10.8			
130 –139	36	+14.7	45	—14.1	14.4	219	14.0	
140 –149	34	+15.8	24	—16.8	16.3			
150 –159	26	+14.1	17	—17.3	15.7			
160 –169	17	+14.2	11	—15.3	14.8	80	15.7	
170 –180	6	+19.6	3	—17.5	18.6			(16.3)

midway between the two vertices are as contained within parentheses in the last column. The average radial velocity, 12.5 km., of the stars midway between the two vertices is about 25 per cent less than the average radial velocity, 16.5 km., at the vertices. The indications for preferential velocities toward and away from the vertices are therefore fairly clear; but quantitatively the preference is very much smaller than we were expecting to find, in view of the proper-motion results. Professor Dyson,[8] for example, assigned a relative velocity of 2.6 times the velocity of the solar motion, or approximately 48 km. per second, to the two drifts into which his important investigation divided the stars having large proper motions.

TABLE XVIII

RESIDUAL VELOCITIES IN TERMS OF ANGULAR DISTANCE FROM ASSUMED VERTEX AT R. A. = 93°, DECL. = + 12°

Distance from Vertex	Number of Stars	Average Radial Velocity
0° to 30°	94	13.72 km.
30 to 60	188	14.56
60 to 90	269	12.78
90 to 120	296	11.92
120 to 150	251	14.51
150 to 180	99	14.55

This table includes all the velocities in TABLE XIII and the velocities of the twelve planetary nebulæ in addition. It does not include the stars whose radial velocities were rejected because in excess of ± 60 km. per second. If the stars with velocities in excess of 60 km. per second had been included in TABLE XVII, the average velocities in the different zones would have been appreciably greater, but the ratios of the velocities would not have been changed more than we should fortuitously expect.[9]

[8] *Proc. R. S. Edinburgh,* **29,** 391, 1909.

[9] Let us examine corresponding data obtained in March, 1911, from 1192 residual velocities, as in TABLE XV.

Indications for preferential velocities to and from the vertex are not so strong as those presented in the Silliman Lecture (TABLE XVII). The

Table V, in Chapter III, page 115, contains a list of all observed radial velocities, with reference to the stellar system, which are in excess of ± 50 km., including the velocities of three planetary nebulæ. (Three stars were added to the list after the date of the lecture.) On this list there are 11 stars within 60° of the vertex at $\alpha = 93°$, $\delta = +12°$; and 15 stars within 60° of the opposite vertex. These two areas make up one-half the area of the sky and contain 26 stars on the ± 50 km. list. The other half of the sky, comprising vertex distances lying between 60° and 120°, contains 14 stars. We have therefore a tolerably clear and strong indication that the high velocity stars have a preference for motion toward the Kapteyn vertices.

There is promise that the observed radial velocities of large numbers of stars afford the best means of determining the average distances of groups of stars; say of the fifth-magnitude stars or of the stars of a given class of spectrum and magnitude; and thus to determine the scale on which the stellar universe is constructed. We shall assume, again, that the real motions of the stars are at random as to speed and direction. After the effects of the solar motion have been eliminated from the measured radial velocities of the stars in question, we should find that the number of approaching stars and the number of receding stars are essentially equal. Likewise the mean corrected velocities of approach and recession should be nearly equal. If perchance they are affected by an apparent systematic error, as in the case of the Class B to B9 stars, Table XIII, such effect can be eliminated by subtracting the algebraic mean of the velocities from the individual velocities, and forming the arithmetical mean of the results. Let this mean velocity, without regard to algebraic sign, be V_m. We have shown (page 215) that the corresponding average velocity of these stars in space must be $2V_m$, and their average velocity at right angles to the line of

differences are probably due largely to the addition, in the intervening fourteen months, of a greater proportion of Classes B, A, and F velocities, which are low, than of G, K, and M velocities, which are high. The large reduction of the average velocity in the zone 0° to 30° is undoubtedly owing to the addition of many Classes B and A velocities in the region of the constellation of *Orion*, which includes Kapteyn's northern vertex of preferential motion.

sight, that is, on the surface of the celestial sphere, $\frac{\pi}{2} V_m$. These velocities, we repeat, are free from the effects of the solar motion. If we could free the observed proper motions of the same stars from the effects of the solar motion, obtaining thus the displacements of the stars on the surface of the sphere due to their own motions, as they would be determined by an observer at rest with reference to the sidereal system, it is evident that the average of these corrected proper motions in angular measure should be very closely equivalent to the linear cross motion, $\frac{\pi}{2} V_m$, as determined by means of the spectrograph, provided the distances of the stars in the group are not too unequal. If the distances were very unequal an error of appreciable size could ensue, for the distances of the stars and their proper motions measured in angle are reciprocal relations; and it is a well-known principle that the reciprocal of the mean of several quantities is not equal to the mean of their several reciprocals. . Strictly, the application of the principle would require us to know the law of stellar distances, which would leave us about where we started. Practically, the proper motions being, in general, simple reciprocal functions of the stellar distances, we could combine in one solution those stars of nearly equal magnitudes and nearly equal proper motions and determine their mean distances; and so on for other stars nearly equal in brightness and in proper motion. Now the effects of a known solar motion upon the observed proper motions can be eliminated in part for individual stars, and for the combined stars in a large group, provided, as we assumed, that the motions are at random. Following Kapteyn's practice and notation, let us draw a great circle through each star and the apex of the Sun's way, and resolve the star's proper motion into its component, υ, along this circle, and its component, τ, at right angles to this circle. The former component, υ, involves the whole of the parallactic effect and a certain part of the motion of the star itself. The other component, τ, is independent of the solar motion and is due entirely to the motion of the star with reference to the stellar system. The values of τ are obtainable from the observed proper motions, by computation, assuming the position of the apex as known. Let these be determined for the group of stars in which we are interested, and let their mean value be τ_m. From the spectrographic velocities in the line of sight we found V_m to be the average radial velocity of the group, after correcting for the Sun's motion. Now the average velocity of the stars toward us—*i.e.*, at right angles to the surface of the celestial sphere—is equal to their average linear velocity at right angles to any other plane, say the plane to which the τ component is perpendicular. Therefore V_m in linear measure must be equivalent to the τ_m component in angular measure, within limits depending upon the homogeneity of the group of stars. If ρ_m is the average distance of the group of stars in kilometers, we shall have, remembering that τ_m is expressed in terms of one mean solar year as the unit of time, and V_m in km. per mean second,

$$\rho_m \sin \tau_m = V_m \, 86400 \cdot 365.25;$$

or, in kilometers,—the brackets indicating logarithms,—

$$\rho_m = \frac{[7.4991]}{\tau_m'' \sin 1''} V_m = \frac{[12.8135]}{\tau_m''} V_m. \tag{39}$$

For convenience, as well as to assist the comprehension, we shall convert the distance in kilometers given by this equation into light years. Dividing both sides of (39) by the number of kilometers traversed by light waves in one year, we obtain ρ_m in light years,

$$\rho_m = \frac{[9.8373]}{\tau_m''} V_m = \frac{0.6875}{\tau_m''} V_m. \tag{40}$$

If π is the parallax of a star we have

$$\rho \sin \pi = 149{,}500{,}000 \text{ km.};$$

or, expressing ρ in light years and replacing $\sin \pi$ with $\pi'' \sin 1''$, we have

$$\pi'' = \frac{3.257}{\rho}. \tag{41}$$

Therefore, the number of light years, ρ_m, expressing the average distance of a fairly homogeneous group of stars having been obtained from (40), the mean parallax of the close group is given very nearly by

$$\pi_m'' = \frac{3.257}{\rho_m}. \tag{42}$$

From (42) and (40) we have

$$\rho_m = \frac{3.257}{\pi_m''} = \frac{0.6875}{\tau_m''} V_m;$$

and therefore

$$\pi_m'' = 4.738 \frac{\tau_m''}{V_m}. \tag{43}$$

Recalling again the random distribution of the proper motions, according to assumption, it must occur that the real motions of the stars are carrying as many in one direction along the circles through the apex-star-antapex as in the opposite direction. The effect of the Sun's motion overcomes the real motions of many of those actually moving toward the position of the apex and makes them seem to move away from the Sun's apex. If we form the arithmetical mean value of the υ components of the proper motions of a large number *of neighboring stars,* this value must be approximately the average actual angular motion of these stars along the great circles intersecting at the Sun's apex. Therefore υ_m in angle must correspond to V_m in km. per second; whence we shall have, nearly,

$$\rho_m = \frac{0.6875}{\tau_m''} V_m = \frac{0.6875}{\upsilon_m''} V_m, \tag{44}$$

in light years; and

$$\pi_m'' = 4.738 \frac{\tau_m''}{V_m} = 4.738 \frac{\upsilon_m''}{V_m}. \tag{45}$$

At the apex and antapex the solar components of the proper motion would be nil, and the τ and υ components would be alike.

Elsewhere the solar effect on each proper motion would vary as the sine of the angular distance from the apex. In a similar manner it can be shown that the algebraic mean value of the v components of a group of stars reasonably homogeneous would be due almost entirely to the solar-motion displacements of the stars. We deduce, very simply, an equation defining the approximate mean distances of the stars, on this basis. For any star, at angular distance D from the apex, whose parallactic component of proper motion is v, we have from (28)

$$q : \rho :: \sin v : \sin D;$$

and for the mean of a large number of stars not too different in magnitude and proper motion,

$$\sin v_m : \sin D_m :: q : \rho \text{ (approximately)},$$

in which v_m is the algebraic mean of the separate v values; or

$$\rho_m = \frac{V_0 \sin D_m}{\sin v_m} = \frac{V_0 \,.\, 86400 \,.\, 365.25 \sin D_m}{v_m'' \sin 1''}$$

$$= \frac{[12.8135]\ V_0 \sin D_m}{v_m''},$$

in kilometers. Transferring to light years, as before,

$$\rho_m = \frac{0.6875\ V_0 \sin D_m}{v_m''}.$$

Replacing V_0 by its value, 17.77 km., we have, in light years,

$$\rho_m = \frac{12.22 \sin D_m}{v_m''}. \qquad (46)$$

If it is found that the real motions of the stars are not at random, and the laws of distribution of the motions become known, these equations would require changes, more or less simple, to make them correspond. To illustrate, the Kapteyn drift theory would require that these relations be applied with different constants to different parts of the sky.[10]

Great caution must be used in attempting to transform from average stellar distance to average parallax, or *vice versa,* for, as stated above, distances and parallaxes are reciprocal func-

[10] For convenience we deduce the formulæ which are employed in computing the values of τ and υ.

Let

α_0, δ_0 = the right ascension and declination of the assumed apex;

α, δ = the right ascension and declination of a star;

χ = the position angle of the apex as viewed from the star; that is, the angle pole-star-apex, measured from the pole in the direction of increasing position angles;

D = the angular distance of the star from the apex;

$\mu\alpha'$ = the right ascension component of the star's proper motion expressed in seconds of time per annum, as quoted in star catalogues;

$\mu\alpha$ = the right ascension component of the star's proper motion converted to the arc of a great circle;

$\mu\delta$ = the declination component of the star's proper motion expressed in seconds of arc per annum;

μ = the total resultant proper motion of the star;

ψ = the position angle of the star's proper-motion vector.

For the spherical triangle whose vertices are at the pole star and vertex we may write

$$\cos D = \sin \delta_0 \sin \delta + \cos \delta_0 \cos \delta \cos (\alpha - \alpha_0)$$
$$\sin D \cos \chi = \sin \delta_0 \cos \delta - \cos \delta_0 \sin \delta \cos (\alpha - \alpha_0)$$
$$\sin D \sin \chi = - \cos \delta_0 \sin (\alpha - \alpha_0)$$

Let

$$n \sin N = \sin \delta_0$$
$$n \cos N = \cos \delta_0 \cos (\alpha - \alpha_0)$$

Then

$$\tan N = \frac{\tan \delta_0}{\cos (\alpha - \alpha_0)}$$
$$\cos D = n \cos (\delta - N)$$
$$\sin D \cos \chi = - n \sin (\delta - N)$$
$$\sin D \sin \chi = - \cos \delta_0 \sin (\alpha - \alpha_0)$$
$$\tan \chi = \frac{\tan (\alpha - \alpha_0) \cos N}{\sin (\delta - N)} \tag{47}$$
$$\tan D = - \frac{\tan (\delta - N)}{\cos \chi} \tag{48}$$

tions, and if the stars are very irregularly distributed as to distance a large error would result. The reciprocal of the mean of several stellar distances may be quite different from the mean of the reciprocals of the same distances. It is safer to express average stellar distances in terms of mean parallax rather than in kilometers or light years. If we are dealing with the τ components of proper motion it does not matter how irregularly the stellar distances may be distributed: equation (43) will give the correct value for the average parallax of the group, whereas equation (44) might give a highly erroneous value of the average distance of the stars in light years.

We also have

$$\mu_\alpha = 15 \cos\delta \, \mu_\alpha'$$

$$\tan\psi = \frac{\mu_\alpha}{\mu_\delta} \tag{49}$$

$$\mu = \frac{\mu_\alpha}{\sin\psi} = \frac{\mu_\delta}{\cos\psi} = \sqrt{\mu_\alpha^2 + \mu_\delta^2} \tag{50}$$

$$\tau = \mu \sin(\chi - \psi) \tag{51}$$

$$\upsilon = \mu \cos(\chi - \psi) \tag{52}$$

The angle N will always lie between 0° and 180°, inasmuch as the apex is certainly in the Northern Hemisphere; the angle χ may vary from 0° to 360°; and the angular distance D will lie between 0° and 180° If μ_α is plus, ψ will lie between 0° and 180°; but if μ_α is minus, ψ will lie between 180° and 360°. μ is always positive.

The sign of τ is determined by the value of $\chi - \psi$. The sign of υ is determined in the same manner; it is negative when the proper motion increases the star's distance from the apex.

If the apex is assumed to lie in the convenient position $\alpha_0 = 270°$, $\delta_0 = +30°$, the equations for determining D and χ take the form

$$n \sin N = [9.6990]$$

$$n \cos N = -[9.9375] \sin\alpha$$

$$\tan N = -\frac{[9.7614]}{\sin\alpha}$$

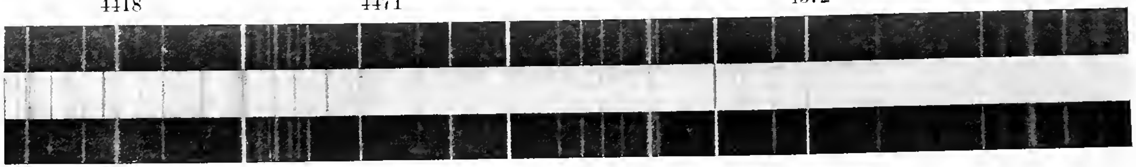
$$\tan\chi = -\frac{\cot\alpha \cos N}{\sin(\delta - N)} \tag{53}$$

$$\tan D = -\frac{\tan(\delta - N)}{\cos\chi} \tag{54}$$

and the accuracy of the computations may be checked in part by the equation

$$[0.0625] \sin D \sin\chi \sec\alpha = -1.$$

Class B1. β *Canis Majoris*, 1910, April 15. Measured displacement, +54 km.



Class B2. γ *Pegasi*, 1910, December 25. Measured displacement, +37 km.

We shall now illustrate the application of the foregoing principles in determining the average parallaxes of the Type I and Type II stars whose proper motions and radial velocities are known. We shall utilize only the τ components of the observed proper motions. The results are condensed in Table XIX, in terms of visual magnitude. The proper-motion data were taken for the most part from Kapteyn's valuable lists in *Publications of the Astronomical Laboratory at Groningen*, No. 7, and the rest were collected from various sources. Stars whose τ components of proper motion exceed 0″.30 per annum were excluded. The numbers of stars of Secchi's Type I, Type II, Types I and II combined, the average τ components, and the average residual radial velocities, are given in the table.

TABLE XIX

Parallaxes by Magnitudes and Types From Radial Velocities and τ Components of Proper Motion

	Type I			Type II			Types I and II		
Vis. Mag.	n	τ_m	V_m km.	n	τ_m	V_m km.	n	τ_m	V_m km.
2.51–3.50	36	0″.0148	9.2	50	0″.0508	15.3	86	0″.0357	12.7
3.51–4.50	76	0 .0279	8.8	150	0 .0545	15.3	226	0 .0455	13.1
4.51–5.50	58	0 .0266	8.6	162	0 .0352	14.0	220	0 .0329	12.5

	π_m, Type I		π_m, Type II		π_m, Types I and II
Vis. Mag.	L.O.	Kapteyn	L.O.	Kapteyn	L.O.
2.51–3.50	0″.0076	0″.0223	0″.0157	0″.0515	0″.0133
3.51–4.50	0 .0150	0 .0157	0 .0169	0 .0357	0 .0163
4.51–5.50	0 .0147	0 .0111	0 .0119	0 .0253	0 .0125

[Note added in 1911.—The substantial accuracy of these results is confirmed by a much more extensive investigation, made in March, 1910, based upon proper motions from Boss's *Preliminary General Catalogue*. See *Lick Obs. Bull.*, 6, 132, 134, 1911.]

Assuming that the average radial velocities in the various compartments of the table are equal to the corresponding average linear τ components of motion, the average parallaxes were computed by equation (43). For comparison, the mean parallaxes assigned by Professor Kapteyn to stars of Secchi's

TYPE I and TYPE II, with reference to the integral stellar magnitudes, are quoted in the table, from *Publications of the Astronomical Laboratory at Groningen,* No. 8, page 24.

We note that the third-, fourth- and fifth-magnitude stars on our list (a large proportion of stars between 5.0 and 5.5 not being included in the discussion) are not at distances in the order of their magnitudes, the brightest group being apparently as far away as the faintest. This, to some extent, is probably a result arising from the limited number of proper motions utilized: 86 for the brighter stars, and 220 for the fainter; but the apparent mixture of the different magnitudes in space is more complete than I had expected to find. In general, the average parallaxes are much smaller than those assigned in Kapteyn's parallax tables.

The conclusions appear to be:

1. That the stars of these three magnitudes are more thoroughly mixed than we had supposed.

2. That the brighter stars of Secchi's TYPE I are not so much farther away than the brighter stars of Secchi's TYPE II as we had supposed.

3. That the scale on which the universe of brighter stars is constructed is apparently a great deal larger than we had supposed.

We shall not consider the υ components of proper motion at the present time, and in other respects also we shall regard the investigation merely as preliminary, chiefly for the reason that Professor Boss's more accurate and more extensive proper motions will be available in a few weeks.

Kobold has published a list[11] of 307 stars whose proper motions are greater than 0″.50 per annum. We have obtained spectrographic velocities for eighty-eight of these stars. They are arranged in TABLE XX, in the order of their angular distances, D, from Kapteyn's preferential vertex at $\alpha = 93°$, $\delta = +12°$. The observed velocities are contained in the next to the last column, and the velocities reduced to the stellar system, on the basis of $V_0 = -17.77$ km. per second, in the last column.

[11] *Bau des Fixsternsystems,* p. 232.

TABLE XX

RADIAL VELOCITIES OF 88 STARS WITH PROPER MOTIONS GREATER THAN 0″.50, TABULATED WITH REFERENCE TO PREFERENTIAL VERTEX AT $\alpha = 93°$, $\delta = 12°$

(Corrected for $\alpha_0 = 272°$, $\delta_0 = +27°.5$, $V_0 = -17.77$ km.)

Kobold No.	Star	α (1900)	δ (1900)	Sp.	D	Obs'd V	Corrected V
			$D = 0° - 30°$				
		h. m.				km.	km.
69	A. G. C. 5700	4 55.9	− 5° 52′	K	26°.1	+ 31.±	+ 15.4
75	Weisse 5h.592	5 26.3	− 3 39	G	19 .4	+ 7.	− 8.8
84	*α Can. Maj.*	6 40.7	−16 35	A	29 .6	− 7.4	− 24.5
93	*α Can. Min.*	7 34.1	+ 5 29	F_5	21 .3	− 3.5	− 16.7
94	*β Gemin.*	7 39.2	+28 16	K	26 .0	+ 3.9	− 4.2
			$D = 30° - 60°$				
47	*ι Persei*	3 1.8	+49 14	G	53 .9	+ 50.5	+ 50.1
57	*ε Eridani*	3 28.2	− 9 48	K	46 .2	+ 16.4	+ 2.9
58	10 *Tauri*	3 31.8	+ 0 5	G_5	41 .5	+ 29.0	+ 16.7
60	*δ Eridani*	3 38.5	−10 6	K	44 .1	− 5.5	− 19.4
62	τ_6 *Eridani*	3 42.5	−23 33	F_8	51 .0	+ 7.3	− 7.9
66	o_2 *Eridani*	4 10.7	− 7 49	G_5	36 .1	− 41.6	− 56.4
71	Cord. Zones 5h.243	5 7.7	−44 58	G-K	58 .8	+242.	+225.1
72	*λ Aurigæ*	5 12.1	+40 1	G	31 .0	+ 66.5	+ 60.7
80	*δ Leporis*	5 47.0	−20 53	K	33 .5	+ 99.2	+ 81.6
97	A. G. C. 10120	7 41.9	−33 59	F_8	50 .7	+105.	+ 88.5
105	A. G. C. 11070	8 13.7	−12 18	F	38 .8	+ 24.	+ 9.5
106	A. G. C. 11499	8 29.0	−31 11	G_2	54 .2	+ 34.	+ 16.9
111	*ι Ursæ Maj.*	8 52.4	+48 26	A_5	49 .3	+ 8.	+ 7.2
112	10 *Ursæ Maj.*	8 54.2	+42 11	F_5	46 .3	+ 27.3	+ 25.0
118	*θ Ursæ Maj.*	9 26.2	+52 8	F_8	55 .9	+ 15.7	+ 16.8
			$D = 60° - 120°$				
2	*β Cassiopeiæ*	0 3.8	+58 36	F_5	80 .8	+ 12.8	+ 20.2
4	*ζ Tucanæ*	0 14.9	−65 28	F_8	100 .6	+ 9.3	+ 0.8
7	*β Hydri*	0 20.5	−77 49	G	101 .3	+ 22.8	+ 13.8
9	A. G. C. 544	0 32.2	−25 19	K	90 .6	+ 18.2	+ 12.5
16	*η Cassiopeiæ*	0 43.0	+57 17	F_8	75 .7	+ 10.0	+ 15.9
19	*μ Cassiopeiæ*	1 1.6	+54 26	G_5	73 .1	− 97.4	− 92.6
22	*ν Phœnicis*	1 10.6	−46 4	G	88 .7	+ 11.8	+ 2.2
29	41 *H. Androm.*	1 35.7	+42 7	F	66 .5	+ 4.9	+ 6.2
31	*τ Ceti*	1 39.4	−16 28	K	73 .1	− 15.7	− 24.4
34	*χ Eridani*	1 52.0	−52 7	G_5	84 .8	− 5.7	− 17.2
36	A. G. C. 2201	2 6.3	−51 19	G	82 .5	+ 49.5	+ 37.5
49	*α Fornacis*	3 7.8	−29 23	F	60 .7	− 20.6	− 34.8
51	ζ_1 *Reticuli*	3 15.6	−62 58	G	82 .3	+ 13.5	+ 0.3
52	e *Eridani*	3 15.9	−43 27	G_5	68 .4	+ 87.3	+ 72.7
53	ζ_2 *Reticuli*	3 16.0	−62 53	F_8	82 .2	+ 12.2	− 1.0
56	*κ Reticuli*	3 27.6	−63 18	F_5	81 .6	+ 12.1	− 1.3
79	*π Mensæ*	5 45.1	−80 34	G_5	92 .6	+ 12.2	+ 0.9
82	A. G. C. 7066	5 53.4	−63 8	K	75 .2	+ 25.4	+ 10.5
130	*α Crateris*	10 54.9	−17 46	K	75 .9	+ 47.9	+ 41.1

Kobold No.	Star	α (1900)	δ (1900)	Sp.	*D*	Obs'd *V*	Corrected *V*
			***D* = 60° — 120° (Cont.)**				
		h. m.				km.	km.
134	ξ *Ursæ Maj.*	11 12.9	+32° 6′	G	71°.2	— 14.3	— 12.2
138	20 *Hydri*	11 29.6	—32 18	G	87 .7	— 24.	— 30.5
141	A. G. C. 16103	11 41.8	—39 57	G	92 .1	+ 17.7	+ 11.1
142	β *Leonis*	11 44.0	+15 8	A_2	80 .2	+ 1.3	+ 2.6
143	β *Virginis*	11 45.5	+ 2 20	F_8	83 .0	+ 4.9	+ 4.3
144	*Groombr.* 1830	11 47.2	+38 26	G-K	77 .7	— 97.	— 92.1
155	β *Can. Ven.*	12 29.0	+41 54	G	85 .1	+ 6.5	+ 13.9
156	γ *Virginis*	12 36.6	— 0 54	F	96 .2	— 20.0	— 17.7
160	δ *Virginis*	12 50.6	+ 3 56	M_a	98 .6	— 17.6	— 13.6
166	β *Comæ*	13 7.2	+28 23	G	96 .7	+ 5.8	+ 14.0
167	61 *Virginis*	13 13.2	—17 45	K	108 .0	— 6.6	— 4.7
172	i *Centauri*	13 40.0	—32 33	F_5	114 .9	— 14.6	— 13.9
177	θ *Centauri*	14 0.8	—35 53	K	118 .9	+ 1.8	+ 2.9
178	α *Bootis*	14 11.1	+19 42	K	112 .8	— 4.7	+ 6.1
182	α *Centauri*	14 32.8	—60 25	G	117 .4	— 22.8	— 25.8
204	χ_1 *Herculis*	15 49.2	+42 44	F	116 .2	— 55.6	— 40.0
235	χ *Draconis*	18 22.9	+72 41	F_8	95 .3	+ 32.4	+ 45.5
245	σ *Draconis*	19 32.6	+69 29	G_5	97 .3	+ 25.5	+ 38.8
263	η *Cephei*	20 43.3	+61 27	K	100 .8	— 87.0	— 73.6
269	61 *Cygni pr.*	21 2.4	+38 15	K_5	115 .9	— 62.	— 48.0
275	γ *Pavonis*	21 18.2	—65 49	F_8	117 .7	— 31.1	— 35.1
278	ε *Indi*	21 55.7	—57 12	K_5	118 .1	— 38.7	— 41.9
281	A. G. C. 29191	22 11.7	—54 7	G	117 .1	— 13.4	— 16.5
283	ν *Indi*	22 16.0	—72 44	F	109 .8	+ 25.6	+ 19.1
285	ξ *Pegasi*	22 41.6	+11 40	F_5	109 .0	— 4.6	+ 2.3
286	σ *Pegasi*	22 47.4	+ 9 18	F	108 .3	+ 13.4	+ 19.6
289	A. G. C. 31353	22 59.4	—36 26	K	111 .6	+ 12.	+ 10.0
296	A. G. C. 31584	23 11.9	—14 22	F	107 .3	— 5.5	— 4.6
297	γ *Piscium*	23 12.0	+ 2 44	K	104 .1	— 13.	— 9.4
300	ι *Piscium*	23 34.8	+ 5 5	F_5	98 .0	+ 6.0	+ 8.5
305	85 *Pegasi*	23 56.8	+26 34	G	88 .0	— 37.	— 32.8
306	A. G. C. 32416	23 59.5	—37 51	G-K	99 .8	+ 23.	+ 17.6
			***D* = 120° — 150°**				
197	5 *Serpentis*	15 14.2	+ 2 9	G	133 .6	+ 53.8	+ 65.7
205	γ *Serpentis*	15 51.8	+15 59	F_8	135 .4	+ 7.3	+ 22.3
232	μ *Herculis*	17 42.5	+27 47	G	139 .7	— 15.6	+ 2.1
246	α *Aquilæ*	19 45.9	+ 8 36	A_5	148 .9	— 33.	— 18.0
248	A. G. C. 27380	19 55.5	—67 35	G_5	121 .8	— 13.5	— 16.6
250	δ *Pavonis*	19 58.9	—66 26	G_5	122 .7	— 21.7	— 24.5
254	A. G. C. 27600	20 4.6	—36 21	K_5	144 .9	—132.	—126.6
256	A. G. C. 27708	20 9.1	—27 20	G_2	148 .5	— 48.5	— 41.0
259	ϕ_2 *Pavonis*	20 31.8	—60 53	F_8	124 .9	— 32.0	— 33.9
264	A. G. C. 28703	20 50.8	—44 29	?	133 .1	— 16.	— 14.1
272	A. G. C. 29191	21 11.5	—39 15	G	132 .0	+ 13.	+ 15.4
			***D* = 150° — 180°**				
217	ε *Scorpii*	16 43.7	—34 7	K	150 .2	— 2.2	+ 4.9
222	A *Ophiuchi*	17 9.2	—26 27	K	159 .3	+ 0.8	+ 10.3
223	A. G. C. 23370	17 10.1	—26 24	G	159 .6	— 8.	+ 1.5
224	A. G. C. 23422	17 12.1	—34 53	K_5	153 .3	— 4.5	+ 2.8
233	70 *Ophiuchi*	18 0.4	+ 2 31	K	165 .2	— 7.4	+ 8.4
234	η *Serpentis*	18 16.1	— 2 55	K	171 .0	+ 9.4	+ 24.3

TABLE XXI

AVERAGE RADIAL VELOCITIES OF THE 88 OBSERVED STARS WITH REFERENCE TO ANGULAR DISTANCES FROM PREFERENTIAL VERTEX

	D	km.	km.	km.
5 stars	0°- 30°	+15.40	−13.55	13.92
15 "	30 - 60	50.08	27.90	45.65
51 "	60 -120	16.10	29.73	22.25
11 "	120 -150	26.38	39.24	34.56
6 "	150 -180	8.70	....	8.70
88 "		+24.02	−29.63	26.38
11 "	(0-30) (150-180)	9.66	13.55	11.07
26 "	(30-60) (120-150)	44.16	35.84	40.96
51 "	(60-120)	16.10	29.73	22.25
	VELOCITIES < 60.0 km.			
5 stars	0 30	+15.40	−13.55	13.92
11 "	30 60	18.14	27.90	20.80
47 "	60 -120	14.01	21.27	17.10
9 "	120 -150	13.23	24.68	20.88
6 "	150 -180	8.70	....	8.70
78 "		+14.02	−21.56	17.21
11 "	(0-30) (150-180)	9.66	13.55	11.07
20 "	(30-60) (120-150)	16.81	25.76	20.84
47 "	(60-120)	14.01	21.27	17.10
	VELOCITIES < 50.0 km.			
5 stars	0 30	+15.40	−13.55	13.92
9 "	30 · 60	13.57	13.65	13.59
47 "	60 -120	14.01	21.27	17.10
9 "	120-150	13.23	24.68	20.88
6 "	150-180	8.70	....	8.70
76 "		+13.20	−20.47	16.26
11 "	(0-30) (150-180)	9.66	13.55	11.07
18 "	(30-60) (120-150)	13.48	21.92	17.23
47 "	(60-120)	14.01	21.27	17.10

The means of all the positive velocities and of all the negative velocities, the numerical mean of the eighty-eight velocities, the numerical mean velocity of seventy-eight stars after rejecting ten velocities greater than ± 60 km. per second, and the numerical mean velocity of seventy-six stars after rejecting twelve velocities greater than ± 50 km. per second, are entered in TABLE XXI.

Let us note next the spectral classes of the stars. Nearly all are of Classes F, G and K; only four are of Class A, one of Class M, and none of Class B or Class O.

The numbers of Class A and Class M stars are too small to have any statistical weight, but the arithmetical mean velocities of the Classes F, G and K stars on the list are much greater than the normal averages for these classes as quoted in TABLE XIV and the accompanying text; even after we remove the velocities in excess of ± 60 km. or ± 50 km., approximately 15 per cent greater. Inasmuch as these stars have in effect been selected at random from a complete list of large proper-motion stars, it must be that the average inclination of their motion vectors with the line of sight is considerably greater than the normal average value, 57°.3. This being true, it follows that the average radial velocities of these stars should be *less* than the normal averages for their spectral classes, provided the space velocities of the stars are of average dimensions. We have found, on the contrary, that the radial velocities of these stars are considerably above normal; and we conclude that the large proper motions of these stars are due not merely to their proximity to the solar system but, in addition, to their possession of space velocities greater than the average. Of course all the stars on the list are comparatively near us.

We have also tabulated the resulting velocities of the eighty-eight proper-motion stars with reference to their angular distances from Kapteyn's preferential vertex as averages for zones 30° wide. Fifty-one of the eighty-eight stars are situated at distances between 60° and 120° from the preferential vertex, and only thirty-seven in the equal areas lying within 60° of the two antipodal vertices. It was expected that the average velocities corresponding to the zones nearest the vertices would exceed those in the zones midway between the vertices, but this expectation has not been realized.

Observing programs for determining the parallaxes of individual stars have usually been made up of stars with large proper motions, and it has necessarily followed that an undue proportion of individual parallaxes refer to stars which are not strictly representative of the stellar system. The resulting parallaxes may be quite correct; but if they and the corresponding proper-motion data are used as samples with which to calibrate the distances of stars in general whose proper motions are of certain magnitudes, the results must necessarily contain an appreciable factor of error. In other words, the resulting scale attributed to the stellar universe must be in error. These facts, and the great increase in our estimate of the scale of the universe resulting from the combination of proper-motion and radial velocity data, in TABLE XIX, make clear some of the difficulties which beset the investigator of stellar distances in the days when he was obliged to depend entirely upon individual parallax and proper-motion data.

CHAPTER VII

VISUAL AND SPECTROSCOPIC BINARY STARS

A study of the composition of the stellar system, based upon the observed motions of the stars, must not overlook the double and multiple stars. These are not sporadic cases in the domain of stellar development; on the contrary, they are so numerous that they must be regarded as direct results of the simplest processes of sidereal evolution. This department of astronomy has had a glorious history. The first discovery of a double star, *Mizar,* at the bend in the handle of the *Big Dipper,* was made by Riccioli of Bologna in 1650. Quite by accident, in the prosecution of other observations, Hooke in 1665 found γ *Arietis* to consist of two stars. Richaud found α *Centauri*, in 1689, to be composed of a first-magnitude and a second-magnitude star, and Bradley about 1750 noted *Castor*, γ *Virginis*, β *Cygni*, and 61 *Cygni* to be double. Apparently the earliest serious search for doubles was that made by Mayer of Mannheim, in 1776, which led to the discovery of thirty-three pairs. The systematic searches of Sir William Herschel, beginning in the year 1779, gave life to the subject. Strange to relate, the nature of double stars was completely misunderstood by Herschel during most of the years he was engaged in observing them. He thought of them as accidentally doubled: two stars at great distances from each other, and from us, which happen to lie nearly in line with us, and are therefore seen as a close double by projection. In fact, Herschel's first paper on determining the solar motion spoke enthusiastically of the valuable assistance we could soon expect, in solving this problem, from the relative parallactic displacement of the two components of the double stars: the nearer star of two stars almost in line would be displaced more than the distant star, and be displaced away from the solar apex. A study of these displacements, he said, would indicate the goal of the solar motion.[1] It was not until 1803 that he was able to detect a motion of revolution of one star around another—to be exact, around their common centre of mass—in the case of *Castor*.[2] That Herschel's philosophical mind should have been so slow to grasp the truth about double stars is all the more surprising when we recall that the contents of Michell's papers were known to him. As early as 1767 Michell applied the doctrine of probabilities to a study of the distribution of the stars over the celestial sphere, and came to the conclusion that the very close proximity of two stars must in general be due to a physical connection existing between them.[3] Again, in a paper read before the Royal Society in 1784,[4] he upheld the probability that some of the great number of double and triple stars which had been observed by Herschel are in reality systems of bodies revolving about each other, and he expressed the opinion that in the distant future we should be able to determine their revolution periods. If the six thousand stars visible to good eyes were distributed at random over the celestial sphere, that is, if they were physically unrelated, the mathematical probabilities would be exceedingly strong against more than two or three of the six thousand lying so nearly in the same directions as two or three others, at the same time, that the unassisted eye could not separate them; and the probabilities that six or eight of the six thousand stars would form three or four pairs as close as the closer pairs in Herschel's list would be so small as to be practically impossible. Of the seven or eight thousand known double stars whose components are at this moment less than five seconds of arc apart, and whose fainter components go down even to the thirteenth magnitude, it can scarcely be true that more than five pairs in the entire list are doubled merely as a projection effect. With not more than this number of exceptions, all the pairs are probably in mutual revolution around their respective centres of mass.

[1] *Phil. Trans.* (Abridged Ed.), **15**, 403-404, 1783.
[2] *Phil. Trans.*, 1803, p. 339.
[3] *Phil. Trans.* (Abridged Ed.), **12**, 428, 1767.
[4] *Phil. Trans.* (Abridged Ed.), **15**, 465, 1784.

Sir William Herschel's discoveries of double stars, extending over several decades, included about eight hundred of the brighter stars visible from English latitudes. In 1825-1827, Wilhelm Struve, of Dorpat, made a systematic survey of the northern three-fifths of the sky and found 2200 additional pairs. He devoted the following ten years to careful observation of these and other known pairs by means of the micrometer. The results, published in 1837 as the *Mensuræ Micrometricæ,* include the position angles, distances, estimated colors, and estimated brightness of 2640 double and multiple stars. This volume is the *magnum opus* of double-star astronomy in the nineteenth century.

In order that the surveys begun by his father should extend over the entire sky, Sir John Herschel sailed with his telescope to the Cape of Good Hope in 1834, where he devoted several years to searching the southern heavens for double stars and nebulæ.

Following the two Herschels and Struve, other observers studied double stars with increasing success, but none more successfully than Burnham. It was he who first developed the full power of modern telescopes in double-star discovery and measurement, and his fruitful researches still continue.

The telescopes of today are much larger and in quality better than those of the Herschels and Struve, and the urgent need for a modern resurvey of the whole sky has been recognized for a full generation.

In 1899, my colleagues Aitken and Hussey began this double-star survey, with our excellent Alvan Clark telescopes, in accordance with most carefully prepared plans.[5] Up to June, 1905, when Hussey assumed other duties, he had discovered and measured about 1300 pairs whose combined light in each case is equal to or brighter than that of a 9.1 magnitude star. Aitken

[5] To be exact, it should be said that the survey was originated in all its essential features by Aitken early in 1899, and the systematic observations were begun by him in April, 1899 (*Astr. Nach.*, **152**, 161, 1900). Coöperative observations on the survey were begun by Aitken and Hussey in July, 1899.

has discovered and measured nearly 2300 close pairs whose combined light is in each case equal to or brighter than that of a 9.0 magnitude star.

All previous observers combined had found that one star in thirty-six in the northern two-thirds of the sky, on the average, is double. The survey by Aitken and Hussey, observing the same regions of the sky, shows that one star in about eighteen is a visual double. In other words, for every double star found by former observers, Aitken is adding another in the same region.

In three years from date, Aitken expects to complete his survey of all the stars, down to the ninth magnitude, that can be observed to advantage from Mount Hamilton. This comprises seven-tenths of the sky. The survey of the remaining three-tenths demands a capable observer in the Southern Hemisphere, equipped with a suitable telescope. This great piece of work should be completed as promptly as possible. If carried out along present lines, it is probable that 500 additional pairs will be found in the northern section, from the North Pole of the sky to —22° declination, and 2000 or more in the remaining southern skies. It should be said that the mere discovery of a double star, or a thousand double stars, is a fact of little consequence. The principal value lies in the opportunity for profitable study of double stars, both individually and as a class, in order that our knowledge of them may be made ready for use in solving the still greater problems of the stellar system.

As telescopes increased in size, and especially as they grew in excellence, the definition of a double star changed. The Herschels catalogued large numbers of pairs whose components were very wide apart. Comparatively few of them have shown change in relative positions, or possess special interest. Wilhelm Struve catalogued no pairs with separations exceeding 32″, and Otto Struve set the limit at 16″. Burnham set a narrower limit; for the fainter stars, at 5″. Aitken and Hussey do not record pairs whose combined magnitude is 7, 8 or 9, if the separation is in excess of 5″. The reason for this can be made clear. The angular separation of the two components of a double-star system, as viewed by terrestrial observers, depends upon three factors: (1) the true linear distance between the components; (2) the distance of the system from the Earth, and (3) the angle between the line joining the components and our line of sight. For our present purpose the third factor may be neglected, which amounts to assuming the true dimensions of the orbits to be identical with their projections on the celestial sphere.

Let us assume that the apparent distances are the *mean* distances, and also that the combined mass of each double-star system is twice the mass of the Sun; then, if we take the year, and the Earth's distance from the Sun, as units of time and distance, we may reduce the formula

$$\frac{D^3}{P^2(M+m)} = \frac{D_1^3}{P_1^2(M_1+m_1)}, \quad \text{to}$$

$$P^2 = \frac{D^3}{2}, \tag{55}$$

in which P is the period of the binary system in years and D is the separation of the two components in astronomical units. From this formula we may compute the roughly approximate revolution periods to be expected for binary systems of different angular distances and different parallaxes. Thus we derive the following table:

Approximate Periods of Binary Stars, with the Arguments Angular Separation and Parallax

Angular Separation	Revolution Period for Parallax 0″.1	0″.05	0″.01	0″.005
0″.25	2½ yrs.	8 yrs.	90 yrs.	250 yrs.
0 .50	8	20	250	700
1 .00	25	65	700	2,000
2 .00	65	180	2,000	5,600
5 .00	250	700	7,900	22,000

If the combined mass be assumed to be eight times the Sun's mass, the periods quoted would be divided by two. The figures are, of course, given in round numbers only. From the results

Class A, with very broad Hydrogen lines. γ *Gruis*, 1907, May 11. Measured displacement, −27 km.

Class A. α *Canis Majoris*, 1910, September 29. Measured displacement, −28 km.

in TABLE XIX we may assume that the parallax of the average double-star system (magnitude 6.5 to 9.0 for the combined components) will be considerably less than a 0″.01. Hence it is not to be expected, *in general,* that a double star fainter than 6.5 magnitude, with distance in excess of 2″, will show decisive orbital motion in less than a century. And it is clearly not worth while, at present at any rate, to catalogue systems with distances in excess of 5″ unless one component be very bright or the proper motion be large, and the system therefore probably relatively near us (*e.g.*, *Sirius*, α *Centauri* or 61 *Cygni*). However, the naked-eye stars in the northern sky have been so thoroughly examined that it is improbable that any wide pair whose secondary component is reasonably bright remains undiscovered. Hence the limit of distance for present discovery with existing telescopes may well be taken at five seconds for all stars. No doubt this limit will here and there cut out individual stars of special interest, but there should be satisfactory compensation in making the observational data more homogeneous.

The chief interest in double stars thus far has been the determination of their periods. This is simplicity itself in the case of such an easy double as α *Centauri*, our nearest neighbor in the stellar system, whose components of the first and second magnitudes have been followed through nearly three complete revolutions since the system was discovered in 1689. The period is 81.2 years. However, the situation is far different with the great majority of doubles. For example, the components of γ *Arietis*, each of 4.8 visual magnitude, 8″.6 apart, have given no evidence of relative motion since Bradley observed them in 1755; yet we cannot doubt that the two stars are physically related, for they have been moving with equal angular speeds, eleven seconds per century, along lines apparently parallel. The linear distances apart of the components must in reality be enormous; yet the linear separation of the two bodies is undoubtedly small in comparison with the distances which separate the members of the *Ursa Major* group of stars, which are known to have essentially equal and parallel motion (TABLE IX). Only twenty-six known double-star systems have completed at least one revolution each since their discovery. This number includes two systems with periods of more than one hundred years, which were discovered by Herschel in the eighteenth century.

The double stars *Sirius* and *Procyon* are in many respects the most interesting of all known pairs, but chiefly because of the mode of their discovery. Along with *Neptune* they were near the starting point, so to speak, of the astronomy of the invisible. Bessel noticed as early as 1834 that the proper motion of *Sirius* is variable, and in 1840 he observed a similar irregularity in the apparent motion of *Procyon.* Following their motions carefully, he was able to announce, in 1844, that these irregularities in the motions of the two stars are due to the attractions of invisible companions, one in each system, the period of revolution in each case being about half a century. He wrote to Humboldt: "I adhere to the conviction that *Procyon* and *Sirius* are genuine binary systems, each consisting of a visible and an invisible star. We have no reason to suppose that luminosity is a necessary property of cosmical bodies. The visibility of numberless stars is no argument against the invisibility[6] of numberless others."[7]

The motions of *Sirius* were investigated by Peters,[8] in 1851, who found in favor of an unseen companion. An exhaustive investigation of all the observed positions of *Sirius,* by Auwers,[9] placed the question beyond doubt, by determining the orbits and relative masses of the bright star and the invisible companion; but before the results were published, Mr. Alvan G. Clark discovered the companion, in 1862, near its predicted place. It has all but completed an entire revolution, meanwhile, and in the predicted period of fifty years. The companion is remarkable for its relatively great mass and relatively feeble radiating power. It is one-half as massive as the bright component, but sends out only 1/10000 as much light. The bright star is only twice as massive as our Sun, but it radiates twenty times as much

[6] Except for the presence of the brilliant primaries, the secondaries in the systems of *Sirius* and *Procyon* would be easily visible in telescopes of moderate size.—*W. W. C.*

[7] Wolf's *Geschichte der Astron.,* p. 743, 1877.

[8] *Astr. Nach.,* **32,** 1-58, 1851.

[9] *Astr. Nach.,* **58,** 35, 1862; **129,** 185, 1892.

light. The faint component is as massive as our Sun and radiates only 1/500 as much light—a condition which we are powerless at present to explain.

A similar investigation of *Procyon* by Auwers,[10] in 1862, assigned a period of forty years to the system. More than half a century following Bessel's prediction, Schæberle[11] found the companion, in 1896, with the Lick telescope. It is of the thirteenth magnitude and exceedingly difficult to observe even under the best atmospheric conditions, although the distance separating primary and secondary is 5″. The companion is usually lost to sight in the glare of the bright body. Here, as in the system of *Sirius,* the companion is very massive in proportion to its brightness.

The application of the Doppler-Fizeau principle to the study of the stars by photographic means has led to the discovery of a different class of stellar systems, known as spectroscopic binaries. This term is applied, in general, to those stars which are apparently single when viewed through our most powerful telescopes, but which the spectrograph has shown to be accompanied by invisible companions. Just as the variable *proper* motions of *Sirius* and *Procyon* led Bessel to suspect, and Peters and Auwers finally to prove, the existence of invisible companions to these stars, so the variations in observed *radial* motions are attributed to the attractions of invisible companions. The presence of two members in a system implies and demands, as we well know, a mutual revolution of the separate bodies, in elliptical orbits, around their common centre of mass; and, if the orbital plane is not at right angles to the line of sight, there must be radial motions alternately of approach and recession, with reference to the velocity of the system as a whole. The corresponding spectral lines of each body must swing alternately toward the red and toward the violet from their mean positions; and if the spectra of both bodies are visible, the two sets of lines must move in opposite directions from their mean positions.

10 *Astr. Nach.,* **58**, 35, 1862.

11 *Astr. Jour.,* **17**, 37, 1896.

The first spectroscopic binary, *ζ Ursæ Majoris*, was discovered by Pickering in 1889. The photographic spectrum of this star shows only a few lines, chiefly those of hydrogen, helium, calcium, etc. It was noted, first, that the calcium K line was alternately double and single. Long-continued observation by Pickering and, later, by Vogel, established that the period of revolution of the two stars, of essentially equal brightness, is 20.5 days. A few months later the Harvard photographs led Miss Maury to the discovery[12] that *β Aurigæ* is a binary system of almost exactly the same sort. Two stars essentially equal in luminosity are in this case revolving around a common centre of mass, in the short period of 3.96 days.

Another discovery made in the same year by Vogel,[13] of Potsdam, relates to *Spica*. The spectrograms of this star yielded velocities varying through about 200 km. per second. Mathematical treatment of these velocities proved that *Spica* is attended by a massive companion, the two bodies, one bright and the other relatively dark, forming a binary system which completes one revolution in a period of 4.01 days. Vogel suspected that the spectrum of the fainter component was faintly visible on some of the photographs. Baker has made a thorough investigation of the system, securing velocity observations of the fainter component as well as of the brighter.[14] He determined the elements

$$\begin{aligned} P &= 4.01416 \text{ days} \\ a \sin i &= 6{,}930{,}000 \text{ km.} \\ a_1 \sin i &= 11{,}400{,}000 \text{ km.} \\ e &= 0.10 \\ m \sin^3 i &= 9.6 \odot \\ m_1 \sin^3 i &= 5.8 \odot \end{aligned}$$

in which the subscript "1" refers to the fainter component.

We may gain some idea of the dimensions of this interesting system. If we were exactly in the plane of the orbit, we should

[12] *Mon. Not. R. A. S.*, **50**, 296, 1890.

[13] *Sitzungsber. der kgl. Akad. Wiss. Berlin*, p. 401, 1890.

[14] *Publ. Allegheny Obs.*, **1**, 65, 1909.

of course observe eclipses partial or complete; once in each period, if the companion star is a dark object, or twice in each period if the companion is fairly luminous in comparison with the brighter star. This condition does not hold, for eclipses have not been observed. We have reason to believe that the orbit is only slightly inclined to the line of sight because of the high radial velocities observed. We are therefore fairly safe in assuming that the maximum distance separating the two components is not greatly in excess of 18,000,000 km. Granting that the parallax of *Spica* is as great as 0″.15, the corresponding maximum separation of the two components would be 0″.02. Now we know that the parallax of *Spica* can scarcely exceed 0″.02, for Gill's heliometer value of the parallax is — 0″.02 ± 0″.01,[15] with reference to comparison stars of magnitude 8.7. It is therefore impossible to hope that the two members of the system, even if of equal brightness, could be seen as an ordinary double star with a telescope of existing forms unless its objective should have a separating power at least thirty- or forty-fold greater than the 36-inch refractor.

In successive years additions to the list of spectroscopic binaries were made by Pickering, Vogel, Belopolsky, Miss Maury, Mrs. Fleming, and Bailey, until, in 1898, thirteen spectroscopic binary systems had been discovered. Since 1898 the number has increased with great rapidity. A *Second Catalogue of Spectroscopic Binaries,* to be issued in a few weeks, will contain fully 300 entries. These exhibit a great variation in lengths of revolution periods, relative luminosities of components, forms of the orbits, etc. There are a few in which the two components must be almost precisely of the same luminosity and spectral type; a great majority are of those for which the spectrum of but one component records itself upon the plates; and there exists a continuous gradation of luminosity-ratios between the two extremes just described. The proportion of spectroscopic binaries discovered to stars observed is extraordinarily large. The 300 binaries thus far announced are from observation programs which probably total fewer than 1600 stars. We may be

15 *Ann. Cape Obs.,* 8 (Part 2), 135 B, 1900.

sure, when we look at the stars, that at *least* one in five, on the average, is attended by a companion, invisible and of mass sufficiently great to swing the bright member of the system rapidly around in a large orbit. Every radial velocity observer has a list of stars whose velocities are suspected to vary, but whose binary characters are awaiting confirmation through continued observation. The number of stars on existing lists of suspects which will eventually reveal their binary nature can hardly be less than 100. These would increase the proportion of variable velocities to constant velocities up to one in four. The observed proportion is plainly greater for certain spectral types than for others. Frost has made a specialty of measuring the radial velocities of those stars whose spectra are of the so-called Orion or helium type, known as Class B stars. He has found that one star in two and a half on his program is a spectroscopic binary. His observations of the stars in Boss's *Taurus* cluster (page 183) have led to the surprising result, as far as the work has been carried, that one star in two, on the average, is a spectroscopic binary.[16] It is certainly a most interesting fact, for a considerable group of stars, to have the chances equal that any star in the group will be double or single.

To bring out certain characteristics of visual and spectrographic methods and limitations, let us consider the well-known double star *α Centauri.*[17]

[16] *Ap. J.*, **29**, 237, 1909.

[17] A critical review of methods employed in the determination of the orbits of spectroscopic binaries is given by Plummer in *Ap. J.*, **28**, 212, 1908, with references to the principal original papers on the subject. A very practical paper by Curtis, in *Publ. Astron. Soc. Pac.*, **20**, 133, 1908, discusses the points of advantage presented by several of the leading methods of determining spectroscopic binary orbits. Schlesinger has published convenient and efficient methods of improving the elements of preliminary orbits by the application of the method of least squares, in *Publ. Allegheny Obs.*, **1**, 33, 1908. A brief discussion of the relations existing between the elements of the orbits of visual and spectroscopic binary systems is given by Campbell in *Lick Obs. Bull.*, **3**, 80, 1905. The elements of spectroscopic orbits as used in the *Second Catalogue of Spectroscopic Binary Stars*, now in course of preparation and soon to be published in *Lick Observatory Bulle-*

tin, Volume VI, are as defined below, including the significance given to the symbol T by the computers in order to meet various conditions.

P=apparent period of revolution in mean solar days, unless specified in mean solar years. The true period of revolution, P_0, may be found from the apparent period by means of the equation

$$P_0 = P/(1 + \frac{V_o}{300,000}).$$

(The apparent period, P, is more frequently used than P_0 in dealing with binary systems; but for triple systems, such as that of *Polaris*, P is variable, and the variations must be taken into account.)

T=Greenwich mean time of periastron passage, expressed in Julian days; except (1) that for variable star orbits the quantity in the column T is the mean solar interval after maximum or minimum brightness, as specified in each case; and (2) that for circular orbits T has been given significance by the computers, as described in the column "Remarks."

ω=angular distance of periastron from the ascending node, measured in the direction of orbital motion. (At the ascending node the radial velocity of the observed body has its maximum value. Spectrographic observations enable the computer to distinguish between the ascending and descending nodes; but micrometer observations of visual double stars do not distinguish between the two nodes, and therefore leave the value of ω uncertain by 180°.)

e=eccentricity of the orbit.

K=semi-amplitude of velocity curve of the primary member of the system, with reference to the centre of mass of the system.

K_1=semi-amplitude of velocity curve of the secondary member of the system, with reference to the centre of mass of the system.

$K+K_1$=semi-amplitude of the velocity curve of one member of the system, with reference to the other member of the system.

a=semi-major axis of the orbit of the primary member of the system, with reference to the centre of mass of the system.

a_1=semi-major axis of the orbit of the secondary member of the system, with reference to the centre of mass of the system.

$a+a_1$=semi-major axis of the orbit of one member of the system, with reference to the other member of the system.

i=inclination of the orbit plane, conveniently defined as the angle between the line of sight and the normal to the orbit plane. (Spectrographic observations leave the value of i undetermined; micrometer observations of a visual binary system leave the quadrant of i undetermined; spectrographic and micrometer observations combined determine i completely, but the number of known systems available for both classes of observation is at present very limited.)

The orbit of α *Centauri* as a double star is probably better determined[18] than that of any other. The period of revolution, the form of the orbit, and the position of the orbit in one of two alternative but definitely stated planes, are known from the micrometer measurements of the relative positions of the two

m=mass of primary member of the system.
m_1=mass of secondary member of the system.
$m+m_1$= mass of the system.
$\odot$ =mass of the Sun.
V_0=radial velocity of the centre of mass of the system.
V=radial velocity of the primary member of the system at any instant.
V_1=radial velocity of the secondary member of the system at the same instant.
r=radius of the primary member.
r_1=radius of the secondary member.
d=mean density of the primary member.
d_1=mean density of the secondary member.
d_0=mean density of the system.
D_0=mean density of the Sun.

Certain relations existing between the semi-major axes, the semi-amplitudes, the velocities, the masses, the periods of revolution, and the inclinations of the orbit planes, are defined in the eight equations which follow.

I. One stellar spectrum observed, with comparison spectra:

$$a \sin i = [4.13833]\ (1-e^2)^{\frac{1}{2}}\ KP, \tag{56}$$

$$\frac{m_1^3 \sin^3 i}{(m+m_1)^2} = [3.01642-10]\ (1-e^2)^{\frac{3}{2}}\ K^3 P \odot. \tag{57}$$

II. Two stellar spectra observed, either without comparison spectra, as by means of objective-prism spectrographs, or with comparison spectra:

$$(a+a_1)\ \sin i = [4.13833]\ (1-e^2)^{\frac{1}{2}}\ (K+K_1)\ P, \tag{58}$$

$$(m+m_1)\ \sin^3 i = [3.01642-10]\ (1-e^2)^{\frac{3}{2}}\ (K+K_1)^3\ P \odot. \tag{59}$$

III. Two stellar spectra observed, with comparison spectra:

$$\frac{m^3 \sin^3 i}{(m+m_1)^2} = [3.01642-10]\ (1-e^2)^{\frac{3}{2}}\ K_1^3\ P \odot, \tag{60}$$

$$m \sin^3 i = [3.01642-10]\ (1-e^2)^{\frac{3}{2}}\ (K+K_1)^2 K_1 P \odot, \tag{61}$$

$$m_1 \sin^3 i = [3.01642-10]\ (1-e^2)^{\frac{3}{2}}\ (K+K_1)^2 K P \odot, \tag{62}$$

$$\frac{m}{m_1} = \frac{K_1}{K} = \frac{(V_1-V_0)}{(V-V_0)}. \tag{63}$$

[18] Roberts, *Astr. Nach.*, **133**, 106, 1893.

companions. Measurements of the varying distances of the two companions from neighboring stars have determined the relative sizes of the orbits, and therefore the relative masses of the components[19] as 51 to 49. Gill's and Elkin's heliometer determinations[20] of the parallax of the system give $\pi = 0''.75 \pm 0''.01$. This parallax is of course relative to the comparison stars, of average magnitude 7.6. Making the usual allowance for the parallax of a 7.6 magnitude star, the absolute parallax of *α Centauri* becomes 0″.76.

From these observational results we know the distance of the system to be 4.4 light years, the linear size of the orbit and therefore the masses of the two bodies. The only remaining orbital uncertainty is as to which of the two orbital planes is correct. Radial velocity observations of the two components by the D. O. Mills Expedition have enabled my colleague Wright to decide between the two planes, keeping one and rejecting the other, and to make a valuable independent determination[21] of the parallax. These measures give a velocity of approach for the system amounting to 22.2 km. per second, and at the epoch 1904.8 a differential velocity of 5.30 km. for the two stars. Now it is apparent that this differential velocity is a direct function of the linear size of the orbit; and as the angular dimensions of the orbit are known from the ordinary double-star observations, it remained only to deduce that value of the distance or parallax which will harmonize the linear and angular dimensions of the system. The spectrographic parallax came out $0''.76 \pm 0''.02$, agreeing perfectly with the Cape heliometer parallax. The linear dimensions of the orbit having been determined in this way, and the *relative* masses being known, it is a simple matter to determine that the combined mass of the two stars is 1.9⊙.[22]

In such favorable cases as *α Centauri* the spectrographic method of determining parallax possesses striking advantages over the heliometer method. The degree of accuracy is inde-

19 Roberts, *Astr. Nach.*, **139**, 10, 1895; see also TABLE XXX.

20 *Ann. Cape Obs.*, **8** (Part 2), 135B, 1900.

21 *Lick Obs. Bull.*, **3**, 4, 1904.

22 *Lick Obs. Bull.*, **3**, 4, 1904.

pendent of the distance of the star, except as this factor influences the apparent brightness of the two components and the micrometer determination of the angular dimensions of the orbit. No assumption as to the relative distance of primary stars and comparison stars is involved. The absolute parallax is determined directly. The differential radial velocity is large in comparison with the observed radial velocity; in this case about 25 per cent. With the heliometer, large angular distances are usually measured, and they involve corresponding uncertainties as to refraction and other similar corrections. A series of twenty-five spectrograms would apparently reduce the probable error of the resulting parallax to equality with that of the heliometer value.

However, it must be made clear that the spectrographic method of determining distances of individual stars is applicable to only a few known systems, in which the velocities of both components are measurable and the visual orbits are well determined; unless, in systems with well-defined orbits, we wait for the bright members to pass over large parts of their orbits.

To illustrate, the orbit of *Sirius* as a double star is extremely well determined, but the velocity of the companion cannot be measured spectrographically. The spectrograph has decided between the two possible orbit planes, accepting the positive value of the inclination[23] and rejecting the negative, and has given us a fair value of the radial velocity of the system, —7.4 km. per second; but we have not sufficient basis for making a spectrographic determination of the parallax of *Sirius,* except by waiting for a large variation of velocities to occur.

The spectrograph has made important contributions to our knowledge of *Castor* as a stellar system, this being the double star in which Herschel first detected orbital motion. Fourteen years ago Belopolsky, of Pulkowa, found that the fainter component of *Castor* is a spectroscopic binary, period 2.93 days, and orbit nearly circular.[24] My colleague Curtis found five years ago that the brighter component of *Castor* is also a spectroscopic

[23] *Lick Obs. Bull.,* 3, 83, 1905.

[24] *Mem. Acad. St. Petersbourg,* 11, No. 4, 83, 1900.

binary, period 9.22 days, and eccentricity 0.50.[25] *Castor* is thus a quadruple system. Curtis has determined the radial velocities of the centres of mass of the two binary members, and finds a difference in their radial speeds amounting to 7 km. per second. Unfortunately, the visual orbit of *Castor* is completely unknown. Numerous published orbits assign periods varying from 232 up to 1001 years. All we can now say is that the period must be several hundred years in length. Not knowing the period and the other elements of the orbit, the spectroscope cannot at present determine the parallax of this system.

We have been interested in measuring the radial velocities of *Procyon* from time to time since 1896; that is, through one-third of the period of the visual companion. The radial velocity appears not to have varied systematically in this interval, though there is perhaps a secondary variation amounting to only 1½ km. per second, in a period of seven years; thus Auwers's conclusion is probably correct, to the effect that the orbital plane of the visual *Procyon* system is tangent to the celestial sphere. The small secondary variation is not certainly established, on account of its smallness and the fact that it has but recently been suspected.

Spectrographic binary stars present a great variety of orbital elements. There is time for only the briefest reference to the more interesting individual features. The binary of shortest known period is *β Cephei*, period only 4 hours 34.2 minutes, discovered by Frost.[26] The orbit is nearly circular and there is a total variation of 34 km. per second in radial velocity. Another interesting short-period system is that of *β Canis Majoris,* for which my colleague Albrecht has determined a period of almost exactly six hours.[27] In this system also the orbit is nearly circular.

Figure 4, Chapter III, reproduces the velocity curve for *η Pegasi*, a solar-type binary, the spectrum of only one component being visible; period 2¼ years, eccentricity 0.15.

25 *Lick Obs. Bull.*, 4, 64, 1906.

26 *Ap. J.*, 24, 259, 1906.

27 *Lick Obs. Bull.*, 6, 22-23, 1910.

We have recently found that α *Ursæ Majoris*, the first star in the *Big Dipper*, has a variable velocity.[28] Observations extending from 1896 to the present time have been needed in order to show that the velocity is certainly variable. So slow is the change and so long the period that it is impossible to predict the length of period at present, but it may include several decades, and perhaps several scores of years. The visual companion discovered by Burnham may be involved.

The Mills spectrograph determined[29] in 1899 that the first-magnitude star *Capella* is a binary through the fact that it has a composite spectrum, consisting of two sets of spectral lines which shift alternately back and forth over each other. The binary character of *Capella* was also discovered three months later, quite independently, by Professor Newall,[30] at Cambridge. The period is 104.2 days. The system as a whole is receding from us at the rate of 30 km. a second. The orbit is nearly circular, eccentricity equal to 0.02. The more massive star varies in radial velocity from + 4 to + 56 km. and the lesser from + 63 to — 3 km. We thus determine that the masses of the two bodies are as 1.26 to 1. The brighter star, of solar type, is a half magnitude more brilliant photographically than the fainter star, of Sirian type. This is equivalent to the solar-type component being a full visual magnitude brighter than the companion.

The position of the orbital plane of *Capella* is totally unknown. If we were exactly in the plane, we should expect to observe an eclipse every fifty-two days. In this case the maximum distance between the two stars would be 83,000,000 km., a little more than one-half the distance separating Earth and Sun. This distance is the value of $(a + a_1) \sin i$. Elkin's value of the parallax, 0″.08, would give in this case an angular separation of the two companions amounting to 0″.045. As no eclipses have been observed, we conclude that we are not in the orbital plane; but the high velocities observed lead us to believe that

28 *Lick Obs. Bull.*, **5**, 174, 1910.

29 *Ap. J.*, **10**, 177, 1899; *Lick Obs. Bull.*, **1**, 34, 1901.

30 *Mon. Not. R. A. S.*, **60**, 2, 1899.

the plane does not pass far to one side or the other of our position in space. If the inclination of the orbital plane to the surface of the celestial sphere were 30°, the corresponding value of $a+a_1$ would be doubled, and the maximum separation would be 0″.09. Of course, if the orbital plane were nearly tangent to the celestial sphere, the dimensions of the orbit would have to be very great in order to give the observed radial velocities. In that case the observation of the spectroscopic binary also as a visual binary should have been accomplished before this, for the star has been most carefully examined. I should say that ten observers at Greenwich, using a 28-inch telescope, made a series of observations[31] of *Capella* as a double star shortly following the announcement of its spectroscopic binary character, the visual observations seeming to be fairly easy to secure, inasmuch as they were made when *Capella* was from 3 hours to 6½ hours from the meridian, and in part by observers inexperienced with double stars. However, the series has been discontinued, and there is little doubt that the observers were misled by elongated images, possibly due either to the lack of adjustment of the object glass of the telescope or to the refraction of the star's light in our atmosphere.

Perhaps as interesting a spectrographic system as we have thus far found is that of the North Pole star, *Polaris*. This system appeals to me no doubt more than to others, for *Polaris* was the first star observed with the Mills spectrograph—in 1896. The velocity in that year was thought to be constant, as the observations, compared with one another, were suprisingly consistent, yielding a probable error of but ¼ km. for a single result. The observed velocities of that year were published as of constant character. There were six of them, with values about —19 km. and —20 km., as plotted near the bottom of Figure 11. Reobserving the star in 1899 to test the adjustments of the apparatus, a quite different velocity was obtained. Following up the indications of variable velocity, by means of an extensive series of observations, we found[32] that in *Polaris* we have a triple

[31] *Mon. Not. R. A. S.*, **60**, 595, 1900.

[32] *Ap. J.*, **10**, 180, 1899; *Lick Obs. Bull.*, **6**, 18-19, 1910.

system, composed of one star bright enough to impress its spectrum upon the plate in a few minutes, and of two invisible companions which leave no traces whatever upon the spectrograms. The salient features of the system are illustrated by means of the velocity curves in Figure 11. It was found that the velocities observed in 1899 repeated themselves in a cycle of 3.9681 days, the variation in velocity amounting to about 6 km. The velocity of the centre of mass of the binary system in 1899,

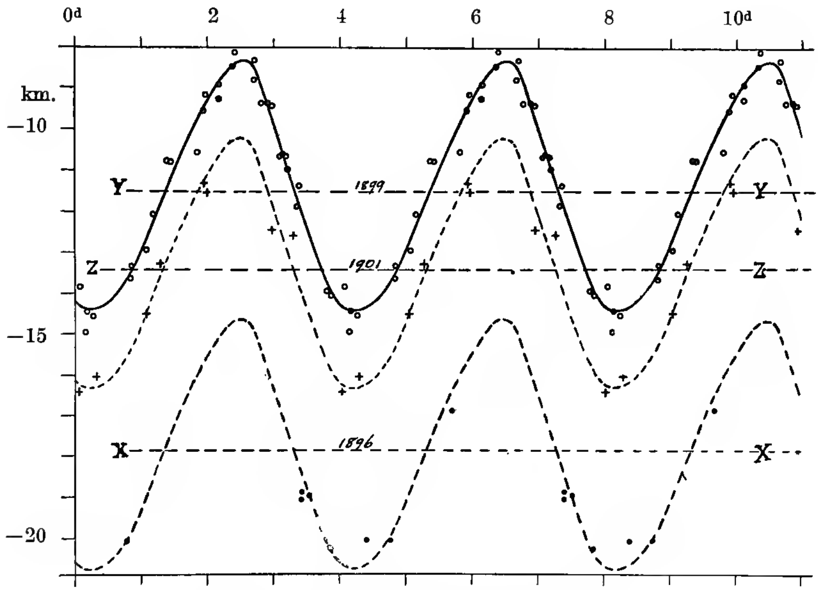


Figure 11.—Some velocity curves of *Polaris*

represented by the straight line YY, amounted to —11.5 km. per second. When the well-determined curve of 1899 was moved down to coincide with the plotted observations of 1896, it was found that the early observations were well represented by a curve of the same form. It had happened that a seventh spectrogram secured in 1896 had been seriously damaged in the darkroom manipulation, the film having been badly torn, and this plate had not been measured. It was measured in 1899, with result $V = -17$ km., and found to conform to the 1896 curve. The velocity of the centre of mass of the binary system in 1896, represented by the line XX, was -17.9 km. per second. The velocity curve of the binary system in 1901 occupied a still different position, with systemic velocity represented by ZZ, -13.4 km. per second. Series of observations in succeeding years have shown that the systemic velocity has been approaching that of 1896, and that the period of revolution of the binary system around the centre of mass of itself and a third member of the system appears to be in the vicinity of twelve years.

Two neighboring binary systems in the constellation of *Orion*, discovered at the Yerkes Observatory, have recently been shown by Plaskett and Harper (*Ap. J.*, **30**, 373, 1909) to possess orbits remarkable for the similarity of their elements, and especially for their high eccentricities, as may be seen from the following tabulation.

	ι Orionis	B. D. — 1°.1004
Right Ascension	$5^h\ 30^m$	$5^h\ 36^m$
Declination	$-5°\ 59'$	$-1°\ 11'$
Spectrum	0e5	B3
Period	29.136 days	27.160 days
Eccentricity	0.74	0.76
Velocity Variation	227 km.	186 km.
Long. of Periastron	112°.4	87°.0
Velocity of System	+21.5 km.	+26.1 km.
$a \sin i$	28,907,000 km.	22,380,000 km.

How serious a complication is injected into the general problems of stellar radial velocities by the discovery of so many spectroscopic binary systems is well illustrated by the stars in the *Big Dipper*. We referred above to Alpha as a long period binary. Ludendorff[33] has announced Beta to be a spectroscopic binary, with period a little over 27 days, though the Mount Hamilton observations do not seem to be confirmatory. Adams[34] has suggested that Epsilon may have variable velocity, and

[33] *Astr. Nach.*, **177**, 235, 1908; **180**, 271, 1909.
[34] *Ap. J.*, **18**, 68, 1903.

Ludendorff suspects[35] that it varies in a period of two years, more or less. We have noted that Zeta was the first spectroscopic binary system to be discovered by Pickering, period 20.5 days. Zeta and its near neighbor, *Alcor,* form an exceedingly interesting multiple group. It is well known that Zeta is a visual double star, magnitudes two and four, with angular separation about 14″, which was discovered by Sir William Herschel. The brighter component is the Pickering spectroscopic binary. The fourth-magnitude component was discovered to be a spectroscopic binary by Frost and Lee,[36] and it was independently suspected of variation by Ludendorff.[37] The near neighbor, *Alcor,* was found by Frost to be a spectroscopic binary.[38] Thus the spectrograph has established that the three stars visible in the telescope are in reality six stars. Of the other bright stars in the *Big Dipper*—Gamma, Delta, and Eta—accurate observations have not been secured, as their spectral lines are very poorly defined; but it would not be surprising to find that some of them are travelling through space accompanied by massive invisible companions.

The number of spectroscopic binary systems discovered to date is more than three hundred, but the orbits have been determined, by various observatories and astronomers, for only sixty-five of the systems, though the approximate periods are known for many additional systems. A study of the relations existing between spectral classes, lengths of periods, eccentricities, and other constants is an engrossing occupation. My colleagues and I have been noticing, for several years, that for the binaries of early spectral classes there is a tendency toward short periods and orbits nearly circular; and for binaries of the older spectral classes, a tendency toward periods relatively long and orbits of considerable eccentricity.[39] Thanks to the shortness of period

[35] *Astr. Nach.,* **180**, 273, 1909.

[36] *Astr. Nach.,* **177**, 171, 1908.

[37] *Astr. Nach.,* **177**, 9, 1908.

[38] *Astr. Nach.,* **177**, 172, 1908.

[39] Professor See has recently called attention to the fact that short period spectroscopic binaries have small eccentricities, and longer period binaries have larger eccentricities, in *Mon. Not. R. A. S.,* **68**, 201, 1908.

TABLE XXII

Spectroscopic Binaries of Classes O and B

H. R.	Star	Class	Period	e	$a \sin i$ km.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	$m \sin^3 i$	$m_1 \sin^3 i$	$m_1 : m$
779	*δ Ceti*	B2	short*						
1088	*τ_5 Eridani*	B8	short						
1347	*υ_4 Eridani*	B9	short						
2781	29 *Can. Maj.*	Oe	short						
4118	*δ Antliæ*	B9	short						
4662	*γ Corvi*	B8	short						
6027	*ν Scorpii*	B3	short						
7248	Y *Aquilæ*	B8	short						
8238	*β Cephei*	B1	$0^d.19$	small	45,000	0.0001⊙			
2294	*β Can. Maj.*	B1	0.25	0.1±	33,510	0.00002			
1567	*π_5 Orionis*	B3	0.87						
3129	V *Puppis*	B1p	1.45	(0.)	6,100,000		17.1⊙	17.1⊙	1.0
6247	*μ_1 Scorpii*	B3p	1.45		4,586,000		7.2	7.2	1.0
5944	*π Scorpii*	B2p	1.57						
6431	*u Herculis*	B3	2.05	0.05	2,800,000	0.205	6.8	2.6	0.39
6431_1	*u Herculis*		2.05	0.05	7,120,000	3.43			
1811	*ψ Orionis*	B2	2.53	0.06	4,995,100	0.780	5.53	4.19	0.76
1811_1	*ψ Orionis*		2.53	0.06	6,570,000	0.343			
8523	2 *Lacertæ*	B5	2.62	0.02	2,890,000	0.141	0.87	0.71	0.82
8523_1	2 *Lacertæ*		2.62	0.02	3,550,000	0.026			
8001	57 *Cygni*	B3	2.85						0.96
936	*Algol*	B8	2.87	0.04	1,600,000	0.020	0.44	0.22	0.50
1239	*λ Tauri*	B3	3.95	0.	3,300,000	0.089			
5056	*α Virginis*	B2	4.01	0.10	6,930,000	0.833	9.6	5.8	0.60
5056_1	*α Virginis*		4.01	0.10	11,400,000	3.68			
226	*ν Androm.*	B3	4.28	0.	4,299,000	0.173			
1131	*ο Persei*	B1	4.42	0.	7,172,000	0.754			
6527	*λ Scorpii*	B2	5.6						
1852	*δ Orionis*	B	5.73	0.10	7,906,600	0.601			
8926	BD+57°.2748	B3	6.07						
1542	9 *Camelopard.*	B	6.33						
3623	*κ Cancri*	B8	6.39	0.15	5,890,000	0.200			
3659	*α Carinæ*	B3	6.74	0.18	1,960,000	0.007			
5984	*β Scorpii*	B1	6.9						
1788	*η Orionis*	B1	7.99	0.02	15,901,000	2.51	11.2	10.6	0.95
1788_1	*η Orionis*		7.99	0.02	16,750,000	2.15			
5231	*ζ Centauri*	B2p	8.02						
1552	*π_4 Orionis*	B3	9.52	0.03	3,393,000	0.017			
7790	*α Pavonis*	B3	11.75	0.01	1,170,000	0.0005			
7106	*β Lyræ*	B2p	12.91	0.07	33,000,000	9.7	9.6	20.9	2.2
7106_1	*β Lyræ*		12.91	0.07	15,800,000	1.0			
1713	*β Orionis*	B8p	21.90	0.30	1,108,900	0.0001			
2653	*o_2 Can. Maj.*	B5p	24.3±		1,670,000±	0.0003±			
1952	BD −1°.1004	B3	27.16	0.76	22,380,000	0.607			
1899	*ι Orionis*	Oe5	29.14	0.75	28,907,000	1.14			
6084	*σ Scorpii*	B1	30.±						
5190	*ν Centauri*	B2	31.±						
3734	*κ Velorum*	B3	116.65	0.19	73,200,000	1.15			
496	*φ Persei*	Bp	126.5	0.43	42,298,000	0.189			
2159	*ν Orionis*	B2	131.4						
1910	*ζ Tauri*	B3	138	0.18	27,900,000	0.046			
154	*π Androm.*	B3	143.72	0.58	76,790,000	0.876			
6812	*μ Sagittarii*	B8p	180.2	0.44	143,500,000	3.63			
936_1	*Algol*	B8	$1^y.90$	0.	89,000,000	0.060			

* The periods marked "short" are unknown.

TABLE XXIII

SPECTROSCOPIC BINARIES OF CLASS A

H. R.	Star	Class	Period	e	$a \sin i$ km.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	$m \sin^3 i$	$m_1 \sin^3 i$	$m_1 : m$
104	BD +43°.92	A2	short*						
897	θ_1 *Eridani*	A2	short						
1380	64 *Tauri*	A2	short						
5971	ι *Cor. Bor.*	A	short						
2124	μ *Orionis*	A2	$0^d.77$						
815	RZ *Cassiop.*	A	1.20		1,170,000	$0.05 \pm \odot$	$0.65\odot$	$0.36\odot$	0.55
1497	τ *Tauri*	A	1.50	0.08	916,130	0.0136			
5586	δ *Libræ*	A	2.33	0.05	2,450,000	0.110	0.8	0.6	0.7
	U *Cephei*	A	2.49						
2890	α_1 *Gemin.*	A	2.93	0.01	1,279,000	0.0097			
7326	U *Sagittæ*	A	3.38						
1568	7 *Cameloprd.*	A2	3.88						
2088	β *Aurigæ*	Ap	3.96	0.00	6,000,000		2.16	2.16	1.0
2088_1	β *Aurigæ*	Ap	3.96	0.00	6,000,000				
6324	ϵ *Herculis*	A	4.02	0.07	2,900,000	0.061	0.37	0.25	0.7
6324_1	ϵ *Herculis*		4.02	0.07	4,286,000	0.194			
2891	α_2 *Gemin.*	A	9.22	0.50	1,485,000	0.0015			
4072	BD +66° 664	A	11.6±	large					
7710	θ *Aquilæ*	A	17.11	0.69	7,665,000	0.061	0 31	0.28	0.89
7710_1	θ *Aquilæ*		17.11	0.69	8,610,000	0.087			
5793	α *Cor. Bor.*	A	17.36	0.33	7,600,000	0.059			
5054	ζ_1 *Urs. Maj.*	Ap	20.54	0.50	17,500,000	3.96	2.0	2.0	1.0
5054_1	ζ_1 *Urs. Maj.*	Ap	20.54	0.50	17,500,000				
8178	β *Equulei*	A	22.7±						
7178	γ *Lyræ*	A	25.6±						
4295	β *Urs. Maj.*	A	27.16	0.79	1,774,000	0.0003			
622	β *Trian.*	A5	37.±						
5291	α *Draconis*	A	51.38	0.40	30,000,000	0.42			
4689	η *Virginis*	A	71.9	0.33	25,500,000	0.126			
15	α *Androm.*	A	96.7	0.51	33,000,000	0.155			
553	β *Arietis*	A5	107.0	0.88	22,880,000	0.042	0.17	0.17	1 0
2421	γ *Gemin.*	A	$3^y.5\pm$	large					
2491	*Sirius*	A	$49^y.3$	0.59					

*The periods marked "short" are unknown.

in most spectroscopic binary systems, and to the energy and skill of spectrographic observers, the time has come to study in greater detail and more accurately the apparent relationships existing between spectral classes, periods of revolution, and eccentricities. The available data on spectroscopic binary orbits are contained in TABLES XXII, XXIII, XXIV and XXV. (These tables have been extended, after the delivery of this lecture, to include all results available up to March 15, 1910, the epoch of the *Second Catalogue of Spectroscopic Binary Stars.*) The tabulations are self-explanatory, and it remains only to draw from them certain evident and simple conclusions of apparently great importance.

TABLE XXIV

SPECTROSCOPIC BINARIES OF CLASS F

(Cepheid variables not included)

H. R.	Star	Class	Period	e	$a \sin i$ km.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	$m \sin^3 i$	$m_1 \sin^3 i$	$m_1 : m$
2788	R *Can. Maj.*	F	$1^d.14$						
142	13 *Ceti*	F	2 .08	0.06	981,460	0.0087$\odot$			
5986	*θ Draconis*	F8	3 .07	0.01	990,000	0.0041			
424	*α Urs. Min.*	F8	3 .97	0.13	164,500	0.00001			
	Z *Herculis*	F	3 .99						
7056	*ζ_1 Lyræ*	F	4 .30	0.00	3,030,000	0.060			
6596	*ω Draconis*	F5	5 .28	0.01	2,632,000	0.0261			
8315	*κ Pegasi*	F5	6 .±						
8430	*ι Pegasi*	F5	10 .21	0.01	6,740,000	0.117			
3852	*ο Leonis*	F5p	14 .50	<0.02	10,775,000	0.238	1.30$\odot$	1.12$\odot$	0.86
3852_1	*ο Leonis*	F±	14 .50	<0.02	12,571,000	0.378			
2693	*δ Can. Maj.*	F8p	¾ yrs.±		5,650,000	0.0001			
6927	*χ Draconis*	F8	$281^d.8$	0.42	63,000,000	0.126			
8123	*δ Equulei*	F5	$5^y.7$	0.36					
424_1	*α Urs. Min.*	F8	$11^y.9$	0.35	166,800,000	0.0098			
3482	*ε Hydræ*	F8	$15^y.7$	0.6	550,000,000	0.18			
3185	*ι Argus*	F5	long*						

* The period marked "long" is unknown.

TABLE XXII contains the orbital data for those spectroscopic binaries of Classes O and B (stars thought to be effectively young) whose periods of revolution have been fairly well determined. These binaries are arranged in the order of the length of period. Except in the case of the last star on the list, which refers, apparently with considerable uncertainty, to the centre of mass of the binary system of *Algol* in revolution around the centre of mass of itself and a suspected third body, the periods are all under one-half year; and two-thirds of them are under ten days. An inspection of column e shows that the eccentricities are, in general, small for the short periods and relatively large for the longer periods; that is, that the orbits are more nearly circular for the short-period binaries than for those whose periods are relatively long.

TABLE XXV

SPECTROSCOPIC BINARIES OF CLASSES G TO M

(Cepheid variables not included)

H. R.	Star	Class	Period	e	$a \sin i$ km.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	$m \sin^3 i$	$m_1 \sin^3 i$	$m_1 : m$
8961	λ *Androm.*	K	20ᵈ.54	0.11	1,900,000	0.0006☉			
1708	*Capella*	G	104 .02	0.02	36,847,900	0.184	1.19☉	0.94☉	0.79
1708_1	*Capella*	F+	104 .02	0.02	46,430,000	0.369			
429	γ *Phœnic.*	K5	190 .±						
6148	β *Herculis*	K	410 .58	0.55	60,280,000	0.052			
5235	η *Bootis*	G	489 .14	0.18	56,010,000	0.0293			
4375	ξ *Urs. Maj.*	G	1ʸ.8						
8650	η *Pegasi*	G	2ʸ.24	0.16	157,800,000	0.234			
7776	β *Capric.*	Gp	3ʸ.77	0.44	377,000,000	1.13			
6134	α *Scorpii*	Map	5ʸ.8	0.20	60,490,000	0.0020			
6212	ζ *Herculis*	G	34ʸ.	0.46					
5459	α_1 *Centauri*	G	81ʸ.18	0.53					
5460	α_2 *Centauri*	K5	81ʸ.18	0.53					
6752	70 *Ophiuchi*	K	87ʸ.86	0.50					
99	α *Phœnic.*	K	long*						
539	ζ *Persei*	K	long						
549	ξ *Piscium*	K	long						
854	τ *Persei*	Gp	long						
915	γ *Persei*	Gp	long						
1066	*f Tauri*	K	long						
2216	η *Gemin.*	Ma	long						
2296	δ *Colum.*	G5	long						
2553	τ *Puppis*	K	long						
2854	γ *Can. Min.*	K	long						
4301	α *Urs. Maj.*	K	long						
8079	ξ *Cygni*	K5	long						
8115	ζ *Cygni*	K	long						

* The periods marked "long" are unknown.

TABLE XXIII contains corresponding facts for the Class A stars; that is, those stars which are believed to be further along in their processes of development than the O and B types. The last star on the list, *Sirius*, is the well-known visual binary; and some of the elements of the visual orbit are included here, for the reason that the spectrograph has measured the velocity of the brighter component in the system and found it to vary in accordance with the requirements of the visual orbit. In this list, also, the eccentricity is seen to be, in general, a function of the period of revolution.

TABLE XXIV contains corresponding data for Class F stars, supposedly of greater effective age. Again the short periods have orbits of small eccentricity, and the long periods have orbits of great eccentricity. The Cepheid-Geminid variables, many of which have Class F spectra, are not included in this tabulation, for these variables appear to belong in a class by themselves.

TABLE XXV contains orbital data for stellar systems of Classes G, K and M, which are believed to represent advanced stellar age. The eccentricities are again a function, in general, of the length of period. This table also does not include the Cepheid-Geminid variables, of Classes G, K and M, for the reason stated in the preceding paragraph.

TABLE XXVI

SPECTROSCOPIC BINARIES.—GENERAL SUMMARY

Periods	"Short"	0^d-5^d	5^d-10^d	10^d+	Years	"Long"
Classes O and B	8	15	10	14	1	
Period	short	$2^d.4$	$6^d.9$	$73^d.2$	$1^y.9$	
Eccentricity		(10)0.04	(5)0.10	(11)0.34	(1)0.0:	
Class A	4	10	1	12	2	
Period	short	$2^d.65$	$9^d.2$	$42^d.2$	$26^y.45$	
Eccentricity		(5)0.04	(1)0.50	(8)0.55	(1)0.59	
Class F	0	6	2	4	3	1
Period		$3^d.1$	$5^d.6$	$145^d.1$	$11^y.1$	long
Eccentricity		(4)0.05	(1)0.01	(3)0.15	(3)0.44	
Classes G to M	0	0	0	3	9	13
Period				$104^d.8$	$24^y.3$	long
Eccentricity				(2)0.06	(8)0.38	
Total	12	31	13	33	15	14
Mean period	short	$2^d.59$	$6^d.90$	$73^d.5$	$20^y.5$	long
Mean eccentricity		(19)0.04	(7)0.14	(24)0.36	(13)0.38	

Reviewing the four tables, we note that two-thirds of the Classes O and B binaries have periods less than ten days; half the Class A and half the Class F have periods less than ten days; and it is a striking fact that there are no known binaries of the Classes G, K and M (excepting possibly H. R. 142, 13 *Ceti*) whose periods are less than twenty days.

The relations existing between the spectral classes, lengths of period and eccentricities are put in a more condensed form in TABLE XXVI. This table is double entry, with known lengths of period divided into four compartments, as described in the first line; and the corresponding spectral classes, average periods, and average eccentricities in the following twelve lines. We note that for the Classes O and B stars, for the Class A stars, and for the Classes G, K and M stars the eccentricity values increase with increasing periods in each division. That this cannot be said for the Class F stars is a fact of small weight, as the exceptional case refers to but a single star. A different division of periods would remove this apparent exception to the rule. Indeed, the remarkable fact is that there are not more apparent exceptions, considering the great number of divisions under which the few orbits are treated statistically.

The decrease in the relative number of short-period binary systems, with advancing stellar age, is graphically evident in the table, from the zero in the F division and the three zeros in the G-K-M division.

The relation between length of period and eccentricity is brought out even more forcibly in the last three lines of the table. The second line from the last, "Total," sums up the number of binaries, listed under the four spectral divisions, for which we have orbital data. The next to the last line contains the average periods of these binary systems. The last line contains the average eccentricity of all those binaries in each division for which the eccentricities have been determined; the number of eccentricity values in each case being indicated in the parentheses. The relationship existing between length of period and eccentricity is unmistakable.

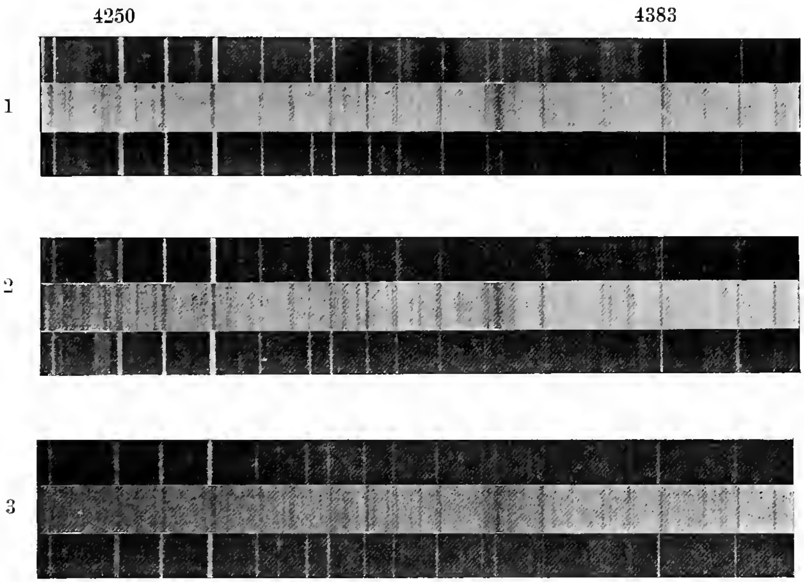
Let us refer again to the first four tables. The second column from the last quotes the mass of the brighter member of each system multiplied by the cube of the sine of the inclination of its orbit ($\sin^3 i$), in terms of the Sun's mass as unit, for those few systems in which the *two* members have been observed spectrographically. The next to the last column quotes similarly the mass of the fainter member of the system. The last column contains the ratio of the mass of the fainter member to that of the brighter member. It is characteristic of the orbits, as determined by means of the spectrograph, that the inclination of the orbit plane to the line of sight cannot be determined. Examining the tabular data for the masses of the two bodies—eighteen systems in all—we note that these masses, multiplied as they are by the sine-cube of the inclination, are on the average manyfold greater than the mass of the Sun. It can be shown for an indefinitely great number of binary systems whose orbital planes are distributed at random, that the average inclination would be 57°.3, in accordance with the formula

$$i_0 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} i \sin i \, di \, d\phi = 1.$$

The average value of $\sin^3 i$, however, would not be $\sin^3 57°.3$ (=.65), but approximately 0.59, in accordance with the formula

$$\sin^3 i_0 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin^4 i \, di \, d\phi = \frac{3}{16}\pi = 0.59.$$

It would not be permissible to use this as the average value of $\sin^3 i$ for the observed systems, as there is the practical consideration that binary systems whose orbital planes have large inclinations are more readily discoverable than those whose inclinations are small. For example, the maximum variation of radial velocity in an orbit whose inclination is 5° will be but one-twelfth the corresponding variation of velocity in the orbit. It is clear, therefore, that many systems whose orbital inclinations are less than 10°, and even less than 20°, will escape detection, in the present state of spectrographic observation. The number of inclinations less than 10° would, however, be relatively small, for the same reason that the area of the surface of a sphere lying within 10° of one of its poles is small in comparison with the area



Class F5. *ι Pegasi*

1.	1899,	October	3.	Measured	displacement,	−39 km.
2.	"	"	17.	"	"	+53 km.
3.	"	"	25.	"	"	+ 1 km.

of a hemisphere. Under ordinary circumstances, and when dealing with a considerable number of orbits, a compromise value of $\sin^3 i = 0.65$ might in fairness be applied. We should use a still larger value in dealing with the eighteen cases before us, for six of the stars are *Algol* or *β Lyræ* variables whose orbital inclinations are in every case probably between 60° and 90°. Assuming

$\sin^3 i = 0.75$ for the present list, we find the average mass of the eighteen principal components to be 5.6 times the Sun's mass, and of the eighteen secondary components 5.7 times the Sun's mass. Excluding *β Lyræ*, on account of very considerable uncertainty in the interpretation of its spectrum, these values reduce to 5.1 and 4.4. The Classes O and B systems appear to be the more massive, but the data are too meagre to venture this as a conclusion.

With only one exception the principal (brighter) component is more massive than its secondary. The exception relates to *β Lyræ*, in which system the fainter member is accredited with a mass 2.2 times that of the brighter member. The two members are believed to be nearly in contact, and Myers has suggested[40] that the greater brightness of the less massive companion may be due to the influence of a heavy absorbing atmosphere which completely encloses both members of the system; the more massive component drawing this atmosphere more densely around itself, leaving the atmosphere overlying the less massive component thinner and less absorptive, thereby permitting "the smaller (less massive) to appear brighter even though it might be intrinsically darker than the larger (more massive) body."

There arises the question whether the masses deduced for the seventeen systems involved are representative of the masses in spectroscopic binary stars in general, discovered and undiscovered. The data are too meagre to answer this question satisfactorily. The spectroscopic binary systems thus far detected and investigated are on the whole those most readily discoverable. They are of stars brighter than the average, and therefore of masses above the average. They are systems in which the range of radial velocity is large, and systems whose periods of revolution are short. Systems with small velocity amplitudes and systems with long periods remain undiscovered in proportions unduly large. It will be seen from equations (61), (62) and (59) that, in the systems to be discovered in the future, the smaller values of K will decrease the corresponding values of m, m_1 and $m+m_1$, and the larger values of P will increase the

[40] *Ap. J.*, 7, 21-22, 1898.

corresponding values of m, m_1 and $m + m_1$. The prevailing tendencies of the *K's* and *P's* in future systems will therefore counterbalance one another in these equations, to some extent, though attention should be called to the fact that the *K's* enter as cubes and the *P's* enter to the first power.

In the great majority of systems thus far investigated we have orbital elements for the brighter components only. There is a relationship expressed in equation (57) which we have computed and entered in the first four tables; namely, $m_1^3 \sin^3 i/(m + m_1)^2$. These quantities convey little definite information. They range from 9.7, in the case of *β Lyræ*, down to .00001 for *Polaris*. If we assume that the average value of $\sin^3 i$ which has entered into these quantities is 0.65, we shall see that the secondary members of the systems are in general of considerably smaller mass than their primaries; and this would appear to be a reliable conclusion, though there are a few probable exceptions. The larger values of this quantity, as may be seen from the tables, seem not to show preference for any spectral type or types.

Dr. Aitken has prepared a list of fifty doubles, arranged in the order of their periods, according to the knowledge of today as in TABLE XXVII. These periods vary from 5.7 years for *δ Equulei*, discovered by my colleague Hussey, up to 194 ± years for *γ Virginis*. Other columns of the table give the computed eccentricities of the orbits, the spectral classes, the mean angular distances of the components, and the visual magnitudes of the components. The number of the pair in Burnham's *General Catalogue of Double Stars* is given in the first column.

Up to date (1910), orbits have been published for 102 binary systems with periods ranging up to 1578 years. At least half of these are too uncertain to be of value, and in only about forty cases have we a fairly exact knowledge of the true orbits. Several of the assigned periods and eccentricities in the tabular list of fifty will undoubtedly be changed as a result of future observations, but it is not expected that a radical change will be required in the elements assigned to the first forty stars on the list.

TABLE XXVII

VISUAL DOUBLE STARS*

Burnham's No.	Star	Period	e	Spectrum	a	Vis. Mag's.
10829	δ *Equulei*	$5^{y}.70$	0.36	F5	0″.25	4.5- 5.0
314	13 *Ceti*	7 .42	0.74	F	0 .21	5.5- 6.2
11222	κ *Pegasi*	11 .37	0.40	F5	0 .29	4.5- 5.3
4771	ϵ *Hydræ*	15 .70	0.68	F8	0 .24	4.0- 6.0
2381	β 883	16 .35	0.48	?	0 .24	7.0- 7.0
8965	ζ *Sagittarii*	21 .17	0.18	A2	0 .56	3.4- 3.6
6578	β 612	22 .8	0.48	A	0 .24	6.4- 6 5
4310	9 *Argus*	23 .3	0.68	F8	0 .61	5.7- 6.3
335	β 395	24 .00	0.15	K	0 .66	6.3- 6.4
6406	42 *Comæ*	25 .56	0.46	F5	0 .64	6.0- 6.0
12701	85 *Pegasi*	25 .70	0.43	G	0 .78	5.8-11.0
10363	β *Delphini*	27 .66	0.36	F5	0 .54	4.6- 5.0
5005	Σ 3121	34 .00	0.33	F?	0 .67	7.2- 7.5
7717	ζ *Herculis*	34 .53	0.46	G	1 .36	3.1- 6.5
1471	20 *Persei*	36	0.75	F	0 .16	5 6- 6 4
4414	β 581	41 .2	0.53	?	0 .61	8.0- 8 0
7251	η *Cor. Bor.*	41 .51	0.28	G	0 .89	5.5- 6 0
7487	ξ *Scorpii*	44 .5	0.77	F8	0 .70	5.0- 5.2
8162	μ_1 *Herculis*	45 .39	0.21	?	1 .37	10.0-10.1
7929	β 416	45 .90	0.62	K5	1 .93	6.0- 8.0
8038	Σ 2173	46 .0	0.20	G	1 .14	6.0- 6.4
3596	*Sirius*	49 .4	0.50	A†	7 .59	−1.4-10.0
7332	OΣ 298	52 .0	0.58	?	0 .80	7.0- 7.3
1036	β 513	53 .0	0.35	A2	0 .61	5.0- 7.5
1070	γ *Andr. BC*	55 .0	0.82	K‡	0 .35	5.0- 6 2

* Note added May 1, 1912.

Additional observations secured since this table was prepared have effected improvements in the orbits of some of the stars listed.

Thus, for β 581 we now have, $P = 46\frac{1}{2}$ years, $e = 0.40$, $a = 0''.53$;
and for OΣ 235, $P = 71.9$ years, $e = 0.40$, $a = 0''.78$.

Good orbits have been computed for two other pairs with periods under one hundred years, viz.:

Secchi 2 (Σ 2481 BC, Burnham No. 9114), $P = 58$ years, $e = 0.50$, $a = 0''.40$, Mags. 8.0 — 8.0;
and

OΣ 79 (Burnham No. 2134), $P = 88.9$ years, $e = 0.62$, $a = 0''.57$, Mags. 7.0 — 8.8.

Minor improvements have been effected in a number of other orbits.

† Star A = Class A.
Star B = yellow?

‡ Star A = 2.28, Class K.
Star BC = blue.

TABLE XXVII (*Continued*)

Burnham's No.	Star	Period	e	Spectrum	a	Vis. Mag's.
10846	τ *Cygni*	53 .±	0.31±	F	1 .16	3.9-10.0
4477	ζ *Cancri*	59 .1	0.38	F	0 .86	5.5- 6.2
5734	ξ *Urs. Maj.*	59 .8	0.41	G	2 .5	4.0- 4.9
8372	99 *Herculis*	64 .5	0.81	F8	1 .28	6.0-11.7
5811	OΣ 235	66 .0	0.50	F	0 .83	6.0- 7 3
5235	8 *Sextantis*	68 .8	0.60	A2	0 .35	5.8- 6.1
7368	γ *Cor. Bor.*	73 .0	0.48	A	0 .74	4.2- 7.0
5805	OΣ 234	77 .0	0.30	?	0 .35	7.0- 7.4
9979	OΣ 400	81 .0	0.46	?	0 .47	7.2- 8.2
....	α *Centauri*	81 .2	0.53	G, K5	17 .71	0.1- 1.9
....	γ *Centauri*	88 .0	0.80	A	1 .02	3.2- 3.2
8340	70 *Ophiuchi*	88 .4	0.50	K	4 .55	4.1- 6.1
9650	OΣ 387	90 .0	0.60	?	0 .66	7.2- 8.2
7001	OΣ 285	97 .9	0.60	?	0 .34	7.1- 7.6
5223	ϕ *Urs. Maj.*	99 .7	0.44	A2	0 .32	5.0- 5.6
12755	Σ 3062	104 .6	0.45	F	1 37	6.9- 8.0
5103	ω *Leonis*	116 .2	0.54	G	0 .88	6.2- 7.0
1144	Σ 228	123 .1	0.31	?	0 .90	6.7- 7.6
7034	ξ *Bootis*	148 .5	0.54	K5p	4 .99	4.7- 6.6
....	γ *Cor. Aus.*	152 .7	0.42	F8	2 .45	5.1- 5.1
21	Σ 2	166 .2	0.40	A	0 .55	6.3- 6.6
2109	o_2 *Eridani BC*	180 .0	0.13	G5§	4 .79	9.2-10.9
6566	25 *Can. Ven.*	184 .0	0.75	F	1 .13	5.0- 8.5
7783	Σ 2107	186 .2	0.39	N?	1 .0	6.5- 8.0
6243	γ *Virginis*	194 .0	0.90	F	3 .99	3.6- 3.6

SUMMARIES FOR VISUAL BINARIES

Spectra	No. of Stars
O-B	0
A	9
F	18
G-K	14
M-N	0

No. of Stars	P_0	e_0
25	$32^y.8$	0.48
25	108 .1	0.51

§ Star A = 4.5, Class G5.
Star BC = blue.

The spectral designations are from the *Revised Harvard Photometry,* excepting that of the forty-ninth star on the list, which is from the *Draper Catalogue.* The class is unknown for ten of the fainter systems. Summarizing the data by spectral class, two facts are striking:

First, there are no visual binaries of known periods, either short or long, belonging to the Classes O and B. This is in strong contrast with the very large proportion of spectroscopic binaries of Classes O and B. It is apparent that the component stars in O or B binary systems are in general too close together to be separated by direct telescopic observation.

Second, there are no visual binary systems of *determined* periods belonging to the Classes M and N, but the apparent explanation is a very different one. The periods in such systems are so long, in comparison with the interval covering accurate micrometer studies of double stars, that we have no reliable information concerning the period of revolution for even one system.

Dr. Aitken has tabulated the spectral classes of 164 double stars on his observing list of the more rapidly moving systems. All assignments of class but one, that of γ *Lupi*, were taken from the *Draper Catalogue.* The results are as follows:

O	0 stars
B	4 stars
A-F	131 stars
G-K	28 stars
M-N	1 star?
Total	164 stars

Visual double stars clearly abhor the Classes O and B, and visual double stars of relatively short periods clearly abhor Classes M and N.

In Table XXVIII let us bring together data for the spectroscopic and visual binaries. As in Table XXVI, the parentheses contain the numbers of individual eccentricities which enter into the mean values of the eccentricities; and the unbracketed numbers denote the numbers of periods represented in the average periods.

TABLE XXVIII

SPECTROSCOPIC AND VISUAL BINARIES

	No. of Stars	P_0	e_0
Spectroscopic binaries	31(19)	2.59 days	0.04
Spectroscopic binaries	13(7)	6.90 "	0.14
Spectroscopic binaries	33(24)	73.5 "	0.36
Spectroscopic binaries	15(13)	20.5 years	0.38
Visual binaries	25(25)	32.8 "	0.48
Visual binaries	25(25)	108.1 "	0.51

The period of revolution in a binary system is, in general, a function of the spectral class; and the eccentricity is, in general, a function of the period. What is the significance of these facts? Darwin and Poincaré have studied the origin of binary stars from theoretical considerations. Not to review their work in technical detail, nor to define the underlying assumptions, they came to the conclusion that a condensing nebulous mass, rotating about an axis, constantly faster and faster, to keep pace with loss of heat through radiation, should eventually separate into two nebulous masses revolving around their mutual centre of mass. These two masses would in the beginning be revolving in contact, in orbits essentially circular. With advancing time, tidal disturbances within the more or less viscous bodies would cause them to draw apart, rapidly at first and less rapidly later. Jeans[41] and others have called attention to certain limitations in these investigations, which their authors recognize. Darwin has, in fact, stated[42] that the assumed conditions in the parent mass are necessarily not in strict accord with probable distribution of density and other circumstances. However, confidence prevails that the deductions are substantially correct.

See's valuable study of visual double stars, *Die Entwickelung der Doppelstern Systeme,* 1892, taking note of the high eccentricities of their orbits, established that their periods and their eccentricities should increase as a gravitational effect resulting from tidal friction in the viscous components of the system.

[41] *Phil. Trans.,* **199** A, 1, 1902.

[42] *Darwin and Modern Science,* pp. 548-549, Cambridge, 1909.

In the spectroscopic and visual binary systems here described in the present paper we have a tolerably complete sequence of systems illustrative and confirmative of the Darwin-Poincaré-See hypotheses. Short-period orbits should be circular or nearly so, and they should appertain preferentially to stars of early spectral classes; long-period orbits should, in general, attach to the more eccentric orbits and to the older spectral classes. These are the facts unquestionably established by spectrographic and micrometer observations of actual binary systems. There are many lines of supporting evidence which we shall consider.

The Harvard College Observatory has announced the discovery of stars with composite spectra, as in TABLE XXIX. (The Harvard list has been revised by Miss Cannon to date, 1911.) The slit spectrograph has confirmed the expectation that the stars with composite spectra would eventually prove to have variable radial velocities, in nearly all cases.

Of the thirty-four stars in this list (*Annals H. C. O.*, **28**, 93 and 229; **50**, 200-25; and **56**, 113):

Six are moderately close visual double stars whose component spectra overlap on the original photographs, and the resulting spectra may appear to be composite for this reason.

Two are fainter than visual magnitude 5.0, and seem not to have been placed on existing programs for radial velocity measurement.

One has been observed only once for radial velocity.

Of the twenty-five remaining, twenty have been found to be spectroscopic binaries, and the remaining five, observed several times each, have not certainly shown variable radial velocity, but this may be due to unfavorable distribution of observation times, or more probably to long periods of revolution. It is probable that the radial velocities of these five stars will be observed to vary in the future.

Omitting the visual doubles in the list, all the repeatedly observed stars of Classes B or A have been shown to be spectroscopic binaries. Those whose velocities appear to be constant, or to have varied but slightly, are of F5 or more advanced spectral classes.

TABLE XXIX

Stars with Composite Spectra (Objective Prism)

H. R.	Star	R. A.	Br. Sp.	Ftr. Sp.	Vis. Mag.	Remarks
595-6	α *Piscium*	1^h 56^m.9	A2p	I type	5.23-4.33	Visual double, s = 3″.6, p = 336°§
603-4	γ *Androm.*	1 57 .8	K	I type	2.28-4.96	Visual triple, BC-A, s = 10″, p = 62° Perhaps small variation in V
854	τ *Persei*	2 47 .2	Gp	I-II type	4.06	Spectroscopic binary
915	γ *Persei*	2 57 .6	Gp	I type	3.08	Spectroscopic binary
1129	— *Camelop.*	3 37 .3	G	A	4.96	Velocity apparently constant§
1211-2	*w Eridani*	3 49 .2	G5	B?	6.33-4.95	Visual double, s = 6″.7, p = 347° Br. component perhaps variable V
1230	— *Cephei*	3 53 .3	F8p		5.25	Observed V approx. constant
1252	36 *Tauri*	3 58 .4	A	G	5.67*	No observations
1306	*f Persei*	4 8 .1	K	A	4.89	Velocity apparently constant
1612	ζ *Aurigæ*	4 55 .5	Kp	Orion	3.94	Spectroscopic binary
3307	ε *Carinæ*	8 20 .5	Kp	Orion	1.74	Perhaps small variation in V
3619	*f Urs. Maj.*	9 1 .9	A3**	F5	4.54	No observations ‖
3624	τ *Urs. Maj.*	9 2 .7	F8p	I-II type	4.74	Spectroscopic binary
3852	ο *Leonis*	9 35 .8	F5p	I type	3.76	Spectroscopic binary
4707	12 *Com. Ber.*	12 17 .5	G	A	4.78	Spectroscopic binary†
5440	η *Centauri*	14 29 .2	B3p	A2	2.65	Spectroscopic binary
5505-6	ε *Bootis*	14 40 .6	K	A	5.12-2.70	Visual double, s = 2″.6, p = 321° Br. component perhaps variable V
5704	γ *Circini*	15 15 .4	B5p	F8	4.54	1 observation only Approx. equal photo. magnitudes
6134	α *Scorpii*	16 23 .3	Map	A3	1.22-7	Spectroscopic binary Visual double, s = 2″.6, p = 273°
6497	— *Ophiuchi*	17 21 .5	F5p		5.98	No observations
6729-0	95 *Herculis*	17 57 .2	A3	G5	5.21-5.13	Visual double, s = 6″.1, p = 262°
6918	*d Serpent.*	18 22 .1	A	G	5.33	Spectroscopic binary
7342	υ *Sagittar.*	19 16 .0	B8p	B ±	4.58	Spectroscopic binary
7417	β *Cygni*	19 26 .7	Kp	I-II type	3.24	Br. component of visual double Velocity apparently constant
7536	δ *Sagittæ*	19 42 .9	Map	I type	3.78	Spectroscopic binary
7735	31 *Cygni*	20 10 .5	Kp		3.95	Spectroscopic binary
7751	o_2 *Cygni*	20 12 .3	G5	I type	4.16	Spectroscopic binary
7776	β *Capricor.*	20 15 .4	Gp	I type	3.25	Spectroscopic binary
7866	47 *Cygni*	20 30 .0	K5	B?	4.85	Spectroscopic binary†
8131	α *Equulei*	21 10 .8	A8p	I type	4.14	Spectroscopic binary
8278	γ *Capricor.*	21 34 .6	Fp	F ±	3.80	Perhaps small variation in V§
8417	ξ *Cephei*	22 0 .9	A3‡	G	4.57-6.47	Visual double, s = 5″.6, p = 289° Perhaps small variation in V
8762	ο *Androm.*	22 57 .3	B3	Orion	3.63	Spectroscopic binary
8817	c_3 *Aquarii*	23 4 .5	Gp	A2	4.94	Observed V approx. constant

* Visual companion, 12½ mag., too faint to affect spectrograms.

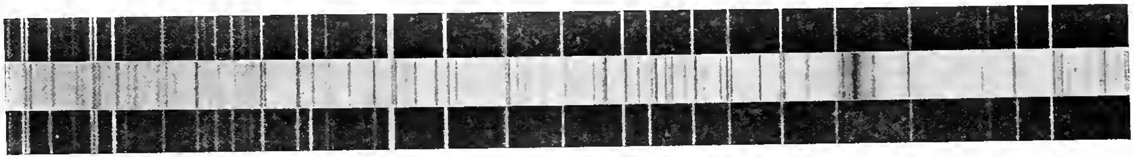
** "Spectrum is peculiar and appears to be composite." *Ann. H. C. O.*, 56 (IV), 106, 1908.

† Variable velocity found at Lick Observatory after January 1, 1910.

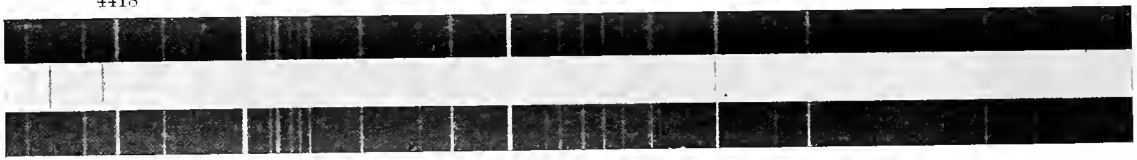
‡ Burnham, No. 11483, assigns colors yellow and blue to br. and ftr. components.

§ In 1910 found to have var. radial velocity.

‖ Four observations in 1910-1911 vary only 1.8 km.



Class F. α *Carinæ*, 1909, January 22. Measured displacement, +27 km.



Class F5. α *Canis Minoris*, 1910, October 22. Measured displacement, −31 km.

Class F8. α *Ursæ Minoris* (*Polaris*), 1911, June 25. Measured displacement, −16 km.

For sixty-two spectroscopic binary systems catalogued in the tables, the spectra of both components have been either observed on the spectrograms, or strongly suspected. These systems are distributed among the spectral classes as follows:

Classes O and B	30 systems
Class A	23 systems
Class F	5 systems
Class G	3 systems
Class K	1 system
Classes M and N	0 system
Total	62 systems

From the published descriptions of the double spectra, it is fairly well established that when the two spectra are substantially equal in brightness they are identical in class; and when one spectrum is considerably fainter than the other, the spectrum of the secondary is apparently of a slightly earlier class than the spectrum of the primary. There appear to be no exceptions to this rule, though the difficulty in the way of giving accurate descriptions of the fainter spectrum must be recognized. Another fact, seemingly of great significance, holds for every one of these systems, so far as they have been investigated, excepting possibly in the very uncertain case of *β Lyræ*: the less massive member of the system is the fainter member, and has the earlier class of spectrum. The wide visual and spectroscopic binary, *α Centauri*, appears to controvert this rule, slightly, in that the fainter component has the older class of spectrum; but according to the data in TABLE XXX, a doubt exists as to which of the components is the more massive.

Attempts have been made to determine the relative masses of the two components in several well-known visual double stars. The most of these investigations are based upon the apparent orbit of the primary, as determined from meridian observations, and upon micrometer observations of the secondary with reference to the primary. Lewis's *Struve Double Stars*, xxi, 1906, contains results for nineteen systems. These were obtained for the most part by himself and other Greenwich investigators, but for very little of this Greenwich work have I been able to find published details. Estimates of the reliability of the results are therefore difficult. In general, they attribute much greater mass to the secondary members than to the primaries, in the case of α *Geminorum* twenty times; but to this I think we need not attach any weight, for would not another value of the assigned proper motion in declination change the resulting value of m_1/m? In fact, was it not necessary to assume a position for the centre of mass of the system and therefore the ratio of the masses before a value of the proper motion could be deduced? Neither does it appear that sufficient attention has been paid to the elimination of the systematic errors of observation, which we should expect to be large and variable for the same observer, as well as for different observers. Doubts[43] as to the reliability of many of the results quoted in Lewis's table arise on comparing them with those secured by other investigators. For example, in the system of ε *Hydræ*, Lewis assigns to m_1/m the value 6; Seeliger's value is 0.9. In the system of σ *Coronæ Borealis*, Lewis's value of m_1/m is 4; Hadley's is 1.1. Prey's value of m_1/m for 70 *Ophiuchi* is 4; whereas, from substantially the same data, Comstock obtains 1.0, and Lau 0.5. In the system of 85 *Pegasi*, three Greenwich observers, using two methods, assign to m_1/m the value 3½, whereas Comstock obtains 1.6. Schorr's value for the system ξ *Scorpii*, 1.36, appears to be uncertain, for it is based upon $P = 105.2$ years, $e = 0.12$; and we now know the correct values to be approximately $P = 44.5$ years, $e = 0.77$. To the system of λ *Ophiuchi* Lewis assigns $m_1/m = 4.3$, but Burnham's comment is, "Nothing whatever is known concerning the orbit" (*General Catalogue*, II, 717, 1906).

On Lewis's list of nineteen systems there are assigned $m_1 > m$ in thirteen systems, $m_1 = m$ in three systems, $m_1 < m$ in three systems. There are eleven in which the fainter companion is said to be the bluer; seven in which the colors are estimated to be equal; and one (*Sirius*) in which the fainter companion is believed to be the redder.

43 See TABLE XXX.

References and Remarks on Table XXX

η Cassiopeiæ	See's *Evolution Stellar Systems*, 76
	Lewis's Σ *Double Stars*, xxi and 18; $m = \frac{2}{3}\odot$, $m_1 = \frac{1}{3}\odot$
	Boss's *Prelim. Gen. Cat.*, xxiii and 264; in R. A. 163 (wt. 60), in Dec. 0.41) wt. 85), adopted 0.76
o_2 *Eridani*, B & C	Lewis's Σ *Double Stars*, xxi and 113-4. Visual triple system
α Canis Maj.	*A. N.*, **129**, 232, 1892; $m = 220\odot$, $m_1 = 1.04\odot$
	Lewis's Σ *Double Stars*, xxi
	Boss's *P. G. Cat.*, xxiii and 265; in R. A. 0.387, in Dec. 0.393, mean 0.39
α Geminorum	Lewis's Σ *Double Stars*, xxi and 215
α Canis Min.	*Astron. Jour.*, **19**, 60, 1898; $m = 4.95\odot$, $m_1 = 0.99\odot$
	Boss's *P. G. Cat.*, xxiii and 267
ζ Cancri, A & B	*Denksch. kk. Akad. Wien*, **44**, 62, 1881. Visual triple system
ε Hydræ, A & B	*A. N.*, **173**, 325, 1906. Visual quadruple system. Max. separation A & B is 0″.3
	Lewis's Σ *Doub. Stars*, xxi and 255
ξ Ursæ Maj.	Lewis's Σ *Doub. Stars*, xxi and 306
	Boss's *P. G. Cat.*, xxiii and 269; in R. A. 1.43, in Dec. 0.75, mean 1.09, adopted 1.0
γ Virginis	Lewis's Σ *Doub. Stars*, xxi and 341
	Boss's *P. G. Cat.*, xxiii and 270-1; in R. A. 0.65 (wt. 1), in Dec. 1.53 (wt. 2), mean 1.1, adopted 1.0
25 *Canum Ven.*	Lewis's Σ *Double Stars*, xxi and 364
α Centauri	*Über die Parallaxe von α Centauri*, 1880, see *Mem. R. A. S.*, **48**, 14, 1884
	Lewis's Σ *Doub. Stars*, xxi; original reference not known, but mass ratio presumably deduced from data in *Mem. R. A. S.*, **48**, 13-83, 1884
	A. N., **139**, 10, 1895; $m = 1.02\odot$, $m_1 = 0.98\odot$
	Boss's *P. G. Cat.*, xxiii and 271
ξ Bootis	Lewis's Σ *Doub. Stars*, xxi
	Boss's *P. G. Cat.*, xxiii and 272; in R. A. 1.5 (wt. 2), in Dec. 0.69 (wt. 5), mean 0.87
ξ Scorpii, A & B	*Untersuch. über die Bewegungsverhältnisse im dreif. Sternsystems ξ Scorpii*, München, 1889. Visual triple system
σ Coronæ Bor.	*Pop. Astr.*, **13**, 264, 1905; in R. A. 1.19, in Dec. 1.01, mean 1.1
	Lewis's Σ *Doub. Stars*, xxi and 446-7
	Boss's *P. G. Cat.*, xxiii and 273; in R. A. 0.85, in Dec. 0.32, adopted 0.47, small wt.
λ Ophiuchi	Lewis's Σ *Doub. Stars*, xxi and 456. Orbit very uncertain. small wt.

ζ Herculis	*Mon. Not.*, **61**, 87, 1900
	Boss's *P. G. Cat.*, xxiii and 274; in R. A. 0.39, in Dec. 0.50, adopted 0.43
70 *Ophiuchi*	*A. N.*, **165**, 158, 1904
	A. N., **178**, 18, 1908; in R. A. 0.84, in Dec. 1.03, mean by wts. 0.91, adopted 1.0
	Bul. Astron., **25**, 141, 1908
	Boss's *P. G. Cat.*, xxiii and 275; in R. A. 0.92, in Dec. 0.72, mean 0.82
85 *Pegasi*	*Ap. J.*, **17**, 220, 1903; in R. A. 1.52, in Dec. 1.71, mean 1.6
	Lewis's Σ *Doub. Stars*, xxi
	Lewis's Σ *Doub. Stars*, xxi
	Mon. Not., **66**, 425, 1906; in R. A. 5.75, in Dec. 3.2, mean 4.5
	Mon. Not., **66**, 429, 1906; in R. A. 2.9, in Dec. 2.5, mean 2.7
	Boss's *P. G. Cat.*, xxiii and 278; in R. A. 1.0, in Dec. 5.2, mean by wts. 1.8, adopted 1.0

[Note added April 1, 1910.—My belief that the data referred to are not of great weight was strengthened by learning of the results secured by Professor Boss for many of the same systems, early in February of the present year. His conclusions, already in type, but communicated to me orally, have reached me before April 1, by virtue of a personal copy of his *Preliminary General Catalogue of Stars for 1910,* in time to be inserted in TABLE XXX. Boss's results are based upon meridian observations from which systematic errors have been eliminated with the utmost thoroughness; and they would seem to form the basis, almost exclusively, upon which we should judge as to the mass ratios in the double star systems concerned.

Boss's general conclusion is, "there appears to be no case in which the fainter companion can be asserted with high probability to have a larger mass than that of the brighter component."—*Prelim. Gen. Cat.*, xxiii, 1910.

Boss's mean for the eleven computed values of m_1/m is 0.81, and the mean of the eleven adopted values is 0.72. The great weight of these individual results, assuming that the eleven systems are representative of double star systems in general, must give them an important place in theories of the development of binary systems.

Boss's investigations of the mass relations in eleven double star systems, while reversing views previously held, that the bluer and whiter secondaries are more massive than their primaries, do not reverse the conclusion generally accepted that the yellower primaries are many-fold more effective as radiators of light than the secondaries are. This may be regarded as an established fact, for it can scarcely be doubted that the secondaries, being in general less massive than the primaries and at the same time bluer and probably of lower average density, have diameters at least as great as those of the primaries.]

Attention should be called to certain radial velocity results obtained from the sharp and narrow H and K calcium absorption lines in several of the hydrogen and helium stars whose other lines are usually broad and hazy. Frost has stated that a very large proportion of the stars of these classes which contain sharp H and K lines are spectroscopic binaries and that this characteristic may almost serve as a criterion of variable radial velocity;* but the remarkable condition appears in several instances that the range of velocity yielded by one or both of the two calcium lines is different from and smaller than velocities obtained from other lines in the same spectra. The first observation bearing upon this subject was made by Hartmann, who noted† for the spectroscopic binary system δ *Orionis* that the calcium K line, faint but sharp, did not share in the periodic displacements of the other lines caused by orbital motion of the star. He attributed this result "with a pretty fair degree of certainty to the presence of an absorbing layer of gas not in immediate connection with the star." Frost and Adams,‡ on the contrary, feel certain that the velocity afforded by the K line in the spectroscopic binary system 9 *Camelopardalis* is variable; and other Yerkes plates show a small range of calcium variation for other stars, but with different average values from those obtained from the broad lines. Frost has suggested that in such stars we may have quaternary systems.

[Added April 1, 1910.—Slipher reports§ similar observations of sharp but faint K lines in the spectra of a number of stars of the helium type, whose other lines are wide and indefinite. Among these are several spectroscopic binaries, but others are not now known to be binary. The radial velocities from the K lines seem to remain stationary within the errors of observation, and when the observed velocities are corrected for the motions of the Earth and Sun the results approach zero within the errors of observation. He believes that the ordinary stellar origin of the calcium line is insufficient to explain the observations.]

With so large a proportion of these stars known spectro-

* *Ap. J.*, **29**, 234-236, 1909.

† *Ap. J.*, **19**, 273, 1904.

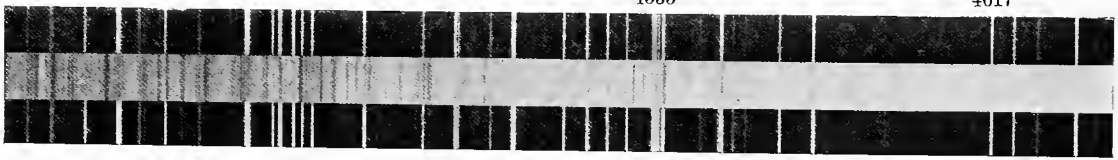
‡ *Ap. J.*, **19**, 350, 1904.

§ Unpublished letter, March 15, 1910.

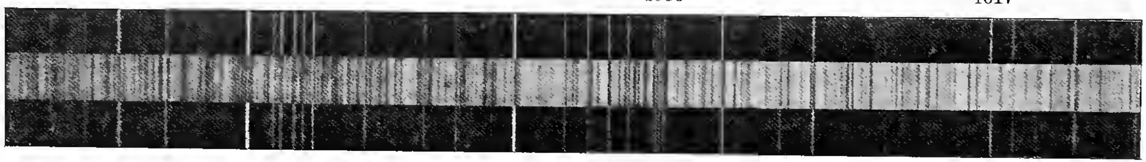
scopic binaries, and with the possibility that all will later prove to be binary, we are hardly justified in attributing these anomalies to causes not associated with binary systems. The idea of a calcium envelope completely surrounding a close binary system, such as Myers suggested in the case of *β Lyræ*, has something to commend itself, but observational data at present available do not justify me in amplifying the suggestion.

The companions of binaries discovered by means of the spectrograph have not *in any case* been observed in our powerful telescopes, although they have been carefully searched for. They are probably so close to the principal star, that, viewed from our great distance, the two images cannot be resolved. The separation of the two components is probably less than 1/100 of a second of arc, for the great majority of the binaries thus far announced. It is doubtful if the separation in any one of the binaries amounts to a twentieth of a second of arc. When the spectrum of the second component is not recorded, which is the case for the great majority of these systems, it cannot be certain that the component is a dark body, but only that it is at least one or two photographic magnitudes fainter than the primary. The fourth-magnitude companion of a second-magnitude star of the same spectral type would scarcely be able to impress itself upon the primary's spectrum. The invisible components in any, and perhaps all, spectroscopic binaries might be conspicuous stars if they stood alone.

In the earlier years of my radial velocity determinations the observing program was composed almost wholly of stars of the later spectral classes; that is, those whose spectra contain accurately measurable lines. Up to the year 1900 the radial velocities of slightly more than 300 stars had been measured. Amongst these were about forty whose radial velocities were observed to be variable: a proportion of one spectroscopic binary system in every seven stars observed. About 1901 Frost and Adams, of the Yerkes Observatory, entered upon extensive determinations of stellar radial velocities, their program consisting of stars in Classes B and A. They found that amongst the Class B stars observed, one out of every two and a half stars, on the average,



Class G. *Venus*, 1907, January 21. Measured displacement, +13 km.



Class K. *a Bootis*, 1907, July 10. Measured displacement, +21 km.

is a spectroscopic binary. With the progress of the Mills spectrographic observations, the proportion of observed spectroscopic binaries has continually increased until at the present time we are able to say that, on the average, one star out of every five or six stars in Classes F, G, K and M is a spectroscopic binary. The reason for the greater proportion of observed binaries amongst the so-called younger stars is apparently this: We have shown that binaries amongst the younger stars are characterized by short periods of revolution. We know that this condition is accompanied by high orbital velocities which vary rapidly through a wide range. Such systems are on this account readily and promptly discoverable. We have also learned the related fact that the orbits of binaries amongst the older stars are characterized by long periods. This condition assures that the orbital velocities are relatively low, and that these velocities vary slowly between narrow limits. Lapse of time and a greater number of observations are therefore necessary to the discovery of a large proportion of such systems. There is considerable probability that we shall eventually be able to show that the proportion of spectroscopic binaries amongst the older stars does not differ appreciably from the proportion existing amongst the younger stars.

It is evident that future catalogues of spectroscopic binaries will contain thousands of entries. We can, by present means, measure the radial velocities down to the eighth photographic magnitude. The increasing size of reflecting telescopes, and improvements in methods, will no doubt extend the limit to fainter stars before all those brighter than the eighth magnitude shall have been observed. Some of us may hope to see the recorded spectroscopic binaries numbered in thousands. We may say that only those systems amongst even the bright stars have thus far been detected whose periods are relatively short and whose variations of radial speed are considerable. All the stars in the sky could have velocities variable up to 1 km. per second, most of the stars could have velocities variable up to 2 km. per second, and a majority of the naked-eye stars could have velocities variable up to 3 km. per second, and still have escaped our notice. The smallest variation definitely established as existent is that of δ *Canis Majoris*, whose velocity passes back and forth over a total range of 3 km. in a period of some five months. Smaller variations than this can be detected by present instruments and methods; but as the range of accidental and unavoidable errors for any type of spectra occasionally exceeds this limit, a large number of observations are required to distinguish between actual variations and error. It is possible, and it seems even probable, that there are more systems with variations of speed under 3 km. per second than there are with larger ones; and all such are awaiting discovery. The velocity of our Sun through space varies slightly, because it is attended by companions—very minute ones compared with the bodies discovered in spectroscopic binary systems. It is revolving, at any instant, around the centre of mass of itself, its planets, and their moons. Its orbit around this centre is small, and variable, and the orbital speed very slight. The total possible range of speed is but .03 km. per second. An observer, favorably situated in another system, provided with instruments enabling him to measure speeds with absolute accuracy, could detect this variation, and in time say that our star is attended by planets. Terrestrial observers have not the present power to measure such minute variations. As the accuracy attainable improves with experience, the proportional number of spectroscopic binaries discovered will undoubtedly be enormously increased. There is a possibility that the stars attended by massive companions, rather than by small planets only, are in a decided majority; suggesting, at least, that our solar system may prove to be an extreme type of system rather than of the prevailing or average type. This is not a casual and passing comment. We do not possess a shred of positive evidence that any other star than our own is attended by small planets: we seem powerless at present to obtain any evidence in favor of or against their existence; and the prevailing belief that planets exist in other systems rests upon analogy to the solar system. We *have* the evidence of visual and spectroscopic binary stars that other systems with two or more massive central bodies are extremely common.

CHAPTER VIII

VARIABLE STARS; SOLAR PARALLAX

In August, 1596, David Fabricius of The Netherlands observed a third-magnitude star, in the constellation *Cetus,* which gradually grew fainter and, in October of the same year, disappeared. Systematic observations of the star were made in 1638-1639 by Holwarda, who discovered that the brightness increases and decreases periodically. Observations extending almost continuously to the present time have determined that the average length of its cycle of change, from maximum brightness down to minimum and up again to maximum, is 332 days, though the intervals are sometimes a fortnight shorter or longer than this. The star usually varies from the third to the ninth magnitude and back to the third in the eleven months; that is, its brightness in this interval first diminishes and then increases 250-fold. There have been times, however, when it has shone four thousand times as brightly as at others. These remarkable variations have been observed for nearly three hundred years, with no indications of abatement; and it is probable that they have been recurring in a similar manner for hundreds of centuries. This was the first "variable star" discovered. Its catalogue name is *o Ceti*; but so unexpected and amazing was its behavior that Hevelius, about the year 1650, called it Mira (The Wonderful), and to this day it is known by that name.

Up to the year 1800 only twelve variable stars were known. Schœnfeld's Catalogue of 113 Variable Stars is dated 1865, and Chandler's catalogue, dated 1888, contains 225 entries. The remarkable progress made by astronomical science in the past twenty-two years, owing principally to the powerful aid afforded by photography, is fairly indicated by the fact that in this interval the number of known variable stars has increased from

225 to more than 3000. To Harvard College Observatory belongs the great credit of discovering fully three-fourths of these objects.

In many respects variable stars constitute the most interesting class of objects in the heavens. The tens of millions of ordinary stars are growing older; and the hundreds of thousands of nebulæ, from which millions of stars will eventually be formed by processes of condensation, are undergoing transformation; but appreciable changes in the ordinary stars and in the nebulæ proceed with extreme deliberation, and no certain changes have yet been noted. Variable stars, on the contrary, are changing before our eyes; and they repeat their fluctuations continually. They present opportunities for discoveries of great interest in themselves, and give promise of remarkable utility in solving the problems concerning the origin of visual and spectroscopic binary stars.

A study of the periods of variable stars brings out most curious relations.[1] The periods clearly have preferences for certain lengths. There are a large number whose variations from maximum to minimum and back to maximum are completed in approximately one day, and many whose periods are half a day or less. As the period lengthens from one day, the number of variables decreases rapidly until we reach the few of eleven-day period. There are relatively very few variables with periods between 11 days and 110 days. Variable-star nature seems to abhor this interval. Beginning with 110 days, the number of variables increases rapidly, with increase of period, up to a maximum at 345 days; and the number of variables then decreases rapidly until we reach periods in length approximately 450 days. There are few with periods longer than 450 days, and our information concerning them or their periods is extremely meagre. It is of interest to note that *o Ceti*, with average period 331 days, is but one of a great number which make up the maximum near 345 days.

Long-period variable stars differ from short-period variables in important particulars. The former vary in brightness

[1] Clerke's *The System of the Stars*, 1905, p. 98.

through a wide range, usually from three to eight visual magnitudes. Short periods, on the contrary, have small ranges of brightness, varying through 0.2 of a magnitude or less up to a maximum of 1½ magnitudes, with few exceptions. The long-period variables are all reddish in color, apparently indicating that the atmospheres of these stars are dense, absorbing the violet rays and transmitting the waves of longer length, or that we are dealing with low-temperature radiations. Short-period variables, on the contrary, are all yellow or white in color. Chandler has found that there exists a relation between the length of period and redness. To quote him: "The redness of variable stars is, in general, a function of the lengths of their periods of light variation. The redder the tint, the longer the period."[2] Whether the conditions which produce redness are also the cause of long periods is an unsettled question. These long-period red variables do not conform to definite time schedules. Their maxima may precede or may follow prediction by a fortnight or a full month, but the average length of twenty-five consecutive periods will differ almost not at all from the average of the twenty-five preceding or following periods.

Of the reasonably bright short-period variables there are about 100 which pass from maximum to minimum and back to maximum within less than thirty days; nearly all of these, within ten days, and some of them within a few hours. In all these cases maxima and minima arrive on time, and the periods of most of them are known within a second. One cycle of change is almost exactly a duplicate of the preceding and following cycles.

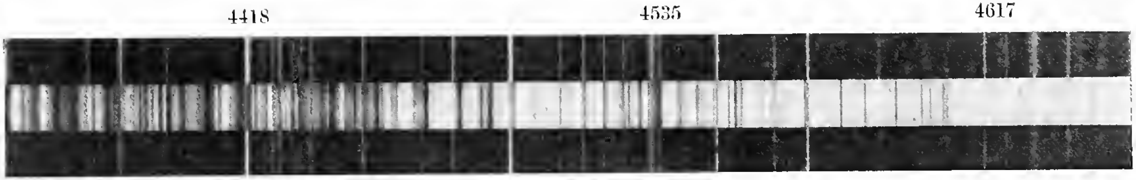
The brighter short-period variables just described have been classified as "Cepheids," following the example of δ *Cephei*, whose brightness increases more rapidly to maximum than it falls away to minimum; as "Geminids," after the prototype ζ *Geminorum*, in which the brightness increases less rapidly to maximum than it falls away to minimum; and as "Algols," with prototype β *Persei* (*Algol*), and as β Lyræ variables, in which the light variations are due to eclipses.

[2] *Astr. Jour.*, **8**, 140, 1888.

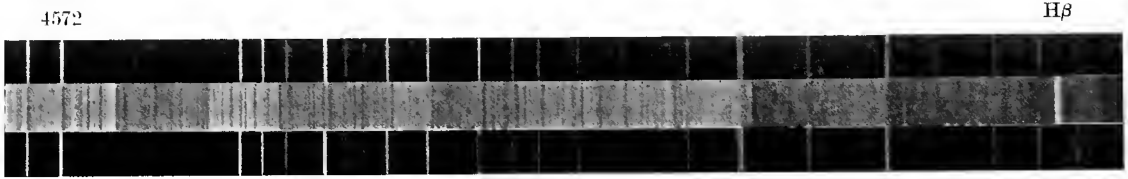
The so-called "cluster" variables are in a class by themselves, and indeed they form a most interesting class. It was found by Bailey that an unusual proportion of the stars in many of the well-known dense clusters are variable; nearly 2000 such variables having been catalogued by Bailey and the other Harvard observers. For example, in the great southern cluster *ω Centauri*, out of 3000 examined, Bailey found[3] 125 stars, approximately of the thirteenth and fourteenth magnitudes, which vary in brightness a full magnitude or more in short periods; 98 at least having periods less than twenty-four hours. In cluster *Messier* 3, Bailey found 132 variables out of 900 stars examined; in cluster *Messier* 5, 85 variables out of 900 stars examined. Of the latter, Bailey has determined the periods of 63. All are short, 39 of the 63 completing their cycle of change in from 10¾ hours to 15 hours. In the *Magellanic Clouds* are fully a thousand of these variables, all faint and probably all of short period. The brightness curves of all the cluster variables thus far investigated repeat themselves with apparent faithfulness and on time, and all exhibit the same general form of curve, descending slowly from maximum to minimum, remaining almost if not quite stationary at minimum for a few hours, and ascending from minimum to maximum with extreme rapidity.

There are other types of variable stars, such as the so-called "new stars," which appear with great suddenness at points where previously no star of catalogue brightness (that is, as bright as the ninth magnitude) was known to exist, and occasionally, according to photographic observations, where no star as bright as the twelfth magnitude was recorded. They reach maximum brilliancy in a few days or a few weeks, pulsate through a considerable range of brightness for a few additional weeks, and thereafter decline more or less continuously until they become comparatively faint stars. In some cases they assume approximate constancy as faint stars, and in others they seem to go beyond the reach of telescopic power, and later become visible again as faint objects. There is the case of *η Carinæ*,

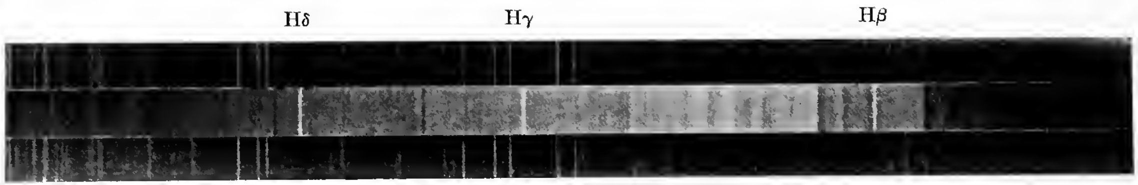
[3] *Ann. H. C. O.*, **38**, 1902.



Class Ma. *α Orionis*, 1907, November 29. Measured displacement, +10 km.



Class Md, variable. *o Ceti*, 1906, December 20. Measured displacement, +87 km.



Class Md. variable. *o Ceti*, 1906, December 17. With one-prism spectrograph located in the densest part of the *Key Hole Nebula,* whose brilliancy waxed and waned in a most irregular manner; almost equalling *Sirius* in 1843, and thence declining irregularly to the 7.6 visual magnitude in 1868, and remaining substantially at that level until the present time. *Nova Aurigæ* of 1892 passed beyond the power of the 36-inch telescope in April of that year. In August of the same year, and later, it was re-observed as of the tenth magnitude.

Not to review the various types further, there remain before us the purposes of all variable star observations, namely: to determine, first, why they vary; and secondly, the more important question, the relations of variable stars to the other units of the sidereal system. Here, as elsewhere, the discovery of an additional variable star is of little consequence, except as it provides wider opportunity for investigation, and thus throws light upon the fundamental problems of astronomy.

We must further limit ourselves, as far as possible, to the bearing of radial velocity observations upon the interpretation of variable star phenomena, calling in other contributing lines of research no more than is essential to the fuller comprehension of the general problem.

Radial velocity observations have contributed to our knowledge of luminosity variations for several types, but we shall do well to begin with the Algol type.

The admirable photometric studies of *Algol,* made before the spectroscopic era, led convincingly to the conclusion that here we are dealing with a binary system, whose members are revolving in a plane which passes close to the solar system, and that the variations observed are completely explained by the eclipses which must ensue. To an observer situated in quite a different part of the universe, *Algol* would not be a variable. Certain peculiarities in the light curve of *Algol,* in particular a gradual shortening of the period by eight seconds between 1790 and 1880, followed by a gradual lengthening of successive periods, were exhaustively discussed by Chandler[4] on the hypothesis that they are due to the influence of a third and invisible

[4] *Astr. Jour.,* 11, 113, 121, 1892; 22, 39, 1901.

body massive enough to swing the binary system around in a large orbit in 131 years, more or less; but they were attributed by Tisserand[5] to other causes, such as a change in the ellipticity of the orbit, a slow revolution of the line of apsides, etc. Upon these interesting details we have not the time to dwell.

The resourceful mind of Pickering[6] suggested that the spectroscope could advantageously be applied to test the general theory of *Algol* by measuring the velocities of approach and recession of the bright star in its supposed orbit. Promptly upon applying photography to the measurement of stellar velocities, Vogel[7] took up the study of this system. He found, in brief, that at $\frac{1}{4}$ period before each minimum of brilliancy, *Algol* was receding from the solar system with a speed of 39 km. per second; and, at $\frac{1}{4}$ period after each minimum the star was approaching the solar system at the rate of 47 km. per second. In all such studies, it is necessary to make assumptions, changing them as successive approximations to the apparent truth may demand. In this case, for example, we do not know that we are exactly in the plane of the orbit of the *Algol* system, but Vogel assumed that we are, so that there is a *central* transit of the darker body over the bright, and of the bright body over the darker, once in each period. He further assumed that the two bodies have the same densities, so that their masses are proportional to their volumes. From the velocities of the bright body, assuming the orbit to be a circle, it was possible for Vogel to estimate the distance between the centres of the bodies, and this came out a little more than 5,000,000 km. From the duration of the eclipse and from the observed brightness curve during eclipse, Vogel estimated the diameters of the two bodies: 1,700,000 km. as the diameter of the bright star, and 1,330,000 km. as the diameter of the faint star. The system, as a whole, is approaching the solar system with a speed of $(47 - 39)/2 = 4$ km. per second. The masses of the two bodies, on the assumptions made, are respectively $\frac{4}{9}$ and $\frac{2}{9}$ the solar mass, and therefore the density of

[5] *Comptes Rendus,* **120,** 125, 1895.

[6] *The Observatory,* 4, 116, 1891.

[7] *Astr. Nach.,* **123,** 289, 1890.

each is but ¼ that of our Sun. The darker body is in effect non-luminous, in comparison with the brighter one, for their combined brightness, when the two bodies are not in eclipse, does not change measurably when the darker body is in maximum eclipse. [Stebbins's extremely accurate observations with the selenium photometer, published after the date of this lecture, in *Ap. J.*, **32**, 199, 1910, have detected a diminution of brilliancy when the companion passes into eclipse.] We repeat, these results are merely approximations to the truth, for they rest upon many assumptions, some of which are undoubtedly close to the facts and others may contain a large percentage of error; for example, the densities of the two bodies are probably not equal.

Investigations of the orbit, more complete than Vogel's, were made in later years by Belopolsky[8] at Pulkowa, and by Schlesinger and Curtiss at Allegheny.[9] Their results for the dimensions of the primary orbit are in close agreement with each other, and in fair agreement with Vogel's. Curtiss considers[10] that the different results for the velocity of the centre of mass of the binary system, as deduced in different years by various investigators, are strong indications that the binary system is revolving around the centre of mass of itself and a third member of the system in a period of approximately 1.9 years.

We take time to describe the results of an extremely interesting investigation, just published,[11] on the Algol variable δ *Libræ*. This star is approximately fifth magnitude at maximum and sixth at minimum, the obscuration by eclipse covering twelve hours in each period of two days eight hours. Schlesinger, at the Allegheny Observatory, has observed the variable radial velocity of the bright component, and finds all the elements of the velocity curve in satisfactory harmony with the eclipse theory. The orbit is nearly circular, with eccentricity 0.05, and the system as a whole is moving toward the solar system with a speed of 45 km. per second. Schlesinger, by successive approxi-

[8] *Mit. Pulkowa,* **2**, 214, 1908; **3**, 72, 1908.

[9] *Publ. Allegheny Obs.*, **1**, 30, 1908.

[10] *Ap. J.*, **28**, 156, 1908.

[11] *Publ. Allegheny Obs.*, **1**, 123, 1909.

mations to the most probable conditions existing in this binary system, has been able to satisfy the photometric observations throughout the twelve-hour period of partial eclipse. He finds, as the most probable conditions and dimensions, that the bright star has a diameter of 4,500,000 km., and the dark companion, 4,000,000 km.; that the two bodies are therefore three times the diameter of our own Sun, and their volumes 33 and 24 times the solar volume; that their masses are respectively .87 and .63, and the combined mass 1.50 times the Sun's mass; further, assuming that the densities in the two components are equal, that the average density is but 1/40 the average density of our Sun; that the bright star revolves around the centre of mass at a mean distance of 2,500,000 km.; that the dark star describes an orbit at a mean distance of 3,400,000 km.; that the plane of the orbits is inclined 81.5° to the plane at right angles to the line of sight; that during one-quarter of the period of revolution, the bright star is more or less eclipsed by the dark one, about one-third of the bright disk remaining uncovered at minimum; and that the distance between the surfaces of the stars is remarkably small compared with their diameters, varying from 1,300,000 km. at periastron, up to 2,000,000 km. at the maximum separation.

It is all but certain that the two stars in a system of this kind must be rotating on their axes once in each revolution, each star constantly presenting the same face to the other star; for the tide-raising forces in such systems must be very great and tidal friction has probably compelled a complete accordance in the periods of rotation and revolution. A rotation once in two days eight hours means that one limb of the bright star is approaching us at the rate of 35 km. per second while the other is receding at the same rate. The relative velocity of the limbs nearest to and furthest from the centre of mass of the system therefore amounts to 70 km. per second, and no doubt an appreciable fraction of the broadening of the few lines in the spectrum is due to this great range of integrated velocities. A curious and perhaps important by-product of the investigation is the discovery that the minimum of the velocity curve, as determined from the blue and violet rays, precedes the photometric minimum

TABLE XXX

Visual Double Stars Whose Mass Ratios Have Been Determined

Star	R. A.	a	P	Colors	Vis. Mag.	m_1/m	Computer
η Cassiopeiæ	0h 43m.0	8″.5			4.0- 7.6	0.37	L. Struve
			233y.3	Y-P	4.6- 7.6	0.5	Lewis
			233 .3		3.6- 7.9	0.76	Boss
o_2 *Eridani* B & C	4 10 .7	6 .2	180 .	B-B	9.2-10.2	1.1	Lewis
α Canis Maj.	6 40 .7	7 .6	49 .4		—1.5- 9.	0.47	Auwers
				W-Y		0.5	See
			48 .8		—2. - 8.	0.39	Boss
*α Geminorum**	7 28 .2	[6±]	[350 .±]	Y-Y	2.7- 3.7	20.	Furner
α Canis Min.	7 34 .1	5±	40 .	Y-G		0 20	See
			39		0.2- 9.0	0.33	Boss
ζ Cancri, A & B	8 6 .5	0 .9	60 .	Y-Y	5.5- 6 2	1.	Seeliger
ε Hydræ, A & B†	8 41 .5	0 .2	15 .7		4.0- 5.5	0.9	Seeliger
		0 .3	15 .	Y-B	3.0- 6.0	6.	Lewis
ξ Ursæ Maj.	11 12 .9	2 .5	60 .	Y-Y	4. - 5.	1.5	Bowyer
			60		4.1- 5.1	1.0	Boss
γ Virginis	12 36 .6	3 .9	182 .	Y-Y	3.0- 3.0	1.0	Lewis
			194		3.6- 3.6	1.0	Boss
25 *Canum Ven.*	13 33 .0	1 .1	220	W-B	5.0- 8.5	2.	Furner
α Centauri‡	14 32 .8	17 .7	81			1.0	Elkin
				Y-Y	1. - 2.	1.05	Gill
						0.96	Roberts
			81 .2		0. - 1.5	0.85	Boss
ξ Bootis	14 46 .8	5 .3	137 .	Y-P	4.5- 6.5	1.25	Bowyer
			148 .5		4.8- 6 7	0.87	Boss
ξ Scorpii, A & B‖	15 58 .9	0 .7	44 .5	Y-	5.0- 5.2	[1.36]	Schorr
σ Coronæ Bor.	16 10 .9					1.1	Hadley
		3 .8	340 .	Y-B	5.0- 6.0	4.	Lewis
			370 .		5.7- 6.8	0.47	Boss
λ Ophiuchi	16 25 .9	1±	134	Y-B	4.0- 6.1	4.3	Lewis
ζ Herculis	16 37 .5	1 .4	34 .	Y-B	3.0- 6.0	1.0	Lewis
			34 .5		2.8- 6.5	0.43	Boss

						1.0	Comstock
		4 .5	87 .5			0.5	Lau
			88 .4		4.3- 5.8	0.82	Boss
85 *Pegasi*§	23 56 .9	0 .8	25 .	Y-B	6.0-10.0	1.6	Comstock
						4.	Furner
						3.	Lewis
			24 .5			4 5	Bowyer & Furner
						2.7	Bowyer & Furner
			25 .7		6.0-11.	1.0	Boss

This table has been brought up to date, April 1, 1910, by the inclusion of Boss's mass ratios published in his *Preliminary General Catalogue of 6188 Stars.*

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* *α Geminorum.* Would not another value of proper motion in Dec. change resulting value of m_1/m? Boss says, "The problem of computing the relative masses of the components from the meridian observations is manifestly indeterminate"; *Prelim. Gen. Cat.*, 266. Lewis deduces $m_1 = 0.01 \odot = 10$ Jupiter's mass; *Struve Doub. Stars*, 216.

† *ε Hydræ.* Seeliger's mass ratio rests on prior deduction that the effective light-centre of AB is at $\frac{1}{7}$ distance A − B from A; if this distance were ⅛ (A − B), the mass ratio would be 0.8.

‡ *α Centauri.* Stone considered $m = m_1$ as close approximation to truth. *Mon. Not.*, **36**, 264-5, 1876. See obtained $m + m_1 = 2.00 \odot$; *Evolution Stellar Systems*, **1**, 148, 1896. Boss's values of ratio: from fainter component, in R. A. 0.67 (wt. 8), in Dec. 0.78 (wt. 55); from brighter component, in R. A. 0.89 (wt. 14), in Dec. 0.91 (wt. 70); adopted 0.85.

‖ *ξ Scorpii.* Schorr's investigation is based upon $P = 105.2$ years, $e = 0.12$, whereas the correct values are approx. $P = 44.5$ years, $e = 0.77$.

§ 85 *Pegasi.* Boss says mass ratio is "extremely uncertain." "The computation is scarcely warranted by weight of meridian observations." "The meridian observations have a total weight . . . . much less than for any other binary treated"; *Prelim. Gen. Cat.*, 278.

as determined by the yellow and orange rays which make up the visual image of the star, by approximately an hour. This is just the opposite result from those obtained by Nordmann and Tikhoff for two other variable stars, for they found that the visual photometric minima preceded the photographic velocity minima. As explained in CHAPTER II, no explanation of the discrepancy in the times of arrival of the two minima is at present apparent.

There are about 72 Algol variables known up to the present time; and the accomplished understanding of the original *Algol* is the key which is rapidly unlocking the mysteries of many members of this class.

The results which have been fairly well established for *β Lyræ* are even more interesting than those for *Algol.* This is a star whose brightness varies through a period of 12.9 days, apparently without maintaining a stationary brilliancy even for a moment, except as this passes through a maximum or minimum value. Photographic spectra by Pickering[12] led him to the conclusion that here again were two revolving stars. The lines in two spectra shifted their relative positions in accordance with the requirements of orbital motions. Belopolsky[13] verified this conclusion and obtained quantitative values of the velocities in the system. Myers[14] correlated the photometric and spectrographic information and established some of the salient features of the system. The two stars are enormous in size, but of very low density. They are so close together as to be almost in contact. The two bodies are in form approximately prolate ellipsoids, with their longer dimensions in the line joining the two bodies. The immediate cause of the variable brightness is due, in large part, to the eclipsing of one body by the other, but there are probably other factors entering to a minor degree, such as tidal ebb and flow, which must exist, as the orbit seems to be slightly eccentric.

Belopolsky assigns a value of 0.07 to the eccentricity, and

12 *Astr. Nach.*, **128**, 40, 1891.
13 *Ap. J.*, **6**, 328, 1897.
14 *Ap. J.*, **7**, 1, 1898.

Myers, 0.11. Both bodies are luminous, but the larger one supplies only 40 per cent as much light as the smaller and more intensely luminous body. The long axis of the larger and fainter star is nearly 50,000,000 km.—thirty-five times our Sun's diameter; and the long diameter of the smaller body is 39,000,000 km. The mean distance of their centres is roughly 50,000,000 km. They appear to be, in fact, like two eggs, one slightly larger than the other, with the small ends almost in contact. The masses of the two bodies, according to Myers, assuming that we are in or nearly in the orbital plane, are respectively ten and twenty-one times the solar mass. Their average densities are comparable with that of our atmosphere at sea level. The spectrum of one of the bodies at least contains strong bright lines, and many of these bright lines have corresponding dark lines in the spectrum of the other body; clear evidence that they are not far removed from nebular conditions.

When the smaller and brighter body is eclipsed by the larger and fainter body, we have the light minimum; when both bodies are broadside to us, we have the maximum; when the smaller body is between us and the larger body, we have the secondary minimum; and so on. As the two bodies are nearly in contact, there is almost continuous eclipse. A short stretch of uniform brightness may well be masked by tidal surgings in the atmospheres.

Roberts obtained[15] a better representation of the photometric curve by assuming that the inclination of the orbit plane is 83°, so that we have only a partial eclipse. He assigns an eccentricity value of 0.02, a density equal to 0.0003 D_0, and a combined mass of the system equal to 64$\odot$.

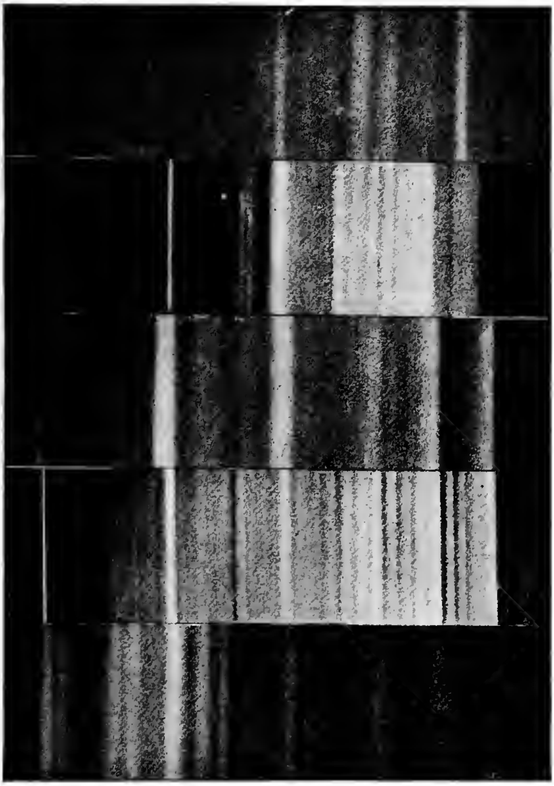
In view of the complexity reported to exist in the spectra, by several of the observers, it must be recognized that considerable uncertainty attaches to the published elements of the orbits.

[Note added May, 1912.—From a very extensive investigation of the spectrum of β *Lyræ*, Curtiss[16] concludes that the composite spectrum of the system consists of two dark-line spectra and a bright-line spectrum. He

[15] *Report B. A. A. S.*, 254-256, 1905.

[16] *Publ. Allegheny Obs.*, 2, 73, 1911.

has attempted to satisfy the observations on the basis of two very different hypothetical binary systems, each of which is consistent with the photometric curve. It seems clear from the radical difference of the two systems that the proper interpretation of the composite spectrum remains in serious doubt.]



R *Lyncis*
Md 1

S *Carinæ*
Md 4

R *Carinæ*
Md 6

o Ceti
Md 9

R *Hydræ*
Md 10

CLASS Md SPECTRA—Harvard College Observatory

Much light is thrown upon conditions existing in the Algol and β Lyræ systems by the data collected in TABLE XXXI, into which the photometric work of Roberts, Dunér, Myers, Hartwig, Wendell, and others enters. The results collected in this table

TABLE XXXI

Data Deduced from Photometric Curves

Star	R. A.	Vis. Mag.	Spectrum	Type of Variable	P	e	i	$br_1 : br$	$r_1 : r$	d_o
RR *Centauri*	14h 9m.9	7.4- 7.8	F	β Lyræ	0d.61	0.02	....	0.8	1.	$0.25D_o$
				Short P.	0 .303	....	....	....	....	......
U *Pegasi*	23 52 .9	9.3- 9.9		β Lyræ	0 .375	0.03	73°	0.7	1.	0.36
				β Lyræ	......	....	90	0.77	0.78	......
				Short P.	......	....	....	....	....	......
WZ *Cygni*	20 49 .3	9.9-10.8		β Lyræ	0 .584	....	....	....	....	......
X *Carinæ*	8 29 .8	7.9- 8.7	A	β Lyræ	1 .082	0.02	84	0.92	1.	0.05
					0 .541	....	....	....	....	......
				Algol	......	....	....	....	....	......
V *Puppis*	7 55 .3	4.1- 4.8	B1	β Lyræ ?	1 .454	0.02	74	0.75	1.	0.02
					......	....	....	....	....	......
				Algol?	......	....	....	....	....	......
u *Herculis*	17 13 .7	4.6- 5.4	B3	β Lyræ	2 .051	....	....	....	....	......
V *Serpentis*	18 11 .1	9.5-11.1	A	β Lyræ	3 .453	....	....	....	....	......
RS *Sagittarii*	18 11 .0	5.9- 6.3		Algol	2 .416	0.25	....	....	....	0.16 & 0.21
Y *Cygni*	20 48 .1	7.1- 7.9		Algol	3 .00	0.14	....	1.	1.	......
					1 .498	....	....	....	....	......
Z *Herculis*	17 53 .6	7.1- 7.9		Algol	3 .993	0.25	90	0.50	1.	......
					......	0.20	90	....	....	......
β *Lyræ*	18 46 .4	3.4- 4.1	B2p	β Lyræ	12 .908	0.02	....	....	....	Atmospheric
					......	0.02	83	0.4	1.	0.0003
					......	0.04	....	....	....	......
SX *Cassiop.*	0 5 .5	8. - 9±		β Lyræ	36 .572	....	....	....	....	......
V *Vulpeculæ*	20 32 .3	8.2- 9.8		β Lyræ	75 .	....	....	0.4	....	0.00002
				Cepheid?	37 .79	....	....	....	....	......
RV *Tauri*	4 41 .0	9.3-10.5		β Lyræ	79 .0	....	....	....	....	......
RT *Sculptoris*	0 32 .5	9. -10±		β Lyræ	?	....	....	....	....	......

Star	Remarks	Authority for Type of Variable, *P*, and other Data
RR *Centauri*	Two stars overlap by .02 $(a + a_1)$	Roberts, *Rep. B. A. A. S.*, 256, 1905, and *Mon. Not.*, **66**, 140, 1906
		V. J. S., **44**, 365, 1909
U *Pegasi*	Dist. between surfaces, .07 $(a + a_1)$	Roberts, *Rep. B. A. A. S.*, 256, 1905; *H. C. O. Circ.*, No. 23
	Dist. between surfaces, 0.	Myers, *Ap. J.*, **8**, 172, 1898
		V. J. S., **44**, 351, 1909
WZ *Cygni*		*V. J. S.*, **44**, 365, 1909
X *Carinæ*	Dist. between surfaces, 0.	Roberts, *Rep. B. A. A. S.*, 253-6, 1905
		Roberts, *An. H. C. O.*, **55** (I), 41, 1907
		V. J. S., **44**, 355, 1909
V *Puppis*	Dist. between surfaces, 0.1 $(a + a_1)$	Roberts, *Rep. B. A. A. S.*, 253-6, 1905
	$r = r_1 = 12{,}500{,}000$ km.; $a + a_1 =$	
	27,000,000 km.	
		V. J. S., **44**, 387, 1909
u *Herculis*		*V. J. S.*, **44**, 364, 1909
V *Serpentis*		Wendell, *An. H. C. O.*, **55** (1), 51, 1907
RS *Sagittarii*		Roberts, *Ap. J.*, **10**, 309-10, 1899
Y *Cygni*	$a = a_1 = 4r = 4r_1$	Dunér, *Ap. J.*, **11**, 190, 1900
		V. J. S., **44**, 370, 1909
Z *Herculis*	$a + a_1 = 6\ (r + r_1)$	Dunér, *Ap. J.*, **1**, 294, 1895
		Hartwig, *A. N.*, **152**, 309, 1900
β *Lyræ*		Myers, *Ap. J.*, **7**, 1, 1898
	$a + a_1 = 67{,}000{,}000$ km., increasing.	Roberts, *Rep. B. A. A. S.*, 254-6, 1905
	Stars in contact	
		v. Hepperger, *Sitz. Akad. Wien*, **108** (6), 938, 941, 946
SX *Cassiop.*		*V. J. S.*, **44**, 364, 1909
V *Vulpeculæ*	Nearly in contact, *e* rather large	Roberts, *Rep. B. A. A. S.*, 255, 1905
	Sears thinks it Cepheid variable	*An. H. C. O.*, **55** (I), 60
RV *Tauri*		*V. J. S.*, **44**, 326, 1909
RT *Sculptoris*		*V. J. S*, **44**, 353, 1909

do not depend in any way upon observed stellar velocities, and on this account no effort has been made to have the data complete. For convenience, all known β Lyræ stars are listed. There are many interesting facts brought out concerning the eccentricities (e) of orbits, relative brightness ($br_1 : br$), and relative size ($r_1 : r$) of the component stars, average density (d_0), distances between components ($a_1 + a$), etc.; though it should be said, I think, that studies of this kind, based exclusively upon photometric data without the illuminating assistance of radial velocity measurements for orbital proportions and scale values, must be considered as roughly approximate. The relatively high eccentricities, 0.14 to 0.25, assigned to three Algol stars, with periods under four days, are surprising, but probably not seriously in error; and the distances between the components, as determined by Dunér, in two of these systems, afford abundance of space for revolution without direct interference between the primaries and their secondaries. The diameters of the secondaries are large relatively to their luminosity, and their mean densities are very low. These facts, however, do not appear to be out of harmony with characteristics supplied perhaps more definitely by other means of binary investigation.

TABLE XXXI includes all known β Lyræ variables, to date 1910.

It is customary to treat the Algol and β Lyræ variables separately, but there is scarcely a doubt that they are closely related. The β Lyræ stars vary in luminosity without pause, except as they pass through maxima or minima, and they seem to repeat their photometric curves with absolute fidelity as to form and period, neglecting possible secular changes. Praiseworthy investigations by Roberts, Myers, and others appear to have established that the observed variations of brightness may be accounted for, in all cases studied, within the unavoidable errors of observation, by the mutual eclipses of two attenuated components nearly or quite in contact. This being true, the β Lyræ variables are Algols of an extreme type. It is not certain, however, that minor brightness variations, due to tidal surgings reasonably to be expected, may not be involved. The

72 Algol stars listed in *V. J. S. Astr. Gesell.*, **44**, 391-392, 1909, are assorted according to the lengths of their periods in the first two columns of TABLE XXXII. Similar data for the β Lyræ stars from the same publication, pages 322-365, are tabulated in columns three and four.

TABLE XXXII

VARIABLES OF ALGOL AND β LYRÆ TYPES

Algols		β Lyræ	
No. of Stars	Periods in Days	No. of Stars	Periods in Days
9	0 to 1	3 or 4	0 to 1
11	1 to 2	2 or 1	1 to 2
17	2 to 3	1	2 to 3
12	3 to 4	1	3 to 4
4	4 to 5	1	13
4	5 to 6	1	37
4	6 to 7	1?	38
1	7 to 8	1	79
1	8 to 9	1	?
2	9 to 10		
0	10	10 to 12	
0	11		
2	12		
1	13		
1	31		
1	32		
1	35		
1	262		
72			

Total		Spectra	
No. of Stars	*P* in Days		
56	0 to 4	Class B	8 stars
16	4 to 10	Class A	15 stars
12	10 to 272	Class F	3 stars
		Classes G-K-M	0 stars
		Unknown class	58 stars
84			84

DENSITIES (ALGOLS)

	No. of Stars	Average Density
Roberts	4	0.13 D_0
Roberts	6	0.06
Russell	17	< 0.20
Ristenpart	10	0.07

SPECTROGRAPHIC ORBITS

	e	*P*
β Persei	.03	2d.87
λ Tauri	v. small	3 .95
V *Puppis*	v. small	1 .45
δ Libræ	.05	2 .33
μ Herculis	.05	2 .05
β Lyræ	.05 ±	12 .91

The great preponderance of short periods is striking. Of the eighty-four stars in both classes, fifty-six have periods less than four days long; sixteen have periods between four and ten days; and only twelve have periods between ten and two hundred sixty-two days.

Most of the stars concerned are faint, and their classes of spectra have not been determined at Harvard College Observatory or elsewhere. Of the twenty-six spectra described, eight are of Class B, fifteen are of Class A, three are of Class F, and none are of Classes G-K-M-N. The preference for early spectral classes is marked, which fact has often been commented upon.

Roberts[17] has computed, upon reasonable assumptions, the densities in eight Algol systems; Russell,[18] in seventeen systems; and Ristenpart,[19] in ten systems. The mean of the densities assigned by the three investigators is 0.13 of the Sun's mean density.

Considering the preference of these stars for short periods, early spectral classes, and low densities, it is easy to reach conclusions as to why we have Algol and β Lyræ variables.

The two members in Algol and β Lyræ systems are, in general, so near each other and so great in diameter that the eclipse phenomena for any one system are observable throughout a wide volume of sidereal space, and the eclipses are of long duration in reference to the periods of revolution, so that even the unsystematic observations of the past have readily detected variable brightness. It is not necessary that observers be in or near the orbital plane of the binary eclipsing system, nor that

[17] *Ap. J.*, **10**, 314, 1899, and *Rep. B. A. A. S.*, p. 256, 1905.
[18] *Ap. J.*, **10**, 317, 1899.
[19] *A. N.*, **178**, 31, 1908.

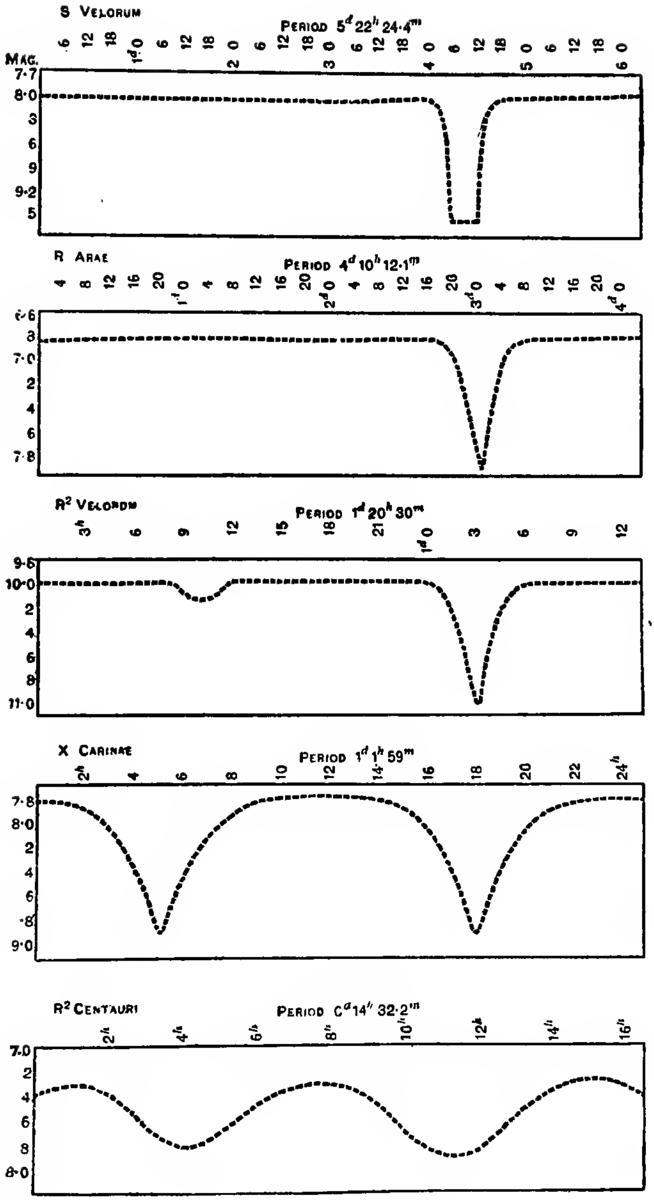

S VELORUM
PERIOD 5d 22h 24·4m
MAG.
R ARAE
PERIOD 4d 10h 12·1m
R2 VELORUM
PERIOD 1d 20h 30m
X CARINAE
PERIOD 1d 1h 59m
R2 CENTAURI
PERIOD Cd 14h 32·2m

Figure 12

highly organized programs of observation be followed, in order to detect a variation in luminosity. As the two members of a system grow farther and farther apart, corresponding to the longer periods of TABLE XXXV, the number of eclipsing pairs decreases very rapidly, as indicated in the same table. Increasing density and diminution in diameter no doubt keep step with increasing length of period. Algol and β Lyræ stars of Classes G-K-M-N have not been observed apparently, for the reason that the binary stars of these types have components, in general, far apart and of relatively small diameters. In order to observe them we should have to be in or nearly in the orbital planes; and even so the eclipses would be of short duration relatively to the period of revolution.

We need scarcely recall that all spectroscopic binary systems would be Algol variables for observers situated in the planes of their orbits.

In Figure 12 are curves for five other Algol stars, which appear to form a chain of connecting links between Algol and β Lyræ variables. We owe these intensely interesting curves[20] to the efficiency of Dr. Roberts and his small telescope, located on the South African frontier. All are of far-south stars. The visual magnitudes are indicated on the left margins of the diagrams, and the times along the horizontal lines.

In S *Velorum* we seem to have a large but very dim star and a small but vastly brighter and more massive star revolving once in a little under six days.[21] The smaller star is completely eclipsed for the 6½ hours represented by the straight-line minimum. This body, having the greater mass, is probably a fairly dense star, whereas the dim but large body is believed to be in a semi-nebulous state.

In R *Aræ*,[22] period 4½ days, we seem to have a body dark in effect eclipsing a bright body but very little larger than itself.

[20] Copied from Miss Clerke's *The System of the Stars*, Second Ed., 1905, p. 124, and taken by Miss Clerke from Roberts's paper read before the South African Asso. Adv. Sci., in 1903.

[21] Roberts, *Astr. Jour.*, 14, 113, 1894.

[22] Roberts, *Astr. Jour.*, 16, 202, 1896.

In the system of R_2 *Velorum*,[23] period forty-five hours, two bodies nearly of the same size suffer mutual eclipse, reducing the light greatly when the brighter is eclipsed and but little when the fainter one is covered.

X *Carinæ*[24] appears to be composed of two very large stars, nearly equally brilliant, and quite close together, but not touching. They suffer mutual eclipse. There are thus two minima and two maxima in the twenty-six hour period.

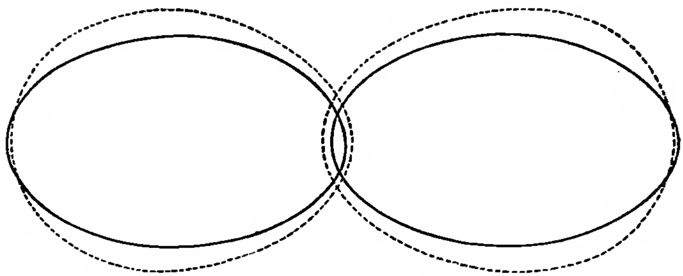


Figure 13

Most interesting of all, perhaps, is the curve for R_2 *Centauri*, as observed and interpreted by Roberts.[25] He has satisfied the observed photometric changes on the basis of two egg-shaped stars equal in size and luminosity, not entirely separate from each other, but their small ends coalescing, and the two revolving around their centre of mass in $14^h\ 32^m$. If the two members of the system are equal in all respects, the photometric variations should repeat themselves in one-half the period of revolution, $7^h\ 16^m$, and photometric observers usually assign this as the length of period. The full curves in Figure 13 represent a section of the system formed by a plane passing through the centres of the two components and at right angles to the orbit plane which Roberts has concluded would satisfy the photometric obser-

23 Roberts, *Astr. Jour.*, **22**, 32, 1901.

24 Roberts, *Astr. Jour.*, **16**, 202, 1896; *Ap. J.*, **10**, 309, 1899.

25 *Ap. J.*, **10**, 312, 1899.

vations. The dotted curves are Darwin's theoretical figures for two bodies just separating by fission, by virtue of accelerated angular rotation.

The foregoing interpretations of the five stellar systems are based purely upon photometric data. There are no spectrographic measurements of their velocities. The third system of the five is too faint for existing telescopes and spectrographs in the Southern Hemisphere, but the other four systems are bright enough for 1-prism dispersion. A knowledge of the radial velocities in these systems would tell us a great deal about the size of the orbits, sizes of the bodies, masses, and densities, and the efficiencies of the separate stars as light radiators. Roberts has estimated some of these factors purely from the photometric data for certain of the more interesting systems.[26] For example, he placed the density of V *Vulpeculæ*, period about seventy-five days, as fixed by the mutual eclipses of primary and secondary, at only 1/50000 the average density of the Sun. According to Roberts, this system consists of two great nebulous masses essentially in contact, in slow revolution about their mutual centre of mass.

These six systems seem to have many properties in common, and in harmony with the conclusions which were drawn in CHAPTER VII as to spectroscopic binary systems in general. All the orbits are essentially circular; the densities are extremely low; the spectra, as far as known, are of very early classes; and the primary and secondary bodies appear to be greatly elongated in the directions of the centres of mass of the systems. The need for radial velocity measurements in these cases, as in numberless others, is pressing, in order to confirm Roberts's deductions or to establish modified results.

Returning to the red variable stars of long period, we may say that measurements of spectrographic velocities have been substantially confined to one star, the brightest member of the class, o *Ceti*; and that the evidence these velocities afford is entirely negative. Observing with 3-prism[27] dispersion when the star

[26] *Report B. A. A. S.*, 1905, p. 249.
[27] *Ap. J.*, **9**, 31, 1899.

was comparatively bright and with 1-prism[28] dispersion when the star was of the seventh and eighth visual magnitudes, constant radial velocities were indicated, to the effect that the variable was receding from the solar system at a uniform rate of 63 km. per second. The bright hydrogen lines, which are intense near the times of maximum brightness, give a very different velocity, usually about 48 km. recession per second. The cause of the discrepancy is not clear, but considering all the facts, there is a suspicion that the bright lines may be due to an outburst of hydrogen gas at the time of the star's greatest activity. It is not impossible that in this star and others of the very red stars, there is a struggle to form a crust over the surface. As the crust forms and covers an increasing proportion of the solar surface, the light must diminish; and it is not unreasonable to suppose that the imprisoned gases and vapors immediately below the crust increase in temperature and pressure, so that when a certain point is reached the crust is broken through, and destroyed, and the brightness of the star thenceforth increases rapidly up to maximum. If this is really the case, it would not be surprising to have the lighter elements, such as hydrogen, projected outwardly from the star's surface with great speed. The discrepancy between the radial velocities afforded by the dark lines and the bright hydrogen lines would correspond to an effective velocity of ejection of 15 km. per second. If gases so ejected return later, under the influence of gravity, to the star, they no doubt do so in a cooled-off condition, but they seem not to make a record on the spectrographic plate, as dark absorption lines.

As our central body loses its heat and assumes a strong red color in the far distant future, it is not impossible that there will be a struggle to form a crust over the surface such as we may be witnessing in the red variable stars.

If the periods of these stars were of definite length, we should be interested to investigate the question of the effect of a very close approach of two bodies, in each system, revolving in very eccentric orbits. Such an approach, through disruptive tidal attractions, could well produce enormous disturbances in the

[28] *Lick Obs. Bull.*, **2**, 78, 1903.

TABLE XXXIII

Cepheid-Geminid Variables

H. R.	Star	Class	Period	e	$a \sin i$ km.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	V_0 km.	V' km.	K km.	Min_v-Max_l	Max_v-Min_l
2332	RT *Aurigæ*	G	$3^d.73$	0.37	856,500	0.0018	+21.5	+11.7	17.3	$+0^d.2$	$+0^d.4$
7518	SU *Cygni*	F5	3 .84	0.21±	1,350,000±	0.0058±	−33.4	−14.2	25.±	+0 .2±	0 .0±
7988	T *Vulpec.*	F	4 .44	0.43	969,180	0.0018	− 1.3	+15.9	17.6	+0 .3	+0 .1
8571	δ *Cephei*	G	5 .37	0.36	1,371,000	0.0037	−18.	− 5.	20.5		
6863	Y *Sagittar.*	G	5 .77	0 16	1,485,000	0.0040	+ 4.0	+17.0	19.0	+0 .78	+0 .1±
6616	X *Sagittar.*	F8	7 .01	0.40	1,334,000	0.0016	−15.±	− 5.±	16.±	−0 .2±	−1 .3±
7402	U *Aquilæ*	G	7 .02							−0 .5	
7570	η *Aquilæ*	G	7 .18	0 49	1,773,000	0.0043	−14.2	+ 4.2	20 6	0 .0±	−0 .6
6742	W *Sagittar.*	F5	7 .60	0.32	1,930,000	0.0050	−28.6	−18.6	19.5	−0 .1	+0 .2
7609	S *Sagittæ*	G	8 .38	0.35±	2,000,000±	0.0049±	−12.5	+ 5.8	19.±	−0 .2±	+0 .4±
7107	κ *Pavonis*	F5	9 .09								
2650	ζ *Gemin.*	G	10 .15	0.22	1,797,800	0.0023	+ 6.8	− 5.7	13.2	−0 .2	−0 .3
7932	X *Cygni*	F5p	16 .39								
6661	Y *Ophiuchi*	G	17 .12	0.10	1,999,000	0.0011	− 5.0	+10.8	8.5	−1 .3	+0 .5±
2310	T *Monocer.*	G	27 .01								
3884	*l Carinæ*	G	35 .53							−2½±	

atmospheres of the two bodies, though we should not expect so great a range of brightness variation to go with so short a period. However, the irregularities in the periods of all these stars substantially deny the existence of companion stars of appreciable masses; and, as explained, the only radial velocity evidence at hand is in harmony with this view. Unfortunately, we cannot assign with much confidence the reasons for variability of the red stars.

Fully as interesting as the Algol stars are the Cepheids and Geminids. These we shall treat briefly together, for the only apparent distinction is that in one case the brightness increases to a maximum more rapidly than it falls off to minimum, whereas the reverse is the case for the other sub-class. Fifty-three of these stars are known, and all have perfectly definite periods. Eleven of them have been extensively investigated on the basis of their spectrographic velocities, ten of these studies having been made with the Mills spectrographs of the Lick Observatory. The principal known facts are contained in TABLE XXXIII. In every case investigated, the radial velocity is variable, indicating clearly, I think, that we are again dealing with binary systems. The period of velocity variation, studied for sixteen stars, is in every case equal in length to the photometric period. We cannot doubt, therefore, that the invisible companion star is in some way, perhaps only indirectly, responsible for the light variations. In the column V_0 of the table are given the velocities of the centres of mass of the systems. In the next column are these velocities after correcting for the solar motion. It is interesting to note in passing that while these systems have their individual velocities of approach and recession, their average is almost exactly zero, being in fact $+1$ km. The range of orbital velocities in these systems is very much less than in the Algol systems. K is the single amplitude of the velocity curve. The corresponding amplitudes for Algols will be several times as great. The orbits have considerable eccentricities, as indicated by the column e. Some of the most interesting and apparently important conclusions for any variable stars are those indicated by the figures in the last two columns. The first of these gives the interval for each system between the instants of minimum velocity and maximum light. The second, the time interval between maximum velocity and minimum light. These intervals are small in comparison with the lengths of the periods. Interpreted, we have the astonishing result that every star investigated has its maximum brilliancy at or very near the time of greatest velocity of approach toward the solar system, and the minimum brilliancy at or very near the time when the bright star in the system has its maximum velocity of recession from the solar system. These are keys which give promise of unlocking many secrets of the Cepheids and Geminids. What can be more remarkable than that variable stars of this class should be at their brightest when they are moving rapidly toward the observer and at their faintest when they are moving rapidly away from him? Let us hold this fact in mind for consideration a little later. Other features should now be made familiar. The quantity of light received from one of these systems varies continuously throughout the period, without stopping, except as the brightness passes through a maximum or minimum. The photometric curves are usually not simple or smooth curves but include irregularities, more or less prominent.

The amount of variation from maximum to minimum and *vice versa* is usually about one magnitude.

These stars, so far as investigated, are of spectral classes approximately solar, whereas the Algol and β Lyræ stars, apparently without exception, are of the newer and simpler classes.

No case has been found in which the spectrum of more than one of the two stars has recorded itself upon the spectrograms.

The values of $a \sin i$ are all less than two million km., which is evidence that the primaries revolve in orbits whose dimensions may be described as minute. The values of $m_1^3 \sin^3 i/(m + m_1)^2$ are also abnormally small, the largest being about 0.005. We have here a tolerably clear indication that the masses of the companion stars are very small in proportion to the masses of their primaries, for it is not probable that smallness of the angle of inclination, i, is a peculiarity of these systems.

Here, as always, the purpose of investigations is to determine why the stars vary. We may dismiss most promptly any eclipse theory accounting for the change in brightness, for the relative positions of the two bodies as determined by the spectrograph are such that eclipses can have no connection with the time of minimum brightness. Nevertheless, the light variation must be connected in some way with orbital revolution in the system. Several hypotheses have been advanced to explain the observed phenomena; but on the whole it must be said that these systems are still involved in mystery. A very interesting hypothesis, one which may have great merit, is that of a resisting medium in which the binary system is supposed to be enveloped, so that the impacts of the advancing side of the bright star with the elements of the resisting medium generate heat, and in this manner build up the maximum of brightness. We can readily see that as a star revolves in its orbit, if it moves at different times with different speeds, or through portions of the medium having different densities, there could result corresponding differences in the rate of light generation and emission; or, looking at the question from a different point of view, if the bright star presents always the same face to its companion, as it probably does, the preceding or forward face of the bright body will by the impacts of the particles be kept constantly hotter than the following side, so that the observed fact of maximum brilliancy at the time of maximum velocity of approach would be explained. The related fact of minimum brilliancy when the following side of the star is turned toward us and receding from us would likewise be explained.

The objections to this hypothesis present some interesting considerations. If the companion's mass is less than the bright star's mass, as it probably or undoubtedly is, its orbital velocity must be greater than that of the bright star. Why is it not visible also, not separately, but in the combined light curve, as a result of correspondingly more severe impacts? We certainly should expect this result. Again, it must occur that the work done by the stars in giving momentum to resisting particles will tend to shorten the period of revolution. On reasonable

Lick Observatory (from the East)

assumptions as to the elements of such collisions, the corresponding diminution in the length of period has been computed by Duncan.[29] He finds for one of the stars, of period six days, that the observed excess of radiation at maximum over that at minimum would probably demand a diminution of the period by as much as 0.04 second in each revolution. This, of course, may be excessive, as it rests upon some uncertain assumptions, but such a shortening of the period could not long escape detection. A diminution of this kind has been noted by Chandler for one and only one star in this class of variables, and the observed rate is very small indeed.

Another apparently important fact noted by several observers of these stars is that the spectrum at maximum is relatively strong in the violet and at minimum relatively weak in the violet. Further, certain of the lines change their apparent wave lengths, with reference to the wave lengths of neighboring lines, in the same way that the wave lengths change as one passes from solar type stars toward the red stars and *vice versa*. We can scarcely doubt that variable *absorption* of the bright star's light is here involved. Duncan has suggested that the rapid motion of the bright star through a resisting medium may cause a partial brushing aside of some of the absorbing strata of the star's atmosphere on the advancing side, that is, the side presented to us at maximum brightness; and a building up of the atmosphere on the following side, that is, the side presented to us at the time of maximum recession and minimum light. It is conceivable that the changes plainly visible in the spectrum, corresponding to these two positions of the revolving body, may have a partial explanation in this theory.

The further study of the stars in this class is an exceedingly promising one, but unfortunately the forty-odd uninvestigated stars are for the most part faint; and, as their spectra are approximately of the solar type, implying comparative weakness in the blue and violet regions, the dispersion employed must be regrettably low. It seems tolerably certain that the problem of interpreting these variables is not exclusively a geometrical one,

29 *Lick Obs. Bull.*, 5, 90, 1908, and 6, 154, 1911.

and it may be that radial velocity methods are not the most efficient, yet we cannot doubt that the observed phenomena have their origin in the influence of the invisible companion over the brighter member of the system.

TABLE XXXIV

CEPHEID-GEMINID VARIABLES

No. of Stars	Period Days	Spectrum	No. of Stars	P_0
7	0– 1	O–B	0	
0	1– 2	A	1	0.6 days
1	2– 3	F	14	8.2 days
5	3– 4	G–K5	26	11.4 days
5	4– 5	Unknown	12	7.9 days
5	5– 6		—	
5	6– 7		53	
8	7– 8			
1	8– 9			
3	9–10			
1	10			
2	12			
1	14			
1	16			
2	17			
1	20			
1	27			
1	35			
1	38			
1	39			
1	41	11 orbits computed		
—		$P_{11}=7.3$ days		
53		$e_{11}=0.31$		

The Cepheid-Geminid variables were not included in the tables of spectroscopic binaries in CHAPTER VII, for the reason that they appear to stand apart from other binary and variable-star systems. Fifty-three of the Cepheid-Geminid variables were known up to the beginning of the present year. These are classified in TABLE XXXIV, in the order of their lengths of period. To seven are assigned periods less than one day. It is probable that some of these extremely short-period variables may later be assigned to the Cluster variables, or it is not impossible that the Cluster variables are, in fact, entitled to classification with the Cepheid-Geminid variables. None of the many hundred Cluster variables discovered by Harvard College Observatory have yet been submitted to radial velocity measurements, owing to their faintness. There are twenty-eight Cepheid-Geminid variables with periods between three and eight days. The small

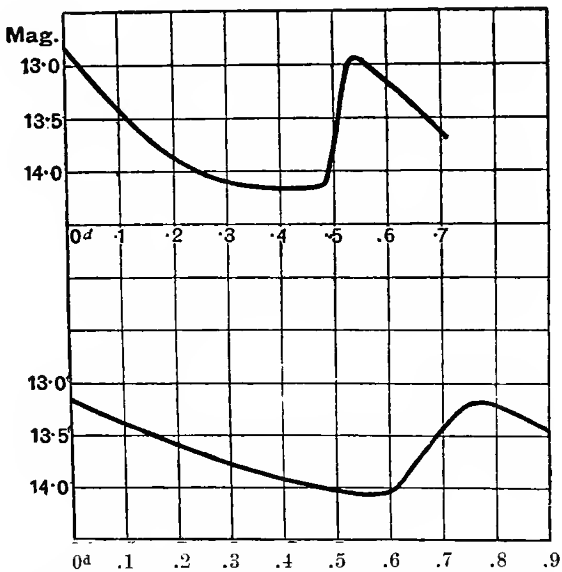


Figure 14.—Typical Light Curves of Cluster Variables

number with periods between one and three days may or may not be significant of different conditions existing in the systems with periods on opposite sides of this minimum. Only eighteen are known to have periods longer than eight days. The numbers and periods corresponding to the different spectral classes are quoted in the table. There are none of Classes O, B, M or N, so far as known. There is only one of Class A, and there are forty of Classes F to K5. The length of period seems from this tabulation to increase slowly with advancing spectral class, but the relationship cannot be regarded as established, because of the small number of stars involved. There is no evidence that the eccentricities are functions of the length of period, as in all other classes of stars discussed in these lectures.

We take a moment for brief reference to the remaining important class of variables, namely, those in the star clusters. Unfortunately the 2000 stars now known to belong to this class are nearly all faint, usually of the twelfth, thirteenth, fourteenth, and fifteenth magnitudes, and are, therefore, quite beyond our present spectrographic power of attack, except in possibly two or three cases. The photometric curves for two stars in this class are given in Figure 14.[30] The curves of all investigated members of this class have the same general features. They resemble the curves for the Cepheid variables in that the decrease from maximum to minimum brightness is relatively slow and the increase from minimum to maximum is relatively rapid. The photometric curves for many of the Cluster variables indicate that the light is constant for a time at minimum, but on account of the faintness of these stars and the consequent difficulties in the way of accurate measurements, it should probably be said that the question of constancy at minimum remains in doubt. Inasmuch as the Cluster variables repeat their photometric cycles exactly on time, I think we cannot seriously doubt that in all these stars we are observing binary systems whose members interact upon each other in such a direct or indirect way as to vary the output of radiations.

Roberts has said that in the study of Algol and β Lyræ systems we are searching for facts concerning stars in their early youth. The investigations of spectroscopic binary systems of all spectral classes confirm and emphasize this view. Our knowledge of these wonderful systems is covered by a span of but twenty years. That it exceeds in fact our knowledge of visual binary systems is due not so much to the energy and admirable instrumental equipment and methods of spectrographic observers, but chiefly to the condition that the periods of revolu-

[30] Taken from *The System of the Stars*, Second Ed., p. 119.

tion are short. There is danger that the fruitfulness of the newer field of labor will withhold attention from the equally important older field of visual binaries. It is of the utmost desirability that available instruments be applied to the accurate observation of the positions of the primary members in double-star systems, with reference to the surrounding stars, by photography, micrometer, heliometer, or other applicable means. It is equally desirable that spectroscopic observers measure the radial velocities in as many visual systems as practicable; the velocities of both members where possible, and of the primaries in other cases. Appreciable variations in the positions and speeds, in many cases, may not occur for centuries; but the securing of these observations, for the benefit of posterity, is a pressing duty.

The wide range of problems to which stellar radial velocity results may be applied is admirably illustrated by the fact that such data may furnish our most valuable method of determining the solar parallax. In the year 1892, I published[31] the following statement concerning this promising method: "By assuming the Earth's mean distance from the Sun to be 92,500,000 miles, which corresponds to a solar parallax of 8″.838, it is probable that the resulting orbital velocities [of the Earth] will not be in error by more than 0.1 mile per second. There is reason to hope that the probable errors of spectroscopic observations will soon reach this low limit, in which case the problem will be reversed and the spectroscope will be used to *measure* the Earth's orbital motion and thus to determine the solar parallax."

The principles involved are of extreme simplicity. If we assume, on the basis of observations continued through several years, that the radial velocity of a star situated in or near the ecliptic is constant with reference to the solar system, and we observe the radial velocity of this star in the evening when its longitude is about 90° greater than the Sun's, and again in the morning when its longitude is about 270° greater than the Sun's, the observed velocities of the evening and morning series should

[31] *Astr. and Astroph.*, 11, 320, 1892.

differ by approximately twice the orbital velocity of the Earth; that is, by about 60 km. per second. Assuming that the velocity of light, the diameter of the Earth, and the times of observation are known, and that the orbital velocity of the Earth is unknown: since the form and position of the Earth's orbit are known to a satisfactory degree of accuracy, the solution of a simple equation will determine that value of the Earth's orbital velocity which will harmonize the evening and morning observed stellar velocities. Inasmuch as a variation in the Earth's assigned velocity implies a corresponding variation in the linear dimensions of the Earth's orbit, the resulting alteration of the solar parallax value follows at once. Plummer has called attention[32] to the fact that the constant of aberration is involved with the solar parallax in this form of solution. It is true that all radial velocity determinations by spectrographic methods depend upon an assumed value of the velocity of light; but as this is thought to be known within narrow limits of error, the uncertainty in a value of the solar parallax determined spectrographically should be correspondingly slight.

The first systematic effort to determine the value of the solar parallax by this method was made by Küstner at Bonn in 1904-1905. From eighteen spectrograms of *Arcturus* he deduced the value[33]

$$\pi = 8''.844 \pm 0''.017.$$

It is a fair conclusion, judging from the minuteness of the probable error here assigned from only eighteen spectrograms, that a really extensive radial velocity program for determining the solar parallax would yield a result comparable with and perhaps superior in value to the best determinations hitherto based upon any of the conventional methods, not excepting the value of the parallax derived from the extremely numerous observations of *Eros*. We might even question whether astronomers will be justified in carrying through another *Eros* parallax program, some fifteen years from now, when that asteroid

[32] *The Observatory*, **31**, 239, 1908.
[33] *Astr. Nach.*, **169**, 262, 1905.

shall again be in a favorable position; except that it is always desirable to arrive at conclusions by two or more independent methods, if they are fortunately available.

[Note added April 1, 1910.—After the delivery of this lecture, there appeared (March, 1910) *A Spectrographic Determination of the Constant of Aberration and of the Solar Parallax*,[34] made by the Royal Observatory, Cape of Good Hope, as based upon 302 measured radial velocities of seven prominent stars situated not far from the ecliptic. The value assigned to the solar parallax by this investigation is

$$\pi = 8''.800 \pm 0''.006.$$

This is in remarkable accord with the results of greatest weight deduced from observations of *Eros*, viz.:

$$\pi = 8''.807 \pm 0''.003;$$

and the small amount of labor involved in reaching the spectrographic result is in most striking contrast with the Herculean labor devoted to the determination based upon observations of *Eros*.]

[34] *Ann. Cape Obs.*, **10** (III), 56C, 1909.

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